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Research Paper

Transient Analysis of Two-Phase N-Policy M/M/1 Queueing System with Server Start-Up and Second Optional Service and Breakdowns

Statistics

KEYWORDS : Two-Phase, Transient behavior, Start-up and Optional service.

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ABSTRACT

Two Phase Single server vacation queues have been extended to different areas. This paper deals with the transient analysis of two-phase N-Policy M/M/1 queue with server start-up and second optional service. The paper differs from the existing literature because it investigates the waiting time of the two-phase queueing system with transient behavior. Numerical illustrations are presented to examine the effect of different parameters in the system.

1 Introduction

In real life, we may encounter the queues where the arriving customers require first phase service and some of them may require subsidiary service provided by the server. For example, students joining in a particular department of a university want to complete their master's program, but some of them may join the Ph.D. program soon after completing the master's program.

The system is utilized as a scheduling problem, for which all arriving ships at a port may need unloading but only some of them require reloading service after the unloading process. There are many other situations like manufacturing processes which involves two phases of service. Where ever the situation arises one can use this type of models. In this paper we investigate the performance measures of the queueing system in different states like expected waiting time in the system, explicit in time. In some situations, the server often requires a random startup time before starting the service. This corresponds to the pre-work of the server before starting the service. The N-policy M/M/1 queuing system with exponential start-up time was first considered by Baker [2]. The concept of N-policy was first introduced by Yadin and Naor (1963)[8]. Yang et.al. [7] studied Steady-state probability of the randomized server with second optional service, server breakdowns and startup. Medhi [4] proposed single server Poisson input queue with Second Optional Service and find the mean queue length and the mean waiting time. Madan [3], considers an M/G/1 system with a second optional service, which some of the units may require second service immediately after completion of the first essential service. Some interesting examples of such situations are given there. He considers second channel service time is to be independent exponential. The case where both the FES and SOS are assumed to be an exponentially distributed random variables which is said to be so-called Coxian distribution. Vasanta Kumar, V. and Chandan, K. (2007)[5] studied the Two-Phase Queueing System with N-Policy where all customers receive batch service in a first phase and individual optional service in the second phase. They derived explicit expressions in different states of the customer in the system. A Mijit and G Gupur[1], studied about the Time-dependent Solution of the M/m/1 Queueing Model with Optional Second Service. V.N.Rama Devi et.al.[6] derived explicit expressions for various performance measures of Two-Phase M/Hk/1 Queueing System with

Transient behavior. Also the sensitivity analysis has been carried out to examine the effect of different parameters in the system.

2. The Model Formulation

In this paper, we consider an M/M/1 queueing system where the arrival occurs according to a Poisson process with arrival λ in a two phase service system.

Every time server is turned off if the system empties. As soon as the queue size exceeds N (threshold) ($\geq$1), the server turned on and begins the service to serve the first phase of essential service (FES) for all the units. The discipline of service is FCFS. After completion of FES of a unit, it may leave the system with certain probability (1-θ) or may wait for a Second phase of Optional Service (SOS) in an additional optional channel with probability θ ($0 \leq \theta \leq 1$). The server turns off the system if the system is empty. The system will be turned on when N customers waiting for the service in the system. Further, it is assumed that the same server serves both the channels.

3. Governing Equations of the Queueing Model

We consider the M/M/1 queueing system with two phases of service where the unreliable server operates under N-policy with the following assumptions:

