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Research Paper

Cost Measures Of Fuzzy Batch Arrival Queuing Model By Ranking Function Method

Mathematics

KEYWORDS : Fuzzy sets, batch arrival queues, Fuzzy ranking, Membership Functions.

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ABSTRACT

In this paper we express a procedure to find the various performance measures in terms of crisp values for the total expected cost of fuzzy batch arrival queuing model where the arrival rate, service rate, batch size, service cost and holding costs are all fuzzy numbers. Here the inter arrival time, service time, batch size, service cost and holding costs are Triangular and also Trapezoidal fuzzy numbers. Our idea is to convert the fuzzy inter arrival rate, service rate, batch size, service cost and holding costs into crisp values by applying ranking function method. Then apply the crisp values in the classical queuing performance measure formulas. Also we find moderate and optimistic values by using the normalized and non-normalized fuzzy numbers. Ranking fuzzy numbers plays a huge role in decision making under fuzzy environment. This ranking method is most reliable method, simple to apply and can be used for all types of queuing problems.

INTRODUCTION

Queuing models are effective methods for performance analysis of computer and telecommunication systems, manufacturing /production systems and inventory control are discussed by Kleinrock [1], Gross and Harris [2]. Kaufmann [3] presented for the theory of fuzzy subsets. Through Zadeh's[4] extension principle, we transform the fuzzy queue with batch arrival into a family of crisp queues . For notational convenience our model in this paper will be denoted by FM[K]/FM/1.Many types of batch arrival queuing models has been investigated so far. Chen [5,6] developed FM/FM/1/L and FM/FM[K]/1/ fuzzy systems. Lin, C.H., Huang, H.I., Ke, J.C.[7,8] developed Parametric Programming Approach for Batch Arrival Queues with Vacation Policies and Fuzzy parameters and also discussed On a Batch Arrival Queue with Set up and Uncertain Parameters Patterns.

Recently Jeeva and Rathnakumari [9] analyzed the bulk arrival single server Bernoulli feedback queue with fuzzy vacations and fuzzy parameters.. For any queuing system cost analysis constitute a very important aspect of its investigation.

Rue and Roshen shine [10] dealt with the control policy on arrivals. Ritha and Lilly Robert [11] developed the profit analysis of fuzzy M/EK/1 queuing system by using non-linear programming techniques. Thus in this paper we develop a method which is able to provide TEC for bulk arrival queues with fuzzified exponential arrival rate, service time, batch size, service cost and holding cost.

Yager [12] proposed four indices which may be employed for the purpose of ordering fuzzy quantities in [0,1].. Campos and Gonzalez [14] proposed a subjective approach for ranking fuzzy numbers. Liou and Wang [15] developed a ranking method based on integral value index. Cheng [16] presented a method for ranking fuzzy numbers by using the distance method. Kwang and Lee [17] considered the overall possibility distributions of fuzzy numbers in their evaluations and proposed a ranking method. Modarres and Nezhad [18] proposed a ranking method based on preference function which measures the fuzzy numbers point by point and at each point the most preferred number is identified. Chu and Tsao [19] proposed a method for ranking fuzzy numbers with the area between the centroid point and original point. Deng and Liu [20] presented a centroid-index method for ranking fuzzy numbers. Liang et al. [21] and Wang and Lee [22] also used the

centroid concept in developing their ranking index. Chen and Chen [23] presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari [24] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α-levels of trapezoidal fuzzy numbers. Ranking methods also have been analyzed by such researchers like F.Choobinesh and H.Li[25], S.H.Chen[26],B.Palpandi, G.Geetharamani [27].

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PRELIMINARIES

Definition 1 : A fuzzy set is determined by a membership function mapping elements of a domain space or universe of discourse Z to the unit interval [0,1]. (i,e) $\widetilde{A} = \{(z, \mu_{\widetilde{A}}(z)); z \in Z\}$.

Here $\mu_{\widetilde{A}} : Z \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set $\widetilde{A}$ and $\mu_{\widetilde{A}}(z)$ is called the membership value of $z \in Z$ in the fuzzy set $\widetilde{A}$. These membership grades are often represented by real numbers ranging from [0,1].

