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A Procedure for Selection of Single Sampling Plans by Variables Based on Inverse Gaussian Distribution

Statistics

KEYWORDS: Inverse Gaussian distribution, Normal distribution, Operating characteristic function, Quality level, Single sampling plan.

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ABSTRACT

Sampling inspection by variables is one of the major classifications of acceptance sampling and consists of procedures for taking decisions about the disposition of a lot of individual units based on sample measurements of units on a quality characteristic under study. Inspection procedures of this kind are defined in general under the condition that the quality characteristic, which is considered as a random variable, is measurable on a continuous scale and the functional form of the probability distribution of the random variable must be known. Several works in variable sampling plan have been done by many researchers under the assumption that the quality characteristics is modeled by a normal distribution and are found in the literature of acceptance sampling. However, the assumption of normality may not be often realized in practice and hence, it is quite inevitable to make an investigation about the properties of sampling inspection plans when the quality variables are modeled by non-normal distributions. In this paper a single sampling plan by variables is devised and evaluated under the assumption that the quality characteristic follows a Inverse Gaussian distribution. A procedure is developed for determining the parameters of the proposed plan when the requirements in terms of quality levels so as to provide protection to the producer and the consumer are specified.

INTRODUCTION

Sampling inspection consists of procedure for taking decisions on one or more lots of finished products which have been submitted for inspection. The decision of either acceptance or rejection of the lots is usually taken by adopting suitable sampling inspection procedures, called sampling plans. Sampling plans are generally categorized into two types, namely, lot-by-lot sampling by attributes and lot-by-lot sampling by variables. In lot-by-lot inspection by attributes, one or more samples of items are drawn from a given lot of manufactured items; each item in the sample(s) is classified as conforming or nonconforming; and the decision of acceptance or rejection of the lot is made based on a specific rule. In lot-by-lot inspection by variables, one or more samples of items are drawn from a given lot; the measurement of a quality characteristic in each sampled item is recorded; and the decision of acceptance or rejection of the lot is made as a function of such measurements. The theory of inspection by variables is applicable when the quality characteristic of sampled items is measurable on a continuous scale and the functional form of the probability distribution is assumed to be known. A variables sampling is advantageous in the sense that it generates more information from each item inspected, requires small sample and provides same protection when compared to attributes sampling. (See, Bowker and Goode [1], and Montgomery [2]).

On the basis of the implicit assumption that the quality characteristic is distributed according to normal with mean μ and standard deviation σ, the concept of variables sampling inspection has been studied by many researchers. Some of the early works on variables sampling inspection are seen in Lieberman and Resnikoff [3], Schilling [4], Owen [5, 6], and Hamaker [7]. Studies relating to sampling plans when the assumption of normality of the quality characteristic fails or the functional form of the underlying distribution deviates from normal or the form of the distribution is not known are also found in the literature of acceptance sampling. Duncan [8], Srivastava [9], Das and Mitra [10], Owen [11], Takagi [12], Guenther [13, 14], Zimmer and Burr [15], Aminzadeh [16], Suresh and Ramanathan [17], Vijayaraghavan and Geetha [18, 19, 20], and Geetha and Vijayaraghavan [21- 25] are few references which deal with variables inspection using non-normal distributions.

When the quality characteristic, X, follows normal distribution with mean μ and standard deviation σ, the problem of designing single sampling plans by variables is addressed with ease. In the industrial situations, quite often, the assumption of normality may not be valid or the quality characteristic may be distributed according to non-normal distributions. In such cases, the selection of variable sampling plans becomes complicated. However, the literature of acceptance sampling traces several works which deal with designing of variables plans when the quality characteristic follows a probability distribution other than normal. Owen [11] presented a detailed survey on various works related to variable sampling plans with emphasis on non-normality.

In this paper a study on single sampling plans by variables is formulated under the assumption that the quality characteristic is assumed to have a Inverse Gaussian distribution. A procedure for determining the parameters of the proposed plan for specified requirements in terms of producer's and consumer's protection is also developed.