1. Customers are assumed to arrive according to Poisson process with mean arrival rate λ. Customers will get the service in the order in which they arrive.
2. The service is in two phases. Here, the first phase is essential batch service to all customers in the queue. On completion of batch service, the server immediately proceeds to the second phase optional service to serve those who are interested customers in the batch individually with probability 'p'. Batch service time is assumed to follow exponential distribution with mean $1/\beta$ which is independent of batch size. Individual service will be provided on completion of batch service and service times according to exponential distribution. On completion of individual service, the server returns to the batch queue to serve those who have arrived. If the customers are waiting, the server starts the batch service followed by individual optional those who required in the batch. If no customer is waiting the server takes a vacation.
3. Whenever the system is empty, the server is turned off. The total number of arrivals in the queue reaches the pre-determined threshold N, the server is turned on each time and is temporarily unavailable for the waiting customers. The server needs a random startup time, follows an exponential distribution with mean $1/\theta$. After finishes the startup, server starts serving the first phase waiting customers.
4. The customers arrive during the batch service are also allowed to join the queue to get the service.
5. The breakdowns are generated by Poisson process with rates ξ_1 for the first phase of service and α_1 for the second phase of service. When the server fails it is immediately repaired at a repair rate ξ_2 in first phase and α_2 in second phase, repair times are assumed to follow an

exponentially distribution. The server immediately resumes the concerned service after repair.

We use the following notations:

$p_{0,m,0}$ = The probability that there are m customers in the batch queue when the server is on vacation, where m= 0,1,2,…,N-1.

$p_{1,n,0}$ = The probability that there are m customers in the batch queue when the server is doing pre-service (startup work), where m = N, N+1,N+2,… ,L.

$p_{2,m,0}$ = The probability that there are m customers in the batch queue when the server is in batch service where m = 1,2, 3,… ,L.

$p_{3,m,0}$ = The probability that there are m customers in batch queue when the server is working but found to be broken down, where m = 1,2,3,…,L.

$p_{4,m,n}$ = The probability that there are m customers in the batch queue and n customers in individual queue when the server is in individual service, where m=0,1,2… ; n =1,2,…,k and m+n=L.

$p_{5,m,n}$ = The probability that there are m customers in the batch queue and n customers in individual queue when the server is working but found to be broken down, where m= 0,1,2,… ; n = 1,2,3, …k and m+n=L.

Thus, at an arbitrary time *t*, the system's behavior is given by the following set of Differential equations:

$$\frac{dp_{0,m,0}(t)}{dt} = -\lambda p_{0,m,0}(t) + \mu p_{4,0,1}(t) + \beta(1-p)p_{2,1,0};\ m = 0. \quad (1)$$

$$\frac{dp_{0,m,0}(t)}{dt} = -\lambda p_{0,m,0}(t) + \lambda p_{0,m-1,0}(t);\quad 1 \le m \le N-1. \quad (2)$$

$$\frac{dp_{1,m,0}(t)}{dt} = -(\lambda+\theta)p_{1,m,0}(t) + \lambda p_{0,m-1,0,0}(t)\ ;\ m=N \quad (3)$$

$$\frac{dp_{1,m,0}(t)}{dt} = -(\lambda+\theta)p_{1,m,0}(t) + \lambda p_{1,m-1,0}(t)\ ; N+1 < m < L-1. \quad (4)$$

$$\frac{dp_{1,m,0}(t)}{dt} = -(\theta)p_{1,m,0}(t) + \lambda p_{1,m-1,0}(t)\ ; m = L. \quad (5)$$

$$\frac{dp_{2,m,0}(t)}{dt} - (\lambda+\beta+\xi_1)p_{2,m,0}(t) + \lambda p_{2,m-1,0}(t) + \beta(1-p)p_{2,m+1,0}(t) + \mu p_{4,m,1}(t) + \xi_2 p_{3,m,0}(t);\ 1 \le m \le N-1. \quad (6)$$

$$\frac{dp_{2,m,0}(t)}{dt} = -(\lambda+\beta+\xi_1)p_{2,m,0}(t) + \lambda p_{2,m-1,0}(t) + \beta(1-p)p_{2,m+1,0}(t) + \mu p_{4,m,1}(t) + \xi_2 p_{3,m,0}(t) + \theta p_{1,m,0}(t);\ N < m < L-1. \quad (7)$$

