ORIGINAL RESEARCH PAPER

Volume-7 | Issue-4 | April-2018 | PRINT ISSN No 2277 - 8179

INTERNATIONAL JOURNAL OF SCIENTIFIC RESEARCH

AN INTRODUCTION TO MATHEMATICAL MODELLING

Microbiology

Dr. Bibhash Nandi* Department of Microbiology, Maulana Azad Medical College, New Delhi, India. *Corresponding Author

Anita Chakravarti Department of Microbiology, Maulana Azad Medical College, New Delhi, India.

ABSTRACT

Current medical research literature is increasingly relying on computer controlled mathematical models of medical outcomes. However, consumers of medical research usually lack the background knowledge of mathematical modelling and an understanding of the methods that are used to construct these models. This often results in readers blindly accepting the interpretations presented with the help of mathematical models. Therefore, a basic knowledge of the concepts of mathematical models will provide the readers of scientific literature with the insight needed to critically evaluate such findings.

KEYWORDS

Mathematical model, Research

Introduction

Modelling is a cognitive activity in which we think about and make models to describe how devices or objects of interest behave. Therefore, a Mathematical Model can be defined as a system of postulates, data and inferences presented as a mathematical description of an entity or a real-world phenomenon.

The role of mathematics and statistics in the clinical and biomedical sciences is increasing and this is reflected in the growing number of journals and articles devoted to the subject. Mathematical models are being used to generate simulated patient populations, and even model their outcomes for therapies of the future. The development of well-informed mathematical models in medicine requires deep knowledge and understanding of both mathematics and medicine. In this process, we translate our beliefs into the language of mathematics. The primary benefit from this approach is that since mathematics is a very precise and concise language, it helps us to formulate ideas and identify underlying assumptions with well-defined rules for manipulation. Another advantage is that computers can be used to perform numerical calculations and carry out simulations. However, the language of mathematics can be intimidating for those unused to it. This can often lead to interesting work being ignored, or, more significantly, the indiscriminating acceptance of the results of models.

To put it simply, Models are abstractions of reality that is, they are representations of a particular thing, idea or condition. Mathematical modelling is the use of mathematics to describe and explain real world phenomenon, investigate important questions about the observed world, test ideas and make predictions about the real world. Here the real world refers to diverse fields of scientific study including the medical sciences. However, it is vital to remember that mathematical modelling complements, but does not replace theory and experimentation in scientific research.

The modelling processes

The steps involved in development of a mathematical model [1] are summarised in Figure 1. The obvious first step in this process is the identification of the real-world problem and the need for a model. This is followed by extensive data gathering which often involves performing background research and conduct investigations if appropriate. The data is then analysed and reviewed to identify and select factors to describe important aspects of the problem and perhaps just as important, to discourage those factors which can be neglected. The formulation of the working model in mathematical terms is now possible, with the help of mathematical equations whose solution describes the working model. Choosing a suitable algorithm and computer software is critical. Commonly used commercial software include Matlab, Excel and Mathematica. Finally, the simulation is run and the results obtained. The user must interpret the solution and draw suitable conclusions. It is important to validate the model which includes verification of the computer program, comparisons with the real world and exploring ranges of validity.

The emphasis is on developing models that describe the behaviour or the results obtained, explain why that behaviour or those results occurred as they did, and allow us to predict future results that are as yet unseen or unmeasured. The keys to the validity of modeling are the known dependent probabilities, associated variances, and coefficients determining the relative significance of each factor to the model. This means that a model must be based on sound research, with actual data that are widely accepted as valid by the medical science community.

Types of Mathematical Models

Models can be categorized [2,3] by a number of key criteria :

- Individual-based and Population average-based simulations

In individual-based simulations, each individual in the population is modelled as a discrete entitity, and characteristics are determined separately for each individual. The major disadvantage of these study models is that they are expensive and require a considerable of time and effort. On the contrary, in models based on Population averages, it is assumed that all individuals in the population have identical characteristics, the obvious disadvantage being that these studies lack accuracies.

The most common modelling approach involve the division of the study population into cohorts of individuals, and individuals in the same cohort are assumed to have same characteristics. For example, cohorts are commonly defined according to the disease status – Susceptible, Infected or Resistant. In models of STI, factors such as age, sex and the level of sexual risk behaviour.

- **Stochastic and Deterministic models**

Stochastic models are often used with individual-based simulations. These models allow events such as infection and death to be simulated by random processes. The characteristics of each individual is taken into account, the set of possible next events is defined with a probability attached to each and a random number generator is used to calculate when the next event will occur and which of the range of possible events it would be. Studies based on stochastic models tend to give dissimilar results each time the simulation is run. This has the advantage that a wide range of possible outcomes can be determined. With recent increases in the speed of computers stochastic models have become more convenient and should have an important role in understanding the persistence and control of the bacterial STIs when they are at a low prevalence.