SINGLE SAMPLING INSPECTION PLANS BY VARIABLES

A single sampling inspection plan by variables is defined under the following assumptions:

(a) The quality characteristic, denoted by, X is measurable on a continuous scale and has a known form of probability distribution, represented by $F_X(x, \mu, \sigma)$; which is the distribution function of X with mean μ mand variance .σ^2.

(b) Each individual unit in a submitted lot has a one-sided specification, say, lower specification, L or upper specification, U. If, for a unit, $X > U$(or $X < L$), the unit is classified as a non- conforming unit.

The operating procedure of a variable sampling plan is as follows:
Step 1: Draw a random sample of n units from a lot and observe the measurements, $x_1, x_2, ..., x_n$ of the quality characteristic, X.

Step 2: Whensis known, accept the lot if $\bar{x} + k\sigma \le U$ (or $\bar{x} + k\sigma \ge L$); otherwise, reject the lot, where $\bar{x}$ is the sample mean. When σ is unknown, accept the lot, If $\bar{x} + ks \le U$ (or $\bar{x} + ks \ge L$), where

$s^2 = \frac{1}{n-1}\sum_{i:1}^{n}(x_i - \bar{x})^2$ is an unbiased estimate of σ^2.

Thus, a single sampling plan by variables is designated by two parameters, namely, the sample size, n, and the acceptability constant, k. When these parameters are known, the plan could be implemented. The explicit expressions for n and k can be derived by specifying two points on the operating characteristic curve of the plan, namely, $(p_1, 1-\alpha)$, and (p_2, β) where p_1 and p_2 are termed as acceptable quality level (AQL) and the limiting quality level (LQL), associated with the producer's risk, α and the consumer's risk, β, respectively. A sampling plan by variables is termed as a known σ or unknown σ plan according as σ is known or unknown.

OPERATING CHARACTERISTIC FUNCTION

An important measure of performance of a variables sampling plan is its operating characteristic function, which is a function of the proportion, p, of non-conforming units, called incoming lot quality, and provides the probability, $P_a(p)$, of acceptance of a lot. The plot of $P_a(p)$ against p results in a curve, called operating characteristic (OC) curve. For a given upper specification limit, U, when σ is known, p and $P_a(p)$ are defined by

$$p = P(X > U|\mu) \tag{1}$$

$$P_a(p) = P(\bar{x} + k\sigma \le U|\mu) \tag{2}$$

AQL and *LQL*, using (1), are defined by

$$AQL = p_1 = P(X > U|\mu_1) \tag{3}$$

and $$LQL = p_2 = P(X > U|\mu_2), \tag{4}$$

where μ_1 and μ_2 are the means of the underlying distribution which results in *AQL* and *LQL*, respectively.

Assume that X be a random variable representing a specific quality characteristic of manufactured items and be distributed according to a Inverse Gaussian distribution with two parameters, namely, the mean, θ, and the shape parameter, λ. The probability density function of the Inverse Gaussian distribution is given by

$$f(x;\theta,\lambda) = \left(\frac{\lambda}{2\pi x^3}\right)^{1/2} \exp\left[\frac{-\lambda(x-\theta)^2}{2\theta^2 x}\right], \quad \theta > 0, \lambda > 0, x > 0 \tag{5}$$

Therefore, the distribution function of the Inverse Gaussian variable whose density function is given as (5) is expressed by

$$F(x;\lambda,\theta) = \Phi\left(\sqrt{\frac{\lambda}{x}}\left(\frac{x}{\theta}-1\right)\right) + \exp\left(\frac{2\lambda}{\theta}\right)\Phi\left(-\sqrt{\frac{\lambda}{\theta}}\left(\frac{x}{\theta}+1\right)\right), \quad \theta > 0, \lambda > 0. \tag{6}$$

where Φ is the cumulative distribution function of the standard normal variable.