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$$\frac{dp_{2,m,0}(t)}{dt} = -(\beta+\xi_1)p_{2,m,0} + \lambda p_{2,m-1,0}(t) + \xi_2 p_{3,m,0}(t) + \theta p_{1,m,0}(t); \quad m = L. \tag{8}$$

$$\frac{dp_{3,m,0}(t)}{dt} = -(\lambda+\xi_2)p_{3,m,0}(t) + \lambda p_{3,m-1,0}(t) + \xi_1 p_{2,m,0}(t); \quad 1 \le m \le L-1. \tag{9}$$

$$\frac{dp_{3,m,0}(t)}{dt} = -(\xi_2)p_{3,m,0} + \lambda p_{3,m-1,0}(t) + \xi_1 p_{2,m,0}(t); \quad m = L. \tag{10}$$

$$\frac{dp_{4,0,n}(t)}{dt} = -(\lambda+\alpha_1+\mu)p_{4,0,n}(t) + \mu p_{4,0,n+1}(t) + \beta p p_{2,n,0}(t) + \alpha_2 p_{5,0,n}; 1 \le n \le L-1, \tag{11}$$

$$\frac{dp_{4,0,n}(t)}{dt} = -(\alpha_1+\mu)p_{4,0,n}(t) + \beta p p_{2,n,0}(t) + \alpha_2 p_{5,0,n}; \; n = L, \tag{12}$$

$$\frac{dp_{4,m,n}(t)}{dt} = -(\lambda+\alpha_1+\mu)p_{4,m,n}(t) + \mu p_{4,m,n+1}(t) + \lambda p_{4,m-1,n}(t) + \alpha_2 p_{5,m,n}; m,n \ge 1, m+n \le L-1. \tag{13}$$

$$\frac{dp_{4,m,n}(t)}{dt} = -(\alpha_1+\mu)p_{4,m,n}(t) + \mu p_{4,m,n+1}(t) + \lambda p_{4,m-1,n}(t) + \alpha_2 p_{5,m,n}; \; m,n \ge 1, m+n = L. \tag{14}$$

$$\frac{dp_{5,0,n}(t)}{dt} = -(\lambda+\alpha_2)p_{5,0,n}(t) + \alpha_1 p_{4,0,n}; 1 \le n \le L-1. \tag{15}$$

$$\frac{dp_{5,0,n}(t)}{dt} = -(\alpha_2)p_{5,0,n}(t) + \alpha_1 p_{4,0,n}; \; n = L. \tag{16}$$

$$\frac{dp_{5,m,n}(t)}{dt} = -(\lambda+\alpha_2)p_{5,m,n}(t) + \lambda p_{5,m-1,n}(t) + \alpha_1 p_{4,m,n}; \; m,n \ge 1, m+n \le L-1. \tag{17}$$

$$\frac{dp_{5,m,n}(t)}{dt} = -(\alpha_2)p_{5,m,n}(t) + \alpha_1 p_{4,m,n}; \quad m,n \ge 1, m+n = L. \tag{18}$$

4. Performance Measures

Some performance measures are calculated to predict the system's behavior using the probabilities obtained in previous section with the help of Runge-Kutta method.

1. Probability of server being idle at time *t*: $I(t) = \sum_{q=0}^{N-1} p_{0,q0,0}(t) + \sum_{q=N}^{L} p_{1,q,0,0}(t)$
2. Probability of server being busy at time *t*: $B(t) = \sum_{q=1}^{L} p_{2,q,0,0}(t) + \sum_{q=1}^{L}\sum_{s=1}^{L-q}\sum_{r=1}^{k} p_{4,q,r,s}(t)$
3. Probability of server under repair at time *t*: $R(t) = \sum_{q=1}^{L} p_{3,q,0,0}(t) + \sum_{q=1}^{L}\sum_{s=1}^{L-q}\sum_{r=1}^{k} p_{5,q,r,s}(t)$
4. Expected number of customers in the system: $L_S = \sum_{q=1}^{L}\sum_{s=1}^{L-q}\sum_{r=1}^{k}\sum_{i=0}^{5}(q+s)p_{i,q,r,s}(t)$
5. Mean waiting time in the system : $W_S = {}^{L_S}/_{\lambda}$

5. Numerical Results

The obtained results are numerically evaluated based on specific parameters. The effect of various parameters on the system performance measures such as expected number of customers in the system and mean waiting time in the system are studied. MATLAB software is used to develop the computational program.