Definition 2 : (*normal*):

A fuzzy set $\widetilde{A}$ of the universe of discourse Z is called a *normal* fuzzy set if there exists at least one $z \in Z$ such that $\mu_{\widetilde{A}}(z) = 1$.

Definition 3 : (*convex*):

A fuzzy set $\widetilde{A}$ is *convex* if and only if for any ,$z \in Z$, the membership function of $\widetilde{A}$ satisfies the condition $\mu_{\widetilde{A}}\{\lambda z_1 + (1-\lambda) z_2\} \geq \min\{\mu_{\widetilde{A}}(z_1), \mu_{\widetilde{A}}(z_2)\}, 0 \leq \lambda \leq 1$.

Definition 4 : (*Trapezoidal fuzzy number*): | *A trapezoidal fuzzy* number $\tilde{A}$ (z) can be represented by $\tilde{A}$ $(a_1,a_2,a_3,a_4;1)$ with membership function $\mu_{\tilde{A}}$ (z) given by

$$\mu_{\tilde{A}}(z)=\begin{cases}\dfrac{z-a_1}{a_2-a_1}, & a_1\le z\le a_2\\ 1 \quad , & a_2\le z\le a_3\\ \dfrac{z-a_4}{a_3-a_4}, & a_3\le z\le a_4\\ 0, otherwise\end{cases}$$

Definition 5 : (*Triangular fuzzy number*):

A *triangular fuzzy number* $\tilde{A}$(z) can be represented by $\tilde{A}(a_1,a_2,a_3;1)$ with membership function $\mu_{\tilde{A}}$(z) given by

$$\mu_{\tilde{A}}(z)=\begin{cases}\dfrac{z-a_1}{a_2-a_1}, & a_1\le z\le a_2\\ 1 \quad , & z=a_2\\ \dfrac{z-a_3}{a_2-a_3}, & a_2\le z\le a_3\\ 0, otherwise\end{cases}$$

Definition 6: (Normalized and Generalized fuzzy number):

Let a fuzzy set $\tilde{A}$, defined on the universal set of real numbers *R* and its membership function has the following characteristics:

1) $\mu_{\tilde{A}}: R\to[0,w]$ is continuous;

2) $\mu_{\tilde{A}}(z)=0$ for all $z\in(-\infty,a_1]\cup[a_4,\infty)$;

3) $\mu_{\tilde{A}}(z)$ strictly increasing on $[a_1,a_2]$ and strictly decreasing on $[a_3,a_4]$;

4) $\mu_{\tilde{A}}(z)=w$, for all $z\in[a_2,a_3]$, where $0<w\le 1$.

Here if w = 1 then the number is called *Normalized fuzzy number* and if w < 1 then the number is called *Non-Normalized fuzzy number* or *Generalized fuzzy number.*

Definition 7 : (L-R type Normalized and generalized fuzzy number):

Let a Fuzzy number $\tilde{A}=(a_1,a_2,a_3,a_4;w)_{LR}$ and its membership function is given by

$$\mu_{\tilde{A}}(z)=\begin{cases}wL\left(\dfrac{a_2-z}{a_2-a_1}\right), & a_1\le z\le a_2\\ w \quad , & a_2\le z\le a_3\\ wR\left(\dfrac{z-a_3}{a_4-a_3}\right), & a_3\le z\le a_4\\ 0, otherwise\end{cases}$$

Where $0<w\le 1$ and *L* and *R* are reference functions.

Here if w = 1 then the number is called *L-R type Normalized fuzzy number* and if w < 1 then the number is called *L-R type Non-Normalized fuzzy number* or *L-R type Generalized fuzzy number.*

Definition 8 : (*Normalized and Generalized Trapezoidal fuzzy number*):

Let a *trapezoidal fuzzy number* $\tilde{A}$(z) it can be represented by $\tilde{A}(a_1,a_2,a_3,a_4;w)$ and its membership function is given by

$$\mu_{\tilde{A}}(z)=\begin{cases}\dfrac{w(z-a_1)}{(a_2-a_1)}, & a_1\le z\le a_2\\ w \quad , & a_2\le z\le a_3\\ \dfrac{w(z-a_4)}{(a_3-a_4)}, & a_3\le z\le a_4\\ 0, otherwise\end{cases}$$

where $0<w\le 1$.

