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β_T open sets in Tri Topological space

Mathematics

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ABSTRACT

The main aim of this paper is to introduce new type of open sets namely β_T open sets in tri topological spaces along with their several properties and characterization .As application to β_T open set we introduce β_T continuous functions and obtain some of their basic properties.

KEYWORDS:

β_T open sets and β_T continuous functions.

Introduction

In 1961 J .C. Kelly [3] introduced the concept of bitopolgical space. In 1986, D. Andrijevic [1] was introduced semi-pre open sets (β open sets) in topological spaces.F.H. Khedr , S.M. Al-Areefi and T. Noiri [4] generalize the notion of semi pre open sets and semi pre continuity in bitopological spaces. Tri topological space is a generalization of bitopological space .The study of tri-topological space was first initiated by Martin M. Kovar
[5]. S. Palaniammal [6] study tri topological space. N.F. Hameed and Moh. Yahya Abid [3] gives the definition of 123 open set in tri topological spaces . U.D. Tapi , R. Sharma and B. Deole [7] introduce semi open set and pre open set in tri topological space. The purpose of the present paper is to introduce β_T open sets and β_T continuity and their fundamental properties in tri topological space. In this paper, tri open set is use in place of 123 open sets.

1. Preliminaries

Definition 1.1 [1]: A subset A of a topological space (X,τ) is called semi-pre open set (β open sets) if $A \subseteq cl(\text{int}(clA))$ .The complement of semi-pre open set is called semi-pre closed set. The class of all semi pre open sets of X is denoted by $SPO(X,\tau)$.

Definition 1.2 [4] : Let (X,τ) and (Y,τ) are topological spaces. A function $f : X \to Y$ is called semi-pre continuous (β continuous) if the inverse image of each open set in Y is a semi-pre open set(β open set) in X.

Definition 1.3 [6]: Let X be a nonempty set and T_1, T_2 and T_3 are three topologies on X .The set X together with three topologies is called a tri topological space and is denoted by (X, T_1, T_2, T_3)

Definition1.4[2] : A subset A of a topological space X is called 123 open set if $A \in T_1 \cup T_2 \cup T_3$ and complement of 123 open set is 123 closed set.

2. β_T open sets in Tri topological space

Definition2.1: Let (X, T_1, T_2, T_3) be a tri topological space. A subset A of X is called β_T open in X, if $A \subseteq spclsp\,\text{int}\,spclA$ The complement of β_T open set is called β_T closed set. The collection of all β_T open sets of X is denoted by $SPO(X, T_1, T_2)$.

Example 2.2 Let $X = \{a,b,c\}, T_1 = \{X,\phi,\{a,b\}\}, T_2 = \{X,\phi,\{a\}\}, T_3 = \{X,\phi,\{b\}\}$

Tri open sets in tri topological spaces are union of all three topologies.

β_T open set of X is denoted by $SPO(X) = \{X,\phi,\{a\},\{a,b\},\{b\}\}$.

Let $A = \{a,b\}$;

$$TclT\,\text{int}\,Tcl\{a,b\} = TclT\,\text{int}\{a,b\}$$
$$= TclX$$
$$= X$$

$A = \{a,b\}$ is β_T open.

Definition2.3: Let (X, T_1, T_2, T_3) be a tri topological space. Let $A \subset X$. An element $x \in A$ is called β_T interior point of A, if there exist a β_T open set V such that $x \in V \subset A$. The set of all β_T interior points of A is called the β_T interior of A and is denoted by $sp\,\text{int}(A)$.

Theorem2.4: Let $A \subset X$ be a tri topological space. $sp\,\text{int}(A)$ is equal to the union of all β_T open sets contained in A.

Note 2.5 1. $sp\text{int}(A) \subset A$

2. $sp\,\text{int}(A)$ is β_T open sets.

Theorem 2.5 $sp\text{int}(A)$ is the largest β_T open sets contained in A.