The mean and the variance of the underlying distribution are given by $\mu=\theta$ and $\sigma^2 = \frac{\theta^3}{\lambda}$. Further, the measures of skewness and kurtosis, which are the measures for identifying the asymmetric nature of the distribution, are expressed as $\alpha_3 = 3\left(\frac{\theta}{\lambda}\right)^{1/2}$ and $\alpha_4 = 3 + \frac{15\theta}{\lambda}$, respectively. From (1), (3) and (4), the lot quality levels, p, *AQL* and *LQL* using Inverse Gaussian distribution are, respectively, defined by

$$p = P(T > K_p^*),$$

$$AQL = p_1 = P(T > K_{p_1}^*) \tag{7}$$

and $$LQL = p_2 = P(T > K_{p_2}^*) \tag{8}$$

where $T = \frac{X-\mu}{\sigma}$, $K_p^* = \frac{U-\mu}{\sigma}$, $K_{p_1}^* = \frac{U-\mu_1}{\sigma}$ and $K_{p_2}^* = \frac{U-\mu_2}{\sigma}$.

The producer's risk, α, and the consumer's risk, β, corresponding to *AQL* and *LQL* are, respectively, defined from (2) as

$$\alpha = P(\bar{x} + k\sigma > U|\mu = \mu_1) \tag{9}$$

and $$1-\beta = P(\bar{x} + k\sigma > U|\mu = \mu_2) \tag{10}$$

When σ is unknown, the estimate s is used in the decision criterion, and hence in the evaluation of α and β.

DESIGNING SINGLE SAMPLING PLANS BY VARIABLES

In the industrial practice the unknown standard deviation variables plans are more realistic than the known standard deviation variables plans. If the distribution is non-normal, then the designing of unknown σ plans is much complicated. Vijayaraghavan and Geetha [18-27] discussed the procedures of designing known and unknown standard deviation sampling plans under the assumption of gamma , Laplace, Rayleigh, Logistic, Pareto, Loglogistic, Lognormal and Weibull distributions for the quality characteristic giving protection to the producer and consumer when the measures of skewness and /or kurtosis are specified In the case of unknown sigma plan, plan, the determination of n and k is usually based on the sampling distribution of . Suppose that $\bar{x} + ks$ or $\bar{x} - ks$. Suppose that $F_X(x)$ is a Inverse gaussian distribution which is a asymmetrical type of distribution is a asymmetrical type of distribution. It is known that if $\bar{x}$ is normally distributed, the distribution of s is nearly normal. It is known, further, that both $\bar{x}$ and s are independent, and hence, $\bar{x} + ks$ and $\bar{x} - ks$, being a linear combination of two independent normal variables, are $N\left(\mu + k\sigma, \sigma\sqrt{\frac{1}{n} + \frac{k^2}{2n}}\right)$ and $N\left(\mu - k\sigma, \sigma\sqrt{\frac{1}{n} + \frac{k^2}{2n}}\right)$ variables, respectively. For specified p_1 and p_2,

$$\mu_1 = U - K_{p_1}^*\sigma \tag{12}$$

and $$\mu_2 = U - K_{p_2}^*\sigma \tag{13}$$

The operating characteristic function $P_a(p)$, for an unknown standard deviation plan is expressed as

$$P_a(p) = P_r[\bar{x} + k_U s \le U \mid p] = P_r(Y \le U \mid p)$$

Assume that p_1 and p_2 are specified with the risks, α, of rejecting the lot and β, of accepting the lot. Then, corresponding to α and β, the normal deviates K_a and K_b are, respectively, defined by

$$K_\alpha = \frac{U - (\mu_1 + k\sigma)}{\sigma\sqrt{\frac{1}{n} + \frac{k^2}{2n}}} \tag{14}$$

and $$-K_\beta = \frac{U - (\mu_2 + k\sigma)}{\sigma\sqrt{\frac{1}{n} + \frac{k^2}{2n}}} \tag{15}$$

On substituting the expressions (12) and (13) for μ_1 and μ_2, equations (14) and (15) would result in the following:

$$K_{p_1}^* = k + K_\alpha\sqrt{\frac{2+k^2}{2n}} \tag{16}$$

$$K_{p_2}^* = k - K_\beta\sqrt{\frac{1}{n} + \frac{k^2}{2n}} \tag{17}$$

The mathematical expressions for nandk of an unknown standard deviation plan for specified requirements in terms of the points $(p_1, 1-\alpha)$ and (p_2, β) on the OC curve such that $P_a(p_1) = 1-\alpha$ and $P_a(p_2) = \beta$, when U is specified, are derived from (16) and (17) as