Deterministic models, however, calculate expected numbers of events in cohorts of individuals. They are mostly used with population average-based or cohort-based simulations. In a deterministic model events are not subject to chance and two realisations of a model using the same parameters and exact starting conditions will give exactly the same results. However, results can diverge in the case of deterministic chaos because it is impossible to exactly specify starting conditions and the value of variables.

- **Distributional and Compartmental models**

In Distributional models, individual characteristics are treated as continous variables that can take on any value within a specified range. In Compartmental models, individual characteristics are treated as discrete variables, which can only take on a finite number of values.[4] For example, in a distributional model, an HIV positive individual's CD4 count may take on any value between 0 to 2000 whereas in a compartmental model, individuals would be classified according to the range in which their CD4 count falls: CD4 < 200, CD4 between 200 and 500, and CD4 > 500.

- **Discrete time and Continuous time models**

In medical studies, certain events of interest such as infection and death can be studied as occurring at a particular point in time or over a time period. In Discrete Time models, population changes or events such as infection or death occur as discrete steps. The choice of time period will depend in practice on the phenomenon being studied. For example, when modelling the risk of disease transmission during acute phase, it will be necessary to use daily or weekly projection intervals whereas in modelling survival studies, it would be acceptable to use yearly projection intervals. In Continuous Time models, events are modelled as occurring at a point in time and as a smooth continuous process. These models are specified in terms of differential equations. For example, the sexual interactions that allow the spread of STI occur continuously in a population over the study time period. [5]

Application of mathematical modelling in scientific studies

A model embodies a hypothesis about a study system and lets us compare that hypothesis with data. Mathematical models and computer simulations are useful experimental tools for building and testing theories, answering specific questions and estimating key parameters from data. This is helpful when limitations like a rare event prohibit repeating actual studies or expanding research on actual patients. Another important aspect is that the model formulation process clarifies assumptions, variables and parameters. A novel application for mathematical modeling is the determination of sample size requirement. Estimates of the population parameters can direct a simulation that increases one patient at a time until a statistically significant difference is detected between the experimental groups. This is most useful in experimental designs with categorical, non-parametric, or otherwise non-normally distributed data.

Medical researchers can use mathematical models built on scientific understanding in order to forecast disease outbreaks, optimize medical treatments, and design systems for biological pest control and bioengineering. Modelling is an integral part of simulated experimentation where realistic experimenting maybe impossible. For example, experiments with infectious disease spread in human populations, which are often impossible, unethical or expensive, can be carried out through well-designed simulations. Simulations are also suitable in studies where it is not possible to determine dosage for clinical trials of new drugs on humans or set safe limits for exposure to toxic substances without the proper knowledge of the consequences.

Limitations of using mathematical models

Mathematical models are only as accurate as the underlying assumptions and the raw data. It is not always possible to find an exact solution since a model is only an abstraction of a real-world problem. Failure to adapt to changing variables is another problem, that is, a mathematical model starts to fail as the variables change and the data gets outdated. Another factor to consider is computational complexity as a model that is sufficiently accurate may require enormous computer power, for example, weather forecasting models.

Conclusion

Mathematical modelling is a relatively novel methodology in medical science research within which we can develop an understanding and communicate infectious disease dynamics. It is important to remember that modeling is not perfect, a model is only as good as its assumptions. The impact of this, or any other, research methodology will only be to the betterment of patient care if the medical community at large undertakes a basic understanding, guiding the application of findings based on reasonable confidence after critical review. Modelling can answer otherwise unanswerable questions and greatly expand our knowledge base from actual study data. It can do all of this efficiently and risk-free, without real patients. However, all logical and mathematical components of a model must be based on valid research of actual patients.

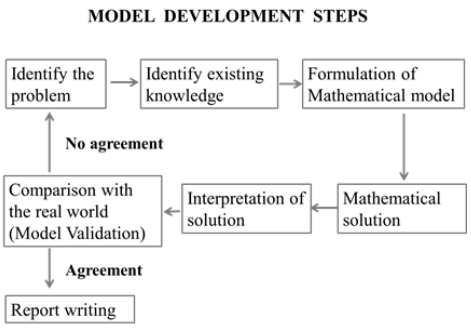


Figure 1: The mathematical modelling process

REFERENCES

1. S. P. Ellner and J. Guckenheimer. Dynamic models in biology. Princeton University Press, 2006.
2. Anderson RM, May RM. Infectious diseases of humans: dynamics and control. Oxford: Oxford University Press, 1991.
3. A. Okubo. Diffusion and ecological problems: mathematical models. Springer-Verlag, Berlin, Heidelberg, 1980.
4. Anderson RM, Garnett GP. Mathematical models of the transmission and control of sexually transmitted diseases. Sex Transm Dis. 2000;27:636–43.
5. Garnett G. An introduction to mathematical models in sexually transmitted disease epidemiology. Sex Transm Dis. 2002;78:7-12.