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String Cosmological Model with Electromagnetic Field in General Relativity

Mathematics

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Nimkar A. S

Department of Mathematics, Shri.Dr.R.G.Rathod Arts & Science College, Murtizapur , Dist. Akola.(M.S.) India.

ABSTRACT

A string cosmological models with magnetic field along X and Z-axis are investigated in the context of Kaluza-Klein space time. To obtain a determinant solution, equation of state for string model and relation between metric potential is considered. The physical and geometrical aspects are also discussed.

Introduction: A unified formulation of Einstein's theory of gravitation and Maxwell's theory of electromagnetism in 4-dimensional space time was first proposed by Kaluza(1921) and Klein(1926) using 5-dimensional geometry.

The cosmological models formulated for higher dimensional space-time play a vital role in many aspects of early stage of cosmological problems, which are one of the frontier areas of research to unify gravity with other forces in nature. The study of higher dimensional space-time provides an idea that our universe was much smaller at an earlier stage of evolution than observed today and extra dimensions were contracted to a very small dimension. The detection of extra dimensions in current experiments is beyond those four dimensions observed so far. The possibility of extra dimensions in the space-time has attracted various researchers to the field of cosmology. Ponce(1998), Chi(1990),Fukui(1993),Liu and Wesson(1994), Coley(1994) have studied Kaluza Klein cosmological models with different matters. Palatnik(2007) constructed Schwarschild solution for 3 space and n time dimensions in Kaluza-Klein theory. Friedman Robertson Walker spatially homogeneous and isotropic models are widely considered as good approximation of the present and early stage of the universe. Two-fluid FRW models of the universe have been investigated where one fluid is the radiation field corresponding to the observed cosmic background radiation, while a perfect fluid is chosen to represent the matter content of the universe Coley and Tupper,1986 ,Reddy D.R.K. and Naidu R.L.(2007) have discuss string cosmological models in Kaluza-Klein five dimensional space time in the framework of Saez and Ballestr (1985) scalar tensor theory of gravitation

Adhav *et.al.* (2008) discussed string cloud and domain wall with quark matter in n-dimensional Kluza-klein cosmological model. Jacobs(1968,1969),Collins(1972) have investigated magnetized cosmological models for perfect fluid distribution in general relativity. Misra *et.al.* (1962) have obtained the analogous relation between some components of metric and electromagnetic potentials for source free Einstein–Maxwell field described by Einstein-Rosen metric. katore S.D. *et.al.*(2013) have discussed Kluza-klein cosmological model four perfect fluid and dark energy. Very recently Sanjay Oli(2016) studied cosmological model variable G and $\wedge$ in Kaluza-Klein space time.

In this paper, we discussed Kaluza-Klein string cosmological model with Electromagnetic field along X and Z-axis in General theory of relativity.

Field Equations:

We consider five dimensional Kaluza-Klein space time in the form

$$ds^2 = dt^2 - R^2\left(dx^2 + dy^2 + dz^2\right) - A^2 d\psi^2 \tag{1}$$

The energy momentum tensor of the source is given by

$$T_{ij} = T_{ij}^S + T_{ij}^E \tag{2}$$

Where T_{ij}^S is the energy momentum tensor for Cosmic String and T_{ij}^E denotes the energy momentum tensor for the Electromagnetic field.

Thus the energy momentum tensor of the cosmic string is

$$T_{ij}^S = \rho\ u_i u^j - \lambda\ x_i x^j \tag{3}$$

Here ρ is the rest energy density of cloud of strings with particles attached to them, $\rho_p + \lambda, \rho_p$ being the rest energy density of particles attached to the strings and λ the tension density of the system of strings. Also

$$u_i u^i = -x^i x_i = 1\ ,\ u_i x^i = 0 \tag{4}$$

We consider $\rho_p + \lambda, \rho_p$ (5)

where ρ_p is the rest energy density of the particles attached to the string. Here p and ρ are the functions of t only.

The energy-momentum tensor T_{ij} in co-moving coordinates for cosmic string is given by

$$T_0^0 = \rho,\ T_1^1 = \lambda,\quad T_2^2 = T_3^3 = T_4^4 = 0 \tag{6}$$

and The energy momentum tensor of the electromagnetic field is

$$T_{ij}^E = \frac{1}{4\pi}\left[-F_{is}F_{jp}g^{sp} + \frac{1}{4}g_{ij}F_{sp}F^{sp}\right] \tag{7}$$

Where F_{ij} is the Electromagnetic field tensor defined by the four potential A_i as

$$F_{ij} = A_{j,i} - A_{i,j} \tag{8}$$

The energy-momentum tensor T_{ij} in co-moving co-ordinates for Electromagnetic field along X-axis is given by

$$T_0^0 = \frac{1}{8\pi}\frac{H^2}{R^4} = T_1^1 = T_4^4,\quad T_2^2 = T_3^3 = -\frac{1}{8\pi}\frac{H^2}{R^4} \tag{9}$$