$$n_s = \left(1 + \frac{k^2}{2}\right)\left(\frac{K_\alpha + K_\beta}{K_{p_1}^* - K_{p_2}^*}\right)^2, \tag{18}$$

and $$k_s = \frac{K_\alpha K_{p_2}^* + K_\beta K_{p_1}^*}{K_\alpha + K_\beta} \tag{19}$$

where K_a and K_b are the standardized deviates exceeded with the probabilities α and β respectively. The known standard deviation plans is obtain with parameters n_σ and k_σ by using the relations $n_\sigma = n_s\left(1 + \frac{k^2}{2}\right)^{-1}$ and $k_\sigma = k_s$ In a similar way, when L is specified, the expressions for n and k are given by

$$n_s = \left(1 + \frac{k^2}{2}\right)\left(\frac{K_\alpha + K_\beta}{K_{1-p_1}^* - K_{1-p_2}^*}\right)^2 \tag{20}$$

and $$k_s = -\frac{K_\alpha K_{1-p_2}^* + K_\beta K_{1-p_1}^*}{K_\alpha + K_\beta}. \tag{21}$$

The known standard deviation plan with parameters n_σ and k_σ could be derived as $n_\sigma = n_s\left(1 + \frac{k^2}{2}\right)^{-1}$ and $k_\sigma = k_s$

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NUMERICAL EXAMPLE

Suppose that it is desired to institute a single sampling plan by variables giving protection to the producer and consumer in terms of quality levels p_1=0.01 and p_2=0.06 associated with producer's risk, α = 0.05 and β = 0.10 when the quality characteristic is distributed according to Inverse Gaussian distribution having the mean θ = 5 and the shape parameter fixed at λ= 10. Corresponding to p_1 and p_2, the values of K_{p_1} and K_{p_2} are determined respectively from equations (7) and (8) as 3.6458 and 1.7621. The normal deviates K_α and K_β are obtained as 1.64485 and 1.28155 by satisfying (9) and (10) for specified values of α and β. Substituting these values in (18) and (19), the parameters of the desired plan are determined as n = 10.4896 which, when rounded, becomes 11 and k = 2.5870 . Thus, the parameters of a known standard deviation plan are computed as

$$n_\sigma = n_s\left(1+\frac{k^2}{2}\right)^{-1} = 2.4134 \approx 3 \text{ and } k_\sigma = 2.5870$$

CONCLUSION

A single sampling plan by variables under the assumption of Inverse Gaussian distribution is proposed and its performance measure, called the operating characteristic function is derived. A procedure for determining the parameters of such plans is discussed through an illustration.