Theorem2.6: A is β_T open if and only if $A = sp\,\text{int}(A)$

Theorem2.7: $sp\,\text{int}(A \cup B) \supset sp\text{int}\, A \cup sp\text{int}\, B$

Definition2.8: Let (X, T_1, T_2, T_3) be a tri topological space. Let $A \subset X$. The intersection of all β_T semi closed set containing A is called a β_T closure of A and is denoted as $spcl(A)$.

Note 2.9: Since intersection of β_T closed sets is β_T closed set, $spcl(A)$ is a β_T closed set.

Note2.10: $spcl(A)$ is the smallest β_T closed set containing A.

Theorem 2.11: A is β_T closed set if and only if $A = spcl(A)$.

Theorem2.12: Let A and B be subsets of (X, T_1, T_2, T_3) and $x \in X$

a) If $A \subset B$, then $spcl(A) \subset spcl(B)$.

b) $x \in spcl(A)$ if and only if $A \cap U \neq \phi$ for every β_T open set U containing x.

Theorem2.13: Let A be a subsets of (X, T_1, T_2, T_3), if there exist an β_T open set U such that $A \subset U \subset spcl(A)$, then A is β_T open.

Theorem2.14: In a tri topological space (X, T_1, T_2, T_3), the union of any two β_T open sets is always an β_T open set.

Proof: Let A and B be any two β_T open sets in X.

Now $A \cup B \subseteq spcl\left(sp\,\text{int}(A)\right) \cup sp\,cl\left(sp\text{int}(B)\right)$

$\Rightarrow A \cup B \subseteq spcl\left(sp\text{int}(A \cup B)\right)$. Hence $A \cup B$ β_T is open sets.

Theorem2.17: Let A and B be subsets of X such that $B \subseteq A \subseteq spcl(B)$. if B is β_T open set then A is also β_T open set.

3. β_T Continuity in Tri topological space

Definition 3.1: A function f from a tri topological space (X, T_1, T_2, T_3) into another tri topological space $\left(Y, W_1, W_2, W_3\right)$ is called β_T continuous if $f^{-1}(V)$ is β_T open set in X for each tri open set V in Y.

Example3.2: Let, $X = \{a, b, c\}$, $T_1 = \{X, \phi, \{a\}\}$, $T_2 = \{X, \phi, \{a, b\}\}$, $T_3 = \{X, \phi, \{b, c\}\}$

Open sets in tri topological spaces are union of all three topologies.

Then tri open sets of $X = \{X, \phi, \{a\}, \{a, b\}, \{b, c\}\}$.

$SPO(X)$ sets of $X = \{X, \phi, \{a\}, \{a, b\}, \{b, c\}\}$.

Let $Y = \{1, 2, 3\}$, $W_1 = \{Y, \phi, \{1, 3\}\}$, $W_2 = \{Y, \phi, \{2\}\}$, $W_3 = \{X, \phi, \{2\}, \{1, 2\}\}$

tri open sets of $Y = \{Y, \phi, \{2\}, \{1, 2\}, \{1, 3\}\}$.

$SPO(Y)$ sets of $Y = \{Y, \phi, \{2\}, \{1, 2\}, \{1, 3\}\}$.

Consider the function $f : X \to Y$ is defined as

$f^{-1}\{2\} = \{a\}$, $f^{-1}\{1, 2\} = \{a, b\}$, $f^{-1}\{1, 3\} = \{b, c\}$, $f^{-1}(\phi) = \phi$, $f^{-1}(Y) = X$.

Since the inverse image of each tri open set in Y under f is β_T open set in X. Hence f is β_T continuous function.

Theorem 3.3: Let (X, T_1, T_2, T_3) and $\left(Y, W_1, W_2, W_3\right)$ be two tri topological spaces. A function $f : X \to Y$ is β_T continuous if and only if the inverse image of every β_T open set in Y is tri open in X.