Here if w = 1 then the number is called *Normalized Trapezoidal fuzzy number* and if w < 1 then the number is called *Non-Normalized Trapezoidal fuzzy number* or *Generalized Trapezoidal fuzzy number.*

Definition 9 : (*Normalized and Generalized Triangular fuzzy number*):

Let a *Triangular fuzzy number* $\tilde{A}$(z) can be represented by $\tilde{A}(a_1,a_2,a_3;w)$ and its membership function is given by

$$\mu_{\tilde{A}}(z)=\begin{cases}\dfrac{w(z-a_1)}{(a_2-a_1)}, & a_1\le z\le a_2\\ w \quad , & z=a_2\\ \dfrac{w(z-a_3)}{(a_2-a_3)}, & a_2\le z\le a_3\\ 0, otherwise\end{cases}$$

where $0 < w \leq 1$.

Here if w = 1 then the number is called *Normalized Triangular fuzzy number* and if w < 1 then the number is called *Non-Normalized Triangular fuzzy number* or *Generalized Triangular fuzzy number.*

COST MEASURES OF FUZZY BATCH ARRIVAL QUEUING MODEL

Consider a fuzzy batch arrival queuing model for to find the TOTAL EXPECTED COST (TEC). It is assumed that the customers arrive at a batch arrival as a Poisson process with fuzzy rate $\widetilde{\lambda}$, the service times as an exponential distribution with fuzzy rate $\widetilde{\mu}$, the expected batch size with fuzzy rate $\widetilde{k}$, the service cost with fuzzy rate $\widetilde{C}_S$ and the holding cost with fuzzy rate $\widetilde{C}_h$ are approximately known and can be represented by convex fuzzy sets. Let $\phi_{\widetilde{\lambda}}(u)$, $\phi_{\widetilde{\mu}}(v)$, $\phi_{\widetilde{k}}(w)$, $\phi_{\widetilde{C}_S}(m)$ and $\phi_{\widetilde{C}_h}(n)$ denote the membership functions of $\widetilde{\lambda}$, $\widetilde{\mu}$, $\widetilde{k}$, $\widetilde{C}_S$ and $\widetilde{C}_h$. Then we have the following fuzzy sets:

$$\widetilde{\lambda} = \left\{(u, \phi_{\widetilde{\lambda}}(u)) \,\middle|\, u \in U\right\}$$

$$\widetilde{\mu} = \left\{(v, \phi_{\widetilde{\mu}}(v)) \,\middle|\, v \in V\right\}$$

$$\widetilde{k} = \left\{(w, \phi_{\widetilde{k}}(w)) \,\middle|\, w \in W\right\}$$

$$\widetilde{C}_S = \left\{(m, \phi_{\widetilde{C}_S}(m)) \,\middle|\, m \in M\right\}$$

$$\widetilde{C}_h = \left\{(n, \phi_{\widetilde{C}_h}(n)) \,\middle|\, n \in N\right\}$$

where U,V,W,M and N are the crisp universal sets of the arrival , service, batch size, service cost and holding cost rates respectively. Let f(u,v,w,m,n) denote the system characteristic of interest. Since u,v,w,m and n are fuzzy numbers. f(u,v,w,m,n) is also a fuzzy number. Let A represents the membership function of the total expected cost.

ie. The total expected cost

$$A = f(u,v,w,m,n) = m + wn\left[\frac{E(w^2)+E(w)}{2[v-uE(w)]}\right] \quad .....(1)$$

RANKING FUNCTION METHOD – ALGORITHM

To solve the problem we defuzzify the fuzzy numbers into crisp ones by a fuzzy number ranking function R : F(R)→ R where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists.