Einstein's field equations in general relativity are

$$R_i^j - \frac{1}{2}R = -T_i^j \tag{10}$$

Here, we consider geometrized units so that $8\pi G = C = 1$

The field equation (10) for the metric (1) with the help of (2) -(9) can be written as

$$3\left(\frac{\dot{R}}{R}\right)^2 + 3\frac{\dot{R}\dot{A}}{RA} = \rho + \frac{1}{8\pi}\frac{H^2}{R^4} \tag{11}$$

$$2\left(\frac{\ddot{R}}{R}\right) + \left(\frac{\dot{R}}{R}\right)^2 + 2\frac{\dot{R}\dot{A}}{RA} + \frac{\ddot{A}}{A} = \lambda + \frac{1}{8\pi}\frac{H^2}{R^4} \tag{12}$$

$$2\left(\frac{\ddot{R}}{R}\right) + \left(\frac{\dot{R}}{R}\right)^2 + 2\frac{\dot{R}\dot{A}}{RA} + \frac{\ddot{A}}{A} = -\frac{1}{8\pi}\frac{H^2}{R^4} \tag{13}$$

$$3\left(\frac{\ddot{R}}{R}\right) + 3\left(\frac{\dot{R}}{R}\right)^2 = \frac{1}{8\pi}\frac{H^2}{R^4} \tag{14}$$

Here over head dot represent partial differentiation with respect to t.

Solution of Field Equations:

The set of field equations (11)– (14) are four independent field equations in five unknown R, A, ρ, λ and H. Hence to get a determinate solution one has to assume relation a physical or mathematical condition. In the literature (Letelier 1983) and the relation between metric potential $A = R^n$

Geometric string ($\rho = \lambda$)

Using geometric string equation, the solution of field equation (11)-(14) are

$$R = k_4(k_2t + k_3)^{\frac{1}{k_1+1}}, \text{where } k_4 = (k_1+1)^{\frac{1}{k_1+1}} \quad (15)$$

$$A = k_5(k_2t + k_3)^{\frac{n}{k_1+1}}, \text{where } k_5 = k_4^{\,n} \quad (16)$$

$$\rho = \lambda = -\frac{H^2}{4\pi}\frac{1}{(k_6)(k_2t + k_3)^{\frac{4}{k_1+1}}}, \text{ where } k_6 = k_4^{\,4} \quad (17)$$

$$H^2 = -\frac{24(n+1)k_6k_2^{\,2}}{(k_1+1)^2}\frac{1}{T\frac{2(k_1-1)}{k_1+1}} \quad (18)$$

The energy condition indicates that $\rho>0$ and $\rho_p>0$. The rest energy denity ρ ,string tension density λ and particle density ρ_p tend to zero as time T increases indefinitely.

Thus Kaluza-Klein cosmological model corresponding to the above solution can be written as

$$ds^2 = dt^2 - k_4^{\,2}(k_2t + k_3)^{\frac{2}{k_1+1}}\left[dx^2 + dy^2 + dz^2\right] - k_5^{\,2}(k_2t + k_3)^{\frac{2n}{k_1+1}}d\psi^2$$

Through a proper choice of coordinates and constants the above model can be written as

$$ds^2 = \frac{dT^2}{k_1^{\,2}} - k_4^{\,2}(T)^{\frac{2}{k_1+1}}\left[dx^2 + dy^2 + dz^2\right] - k_5^{\,2}(T)^{\frac{2n}{k_1+1}}d\psi^2 \quad (19)$$

Physical and kinematical property:

The tension density ρ and string density λ in the model are given by equations (17). The model given by equation (19) represents an exact cosmological model in Kaluza-Klein five dimensional in the framework of cosmic string. The model has no initial singularity.

Spatial volume: $$V = k_4^{\,3}k_5(T)^{\frac{n+3}{k_1+1}} \quad (20)$$

Scalar Expansion: $$\theta = \frac{1}{3}\left[\frac{(3+n)k_2}{k_1+1}\right]\frac{1}{T} \quad (21)$$

Shear Scalar: $$\sigma^2 = \frac{2}{27}\frac{(n+3)^2k_2^{\,2}}{(k_1+1)^2}\frac{1}{T^2} \quad (22)$$

Deceleration parameter: $q>0$ (23)

The spatial volume of the model given by (20) shows that the anisotropic expansion of the universe (19) with time. For the model (19), the expansion scalar θ and shear scalar σ^2 tends to zero as $t\to\infty$. The positive value of the deceleration parameter indicates the model decelerates in the standard way.

Also

$$\lim_{t\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0 \quad (24)$$

The model does not approach isotropy for large values of t

Conclusion:

In this paper, we have obtained a Kaluza-Klein cosmological model in presence of cosmic string and Electromagnetic field in general relativity. It is observed that the models in Geometric string ($\rho=\lambda$), Reddy string ($\rho+\lambda=0$) and ($\rho=3\lambda$) are similar and behave alike.

Along the X-axis, tension density and string density of cosmic string are present but along Z-axis tension density and string density of cosmic string are absent. Therefore in a Kaluza-Klein space time only cosmological model with magnetic field are presents along X-axis.The models are free from initial singularities and they are expanding, shearing and non-rotating in the standard way.

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