Proof: (Necessary): Let $f : (X, T_1, T_2, T_3) \to \left(Y, W_1, W_2, W_3\right)$ be β_T continuous function and U be any β_T open set in Y. Then Y-U is β_T closed in Y. Since f is β_T continuous function, $f^{-1}(Y - U) = X - f^{-1}(U)$ is β_T closed in X and hence $f^{-1}(U)$ is β_T open in X.

(Sufficiency): Assume that $f^{-1}(V)$ is β_T open in X for each tri open set V in Y. Let V be a closed set in Y.Then $Y-V$ is β_T open in Y.By assumption $f^{-1}(Y-V) = X- f^{-1}(V)$ is β_T open in X which implies that $f^{-1}(V)$ is β_T closed in (X, T_1, T_2, T_3). Hence f is β_T continuous function.

Theorem 3.4 Let (X, T_1, T_2, T_3) and (Y, W_1, W_2, W_3) be two tri topological spaces. A function $f: X \to Y$ is β_T continuous function if and only if $f^{-1}(U)$ is β_T closed set in X whenever V is tri closed in Y.

Theorem 3.5: Let $f: (X, T_1, T_2, T_3) \to (Y, W_1, W_2, W_3)$ be a β_T continuous open function .If V is an β_T open set of Y, then $f^{-1}(V)$ is β_T open in X.

Proof : First ,let V be β_T open set of Y.There exist an β_T set W in Y.such that $V \subset W \subset spcl(V)$.Since f is tri open set ,we have $f^{-1}(V) \subset f^{-1}(W) \subset f^{-1}(\text{spcl}(V)) \subset \text{spcl}(f^{-1}(V))$ .since f is β_T continuous, $f^{-1}(W)$ is β_T open set in X .By theorem 2.13, $f^{-1}(V)$ is β_T open set in X .The proof of the second part is shown by using the fact of first part.

Theorem3.6 : The following are equivalent for a function $f: (X, T_1, T_2, T_3) \to (Y, W_1, W_2, W_3)$:

a) f is β_T continuous function ;

b) The inverse image of each β_T closed set of Y is β_T closed in X ;

c) For each $x \in X$ and each tri open set V in W containing $f(x)$,there exist an β_T open set U of X containing x such that $f(U) \subset V$;

d) $spcl(f^{-1}(B)) \subset f^{-1}(spcl(B))$ for every subset B of Y.

e) $f(spcl(A)) \subset spcl(f(A))$ for every subset A of X.

Theorem 3.7: If $f: (X, T_1, T_2, T_3) \to (Y, W_1, W_2, W_3)$ and $g: (Y, W_1, W_2, W_3) \to (Z, \eta_1, \eta_2, \eta_3)$ be two β_T continuous function then $gof: (X, P_1, P_2, P_3) \to (Z, \eta_1, \eta_2, \eta_3)$ may not be β_T continuous function .

Theorem3.8: Let $f^{-1}: (X, T_1, T_2, T_3) \to (Y, W_1, W_2, W_3)$ be bijective .Then the following conditions are equivalent:

i) f is a tri β_T open continuous function.

ii) f is β_T closed continuous function and

iii) f^{-1} is β_T continuous function.

Proof: (i)$\to$(ii) Suppose B is a tri closed set in X .Then $X-B$ is an tri open set in X .Now by (i) $f(X-B)$ is a β_T open set in Y .Now since f^{-1} is bijective so $f(X-B) = Y- f(B)$ .Hence $f(B)$ is a β_T closed set in Y .Therefore f is a β_T closed continuous function.

(ii)$\to$(iii) Let f is an β_T closed map and B be tri closed set of X .Since f^{-1} is bijective so $(f^{-1})^{-1}(B)$ which is an β_T closed set in Y. Hence f^{-1} is β_T continuous function.