(i) . Let a convex trapezoidal fuzzy number $\widetilde{A}(z) = \widetilde{A}(a_1, a_2, a_3, a_4; w)$,

Then the Ranking Index is defined by

$$R(\widetilde{A}) = \int_0^w \frac{(L^{-1}(z) + R^{-1}(z))}{2} dz$$

Where $L^{-1}(z) = a_1 + (\frac{a_2 - a_1}{w})z$ and

$$R^{-1}(z) = a_3 + (\frac{a_4 - a_3}{w})z$$

$$\Rightarrow R(\widetilde{A}) = \frac{w(a_1 + a_2 + a_3 + a_4)}{4} \quad \text{……..…..(2)}$$

(ii) . Let a convex triangular fuzzy number $\widetilde{A}(z) = \widetilde{A}(a_1, a_2, a_3; w)$,

Then the Ranking Index is defined by

$$R(\widetilde{A}) = \int_0^w \frac{(L^{-1}(z) + R^{-1}(z))}{2} dz$$

Where $L^{-1}(z) = a_1 + (\frac{a_2 - a_1}{w})z$ and

$$R^{-1}(z) = a_2 + (\frac{a_3 - a_2}{w})z$$

$$\Rightarrow R(\widetilde{A}) = \frac{w(a_1 + 2a_2 + a_3)}{4} \quad \text{…….(3)}$$

NUMERICAL EXAMPLE

Consider a car service station system in which cars are arrived by batches according to a Poisson process. In that situation, all cars except service processing car are needed to hold in service station. So the holding cost exists. The manager of the system wishes to evaluate how much the total expected cost for whole service. It is evident that this system follows, $FM^{[K]}/FM/1$ and the total expected cost of the system can be derived by the proposed procedure.

A . For Trapezoidal fuzzy number

(i). (When w =0.5,that is generalized fuzzy numbers will give moderate value.)

Suppose the arrival rate, service rate, batch size rate, service cost rate and holding cost rates are trapezoidal fuzzy numbers represented by

$\widetilde{\lambda}$ = [1,2,3,4;0.5], $\widetilde{\mu}$ = [2,3,4,5;0.5],

$\widetilde{k}$ = [5,6,7,8;0.5], $\widetilde{C}_S$ = [1000,2000,3000,4000;0.5] and $\widetilde{C}_h$ = [30,40,50,60;0.5] .

According to (2)

The ranking index of $\widetilde{\lambda}$ is R($\widetilde{\lambda}$) = R(1,2,3,4;0.5)

$$= \frac{0.5(1+2+3+4)}{4} = 1.25$$

Proceeding similarly we get

$R(\widetilde{\mu}) = R(2,3,4,5;0.5) = \frac{0.5(2+3+4+5)}{4} = 1.75$

$R(\widetilde{k}) = R(5,6,7,8;0.5) = \frac{0.5(5+6+7+8)}{4} = 3.25$

$R(\widetilde{C}_S) = R(1000,2000,3000,4000;0.5)$

$$= \frac{0.5(1000+2000+3000+4000)}{4} = 1250$$

$R(\widetilde{C}_S) = R(30,40,50,60;0.5)$

$$= \frac{0.5(30+40+50+60)}{4} = 22.5$$

According to (1)

The total expected cost

$$= m + wn\left[\frac{E(w^2)+E(w)}{2[v-uE(w)]}\right]$$

$$= 1250 + 3.25(22.5)\left[\frac{(3.25^2)+3.25}{2[1.75-1.25(3.25)]}\right]$$

= Rs 1031.43

(ii). (When w = 1,that is normalized fuzzy numbers will give optimistic value.)

Suppose the arrival rate, service rate, batch size rate, service cost rate and holding cost rates are trapezoidal fuzzy numbers represented by

$\widetilde{\lambda}$ = [1,2,3,4;1], $\widetilde{\mu}$ = [2,3,4,5;1], $\widetilde{k}$ = [5,6,7,8;1], $\widetilde{C}_S$ = [1000,2000,3000,4000;1] and $\widetilde{C}_h$ = [30,40,50,60;1] .

According to (2)

The ranking index of $\widetilde{\lambda}$ is $R(\widetilde{\lambda}) = R(1,2,3,4;1)$

$$= \frac{1(1+2+3+4)}{4} = 2.5$$

Proceeding similarly we get

$R(\widetilde{\mu}) = R(2,3,4,5;1) = \frac{1(2+3+4+5)}{4} = 3.5$

$R(\widetilde{k}) = R(5,6,7,8;1) = \frac{1(5+6+7+8)}{4} = 6.5$

$R(\widetilde{C}_S) = R(1000,2000,3000,4000;1)$

$$= \frac{1(1000+2000+3000+4000)}{4} = 2500$$

$R(\widetilde{C}_S) = R(30,40,50,60;1)$

$$= \frac{1(30+40+50+60)}{4} = 45$$

According to (1)

The total expected cost

$$= m + wn\left[\frac{E(w^2)+E(w)}{2[v-uE(w)]}\right]$$

$$= 2500 + 6.5(45)\left[\frac{(6.5^2)+6.5}{2[3.5-2.5(6.5)]}\right]$$

= Rs 1941.30

B . For Triangular fuzzy number

(i). (When w =0.5,that is generalized fuzzy numbers will give moderate value.)