REFERENCES

1. A. H. Bowker and H. P. Goode, Sampling Inspection by Variables, McGraw-Hill, New York, USA, 1952.
2. D. C. Montgomery, Introduction to Statistical Quality Control, John Wiley & Sons, Inc., New York, USA, 2004.
3. G. J. Lieberman and G. J. Resnikoff, 'Sampling Plans for Inspection by Variables', Journal of the American Statistical Association, 50, pp. 457 – 516, 1955.
4. E. G. Schilling, Acceptance Sampling in Quality Control, Marcel Dekker, New York, USA, 1982.
5. D. B. Owen, 'One-Sided Variables Sampling Plans', Industrial Quality Control, 22, pp. 450 – 456, 1966.
6. D. B. Owen, 'Variables Sampling Plans Based on the Normal Distribution', Technometrics, 9, pp. 417 – 423, 1967.
7. H. C. Hamaker, 'Acceptance Sampling for Percent Defective by Variables and by Attributes', Journal of Quality Technology, 11, pp. 139 – 148, 1979.
8. A. J. Duncan, Quality Control and Industrial Statistics, Richard D. Irwin, Homewood, Illinois, USA, 1986.
9. A. B. L. Srivastava, 'Variables Sampling Inspection for Non-Normal Samples', Journal of Science and Engineering Research, 5, pp. 145 – 152, 1961.
10. N. G. Das and S. K. Mitra, 'The Effect of Non-Normality on Plans for Sampling Inspection by Variables', Sankhya - A, 26, pp.169 – 176, 1964.
11. D. B. Owen, 'Summary of Recent Work on Variables Acceptance Sampling with Emphasis on Non-Normality', Technometrics, 11, pp. 631 – 637, 1969.
12. K. Takagi, 'On Designing Unknown-Sigma Sampling Plans based on a Wide Class of Non-Normal Distributions', Technometrics, 14, pp. 669 – 678, 1972.
13. W. C. Guenther, 'Variables Sampling Plans for the Poisson and the Binomial', Statistica Neerlandica, 26, pp. 113 – 120, 1972.
14. W. C. Guenther, 'LQL Like Plans for Sampling by Variables', Journal of Quality Technology, 17, pp.155 – 157, 1985.
15. W. J. Zimmer and I. W. Burr, 'Variables Sampling Plans based on Non-Normal Populations', Industrial Quality Control, 21, pp. 18 – 26, 1963.
16. M. S. Aminzadeh, 'Inverse-Gaussian Acceptance Sampling Plans by Variables', Communications in Statistics – Theory and Methods, 25, pp. 923 – 935, 1996.
17. R. P. Suresh and T. V. Ramanathan, 'Acceptance Sampling Plans by Variables for a Class of Symmetric Distributions', Communications in Statistics – Simulation and Computation, 26, pp.1379 – 1391, 1997.
18. R. Vijayaraghavan and S. Geetha, 'Evaluation of Single Sampling Plans by Variables using Single Parameter Gamma Distribution', National Conference on Quality Improvement Concepts and their Implementation in Higher Educational Institutions, December 11 – 12, Amrita University, Coimbatore, INDIA, 2009.
19. R. Vijayaraghavan and S. Geetha, 'Procedure for Selection of Single Sampling Plans Variables Based on Gamma Distribution', UGC National Level staff Seminar on Applied Statistics, December 24, Sri Sarada College for Women, Salem, INDIA, 2009.
20. R. Vijayaraghavan and S. Geetha, 'Procedure for Selection of Single Sampling Plans Variables Based on Laplace Distribution', Recent Trends in Statistical Research, Publication Division, Mononmaniam Sundaranar University, 209-218, 2010.
21. S. Geetha and R. Vijayaraghavan, 'Evaluation of Single Sampling Plan by Variables Based on Rayleigh Distribution', National Conference on Statistics for Twenty First Century (NCSTC) & Annual Conference of Kerala Statistical Association, March 17 - 19, 2011, University of Kerala, Trivandrum, INDIA, 2011.
22. S. Geetha and R. Vijayaraghavan, 'Selection of Single Sampling Plans by Variables Based on Logistic Distribution', UGC sponsored National Conference on Recent Trends in Statistical and Computer Applications, March 23 - 24, 2011, Mononmaniam Sundaranar University. Tirunelveli, INDIA, 2011.
23. S. Geetha and R. Vijayaraghavan, 'A Procedure for Selection of Single Sampling Plans by Variables Based on Log-logistic Distribution', International Conference on Frontiers of Statistics and its Applications, December 21 - 23, 2012, Department of Statistics, Pondicherry University, Puducherry, INDIA., 2012.
24. S. Geetha and R. Vijayaraghavan, 'A Procedure for the Selection of Single Sampling Plans by Variables Based on Pareto Distribution', Journal of Quality and Reliability Engineering, 2013, pp. 1-13, 2013.
25. S. Geetha and R. Vijayaraghavan, 'Evaluation of Single Sampling Plans by Variables using Weibull Distribution', National Conference on Advances in Statistical Theory, Modeling and Applications, March 27 - 28, 2014 at Department of Statistics, Bharathiar University, Coimbatore, INDIA, 2014.
26. S. Geetha and R. Vijayaraghavan, 'Evaluation of Single Sampling Plans by Variables using Lognormal Distribution', National Seminar RDSMM-2014, August 27-28, 2014 at Department of Statistics, Government Arts College, Coimbatore, INDIA, 2014.
27. S. Geetha and R. Vijayaraghavan, 'A Procedure for Selection of Single Sampling Plans by Variables Based on Logistic Distribution', International Conference on New Horizons In Statistical Modelling and Applications, August February 27-28, 2015 at Department of Statistics, Presidency College, Chennai, INDIA, 2015.