(iii)$\to$(i) Let A be a tri open set in X .Since f^{-1} is a β_T continuous function so $(f^{-1})^{-1}(A) = f(A)$ is a β_T open set in Y. Hence f is β_T open continuous function.

Theorem3.9: Let X and Y are two tri topological spaces. Then $f: (X, P_1, P_2, P_3) \to (Y, W_1, W_2, W_3)$ is β_T continuous function if one of the followings holds:

i) $f^{-1}(spint(B)) \subseteq spint(f^{-1}(B))$, for every tri open set B in Y.

ii) $spcl(f^{-1}(B)) \subseteq f^{-1}(spcl(B))$, for every tri open set B in Y.

Proof: Let B be any tri open set in Y and if condition (i) is satisfied then $f^{-}(spint(B)) \subseteq spint(f(B))$.

We get $f^{-1}(B) \subseteq spint(f^{-1}(B))$.Therefore $f^{-1}(B)$ is a β_T open set in X .Hence f is β_T continuous function. Similarly we can prove (ii).

Theorem 3.10: A function $f: (X, P_1, P_2, P_3) \to (Y, W_1, W_2, W_3)$ is called β_T open continuous function if and only if $f(spint(A)) \subseteq spint(f(A))$, for every tri open set A in X .

Proof: Suppose that f is a β_T open continuous function.

since $sp\ int(A) \subseteq A$ so $f(sp\,int(A)) \subseteq f(A)$.

By hypothesis $f(spint(A))$ is an β_T open set and $spint(f(A))$ is largest β_T open set contained in $f(A)$ so $f(spint(A)) \subseteq spint(f(A))$.

Conversely, suppose A is an tri open set in X .So $f(spint(A)) \subseteq spint(f(A))$.

Now since $A = spint(A)$ so $f(A) \subseteq spint(f(A))$ .Therefore $f(A)$ is a β_T open set in Y and f is β_T open continuous function.

Theorem 3.11: A function $f:(X, P_1, P_2, P_3) \to (Y, W_1, W_2, W_3)$ is called β_T closed continuous function if and only if $spcl(f(A)) \subseteq f(spcl(A))$, for every tri closed set A in X .

Proof: Suppose that f is a β_T closed continuous function. since $A \subseteq spcl(A)$ so $f(A) \subseteq f(spcl(A))$ .By hypothesis, $f(spcl(A))$ is a β_T closed set and $scl(f(A))$ is smallest β_T closed set containing $f(A)$ so $scl(f(A)) \subseteq f(scl(A))$.

Conversely, suppose A is an tri closed set in X .So $scl(f(A)) \subseteq f(spcl(A))$.

Since $A = spcl(A)$ so $spcl(f(A)) \subseteq f(A)$ .Therefore $f(A)$ is a β_T closed set in Y and f is β_T closed continuous function.

Theorem 3.12: Let (X, T_1, T_2, T_3) and (Y, W_1, W_2, W_3) be two tri topological space .Then,
$f: X \to Y$ is β_T continuous function if and only if $f^{-1}(V)$ is β_T closed in X whenever V is tri closed in Y.

Theorem3.13: Let (X, T_1, T_2, T_3) and (Y, W_1, W_2, W_3) be two tri topological space. Then, $f: X \to Y$ is β_T continuous function if and only if $f(spcl(A)) \subset spcl(f(A)) \ \forall A \subset X$.