Suppose the arrival rate, service rate, batch size rate, service cost rate and holding cost rates are trapezoidal fuzzy numbers represented by

$\widetilde{\lambda}$ = [1,2,4;0.5], $\widetilde{\mu}$ = [2,3,5;0.5], $\widetilde{k}$ = [5,6,8;0.5], $\widetilde{C}_S$ = [1000,2000,4000;0.5] and $\widetilde{C}_h$ = [30,40,60;0.5]

According to (2)

The ranking index of $\widetilde{\lambda}$ is $R(\widetilde{\lambda}) = R(1,2,4;0.5)$

$$= \frac{0.5(1+4+4)}{4} = 1.125$$

Proceeding similarly we get

$R(\widetilde{\mu}) = R(2,3,5;0.5) = \frac{0.5(2+6+5)}{4} = 1.625$

$R(\widetilde{k}) = R(5,6,8;0.5) = \frac{0.5(5+12+8)}{4} = 3.125$

$R(\widetilde{C}_S) = R(1000,2000,4000;0.5)$

$$= \frac{0.5(1000+4000+4000)}{4} = 1125$$

$R(\widetilde{C}_S) = R(30,40,60;0.5) = \frac{0.5(30+80+60)}{4}$

$= 21.25$

According to (1)

The total expected cost

$$= m + wn\left[\frac{E(w^2)+E(w)}{2[v-uE(w)]}\right]$$

$$= 1125 + 3.125(21.25)\left[\frac{(3.125^2)+3.125}{2[1.625-1.125(3.125)]}\right]$$

= Rs 898.58

(ii). (When w = 1,that is normalized fuzzy numbers will give optimistic value.)

Suppose the arrival rate, service rate, batch size rate, service cost rate and holding cost rates are trapezoidal fuzzy numbers represented by

$\widetilde{\lambda}$ = [1,2,4;1], $\widetilde{\mu}$ = [2,3,5;1], $\widetilde{k}$ = [5,6,8;1],

$\widetilde{C}_S$ = [1000,2000,4000;1] and $\widetilde{C}_h$ = [30,40,60;1] .

According to (2)

The ranking index of $\widetilde{\lambda}$ is R($\widetilde{\lambda}$) = R(1,2,4;1)

$$= \frac{1(1+4+4)}{4} = 2.25$$

Proceeding similarly we get

$$R(\widetilde{\mu}) = R(2,3,5;1) = \frac{1(2+6+5)}{4} = 3.25$$

$$R(\widetilde{k}) = R(5,6,8;1) = \frac{1(5+12+8)}{4} = 6.25$$

$R(\widetilde{C}_S) = R(1000,2000,4000;1)$

$$= \frac{1(1000+4000+4000)}{4} = 2250$$

$$R(\widetilde{C}_S) = R(30,40,60;1) = \frac{1(30+80+60)}{4} = 42.5$$

According to (1)

The total expected cost

$$= m + wn\left[\frac{E(w^2)+E(w)}{2[v-uE(w)]}\right]$$

$$= 2250 + 6.25(42.5)\left[\frac{(6.25^2)+6.25}{2[3.25-2.25(6.25)]}\right]$$

= Rs 1693.26

CONCLUSION

In this paper, Fuzzy set theory has been applied to a batch arrival queuing model. Cost measures of fuzzy batch arrival queuing model have been used in operations and service mechanism for evaluating system performance. Moreover, the fuzzy problem has been transformed into crisp problem using ranking indices. Since the performance measures such as the total expected cost(TEC) are crisp values, the manager can take the best and optimum decisions. We conclude that the solution of fuzzy problems can be obtained by Ranking function method very effectively. The approach proposed in this paper provides practical information for system manager and practitioners.

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