Proof: Suppose $f: X \to Y$ is β_T continuous function. Since $spcl[f(A)]$ is tri closed in Y. Then
by theorem (3.4) $f^{-1}[spcl(f(A))]$ is tri closed in X ,
$spcl(f^{-1}(spclf(A))) = f^{-1}(spclf(A))$ ----(1).
Now : $f(A) \subset spcl(f(A))$, $A \subset f^{-1}(f(A)) \subset f^{-1}(spclf(A))$.
Then $spcl(A) \subset spcl(f^{-1}(spclf(A))) = f^{-1}(spclf(A))$ by (1).
Then $f(spcl(A)) \subset spcl(f(A))$.
Conversely, Let $f(spcl(A)) \subset spcl(f(A)) \ \forall A \subset X$.
Let F be β_T closed set in Y , so that $spcl(F) = F$. Now $f^{-1}(F) \subset X$, by hypothesis,
$f(spcl(f^{-1}(F))) \subset spcl(f(f^{-1}(F)) \subset spcl(F) = F$.
Therefore $spcl(f^{-1}(F) \subset f^{-1}(F)$. But $f^{-1}(F) \subset spcl(f^{-1}(F)$ always.
Hence $spcl(f^{-1}(F) = f^{-1}(F)$ and so $f^{-1}(F)$ is β_T closed in X .
Hence by theorem (3.6) f is β_T continuous function.

Theorem 3.14 : Let (X, T_1, T_2, T_3) and (Y, W_1, W_2, W_3) be two tri topological spaces. Then, $f: X \to Y$ is β_T continuous function if and only if $spcl(f^{-1}(B)) \subset f^{-1}(spcl(B)) \ \forall B \subset Y$.
Proof: Suppose $f: X \to Y$ is β_T continuous. Since $spcl(B)$ is β_T closed in Y ,then by theorem (3.6) $f^{-1}(spcl(B))$ is β_T closed in X and therefore , $spcl(f^{-1}(spcl(B))) = f^{-1}(spcl(B))$.....(2)
Now, $B \subset spcl(B)$, then $f^{-1}(B) \subset f^{-1}(spcl(B))$,then $spcl(f^{-1}(B)) \subset spcl(f^{-1}(spcl(B))) = f^{-1}(spclB)$ by (2)
Conversely: Let the condition hold and let F be any tri closed set in Y so that $spcl(F) = F$. By hypothesis,
$spcl(f^{-1}(F)) \subset f^{-1}(spcl(F)) = f^{-1}(F)$ .But $f^{-1}(F) \subset spcl(f^{-1}(F))$ always. Hence $spcl(f^{-1}(F)) = f^{-1}(F)$ and so $f^{-1}(F)$ is β_T closed in X . It follows from theorem (3.6) that f is β_T continuous function.

Theorem3.15 : Let (X, T_1, T_2, T_3) and (Y, P_1, P_2, P_3) be two tri topological spaces. Then, $f: X \to Y$ is β_T continuous open function if and only if $f^{-1}(spint(B)) \subset spint(f^{-1}(B)) \quad \forall B \subset Y$.
Proof: Let $f: X \to Y$ be a β_T continuous. Since $spint(B)$ is β_T open in Y, then by theorem (3.3) $f^{-1}(spint(B))$ is β_T open in X and therefore , $spint(f^{-1}(spint(B))) = f^{-1}(spint(B))$.....(3)
Now, $spint(B) \subset B$, then $f^{-1}(spint(B)) \subset f^{-1}(B)$,then $spint(f^{-1}(spint(B))) \subset spint(f^{-1}(B))$ by (3)

Conversely: Let the condition hold and let G be any β_T open set in Y so that $spint(G) = G$. By hypothesis, $f^{-1}(spint(G)) \subset spint\ f^{-1}(G)$ .Since $f^{-1}(spint(G)) = f^{-1}(G)$ then $f^{-1}(G) \subset spint(f^{-1}(G))$ But

$sp\text{int}(f^{-1}(G)) \subset f^{-1}(G)$ always and so $sp\text{int}(f^{-1}(G)) = f^{-1}(G)$.Therefore $f^{-1}(G)$ is β_T open in X. Consequently by theorem (3.3) f is β_T continuous function.

CONCLUSION: In this paper We studied new form of β_T open set in tri topological space. We also studied β_T continuous function in tri topological space. It is established that composition of any two β_T open sets is again a β_T open set in tri topological space .In β_T continuity, inverse image of every tri open is β_T open set .

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