The effect of different parameters on the mean waiting time is summarized in Tables 5.1-5.11. In all numerical computations, the model parameters are taken as

$N = 2$, s=7, λ =1.0, β=1.8, ξ_1=1.2, ξ_2=2, α_1=1.6, α_2=2.0, θ=2.0, μ=2.1 and p=0.5.

Table 5.1: Effect of λ on Mean Waiting time

t \ λ	1.0	1.2	1.4	1.6	1.8
0.2	0.799	0.825	0.8508	0.8763	0.9016
0.4	0.9246	0.9729	1.0199	1.0654	1.1094
0.6	1.0373	1.102	1.1631	1.2206	1.2746
0.8	1.1334	1.2078	1.2758	1.3377	1.3939
1	1.2111	1.2892	1.3581	1.4188	1.4721
1.2	1.2709	1.3478	1.4135	1.4696	1.5174
1.4	1.3139	1.3866	1.4466	1.4966	1.5385
1.6	1.3424	1.4088	1.4624	1.5062	1.5428
1.8	1.3586	1.418	1.4652	1.5036	1.536
2	1.364	1.4172	1.4587	1.4928	1.522

Table 5.2: Effect of θ on Mean Waiting time

t \ θ	1	2	3	4	5
0.2	0.7991	0.799	0.7988	0.7987	0.7987
0.4	0.9259	0.9246	0.9234	0.9224	0.9215
0.6	1.0423	1.0373	1.0334	1.0301	1.0275
0.8	1.1451	1.1334	1.1247	1.1181	1.113
1	1.2324	1.2111	1.1964	1.1858	1.1779
1.2	1.3039	1.2709	1.2494	1.2346	1.2241
1.4	1.36	1.3139	1.2857	1.2671	1.2543
1.6	1.4021	1.3424	1.3078	1.286	1.2713
1.8	1.4315	1.3586	1.3185	1.2941	1.278
2	1.4501	1.3648	1.3201	1.2937	1.2766

Table 5.3: Effect of β in Mean Waiting time

t \ β	1.4	1.6	1.8	2	2.2
0.2	0.799	0.799	0.799	0.7989	0.7989
0.4	0.9251	0.9248	0.9246	0.9243	0.924
0.6	1.0395	1.0384	1.0373	1.0363	1.0354
0.8	1.1384	1.1358	1.1334	1.1311	1.1289
1	1.2203	1.2156	1.2111	1.207	1.2031
1 .2	1.285	1.2777	1.2709	1.2645	1.2586
1.4	1.3338	1.3234	1.3139	1.3052	1.2971
1.6	1.3682	1.3547	1.3424	1.3312	1.321
1.8	1.3904	1.3737	1.3586	1.345	1.3326
2	1.4022	1.3825	1.3648	1.3489	1.3346

Table 5.4: Effect of ξ_1 in Mean Waiting time

t \ ξ_1	0.4	0.6	0.8	1	1.2
0.2	0.7989	0.7989	0.799	0.799	0.799
0.4	0.9244	0.9244	0.9245	0.9245	0.9246
0.6	1.0365	1.0367	1.037	1.0371	1.0373
0.8	1.131	1.1317	1.1323	1.1328	1.1334
1	1.2061	1.2075	1.2087	1.21	1.2111
1.2	1.262	1.2644	1.2666	1.2688	1.2709
1.4	1.3003	1.304	1.3075	1.3108	1.3139
1.6	1.3234	1.3285	1.3334	1.338	1.3424
1.8	1.3338	1.3405	1.3469	1.3529	1.3586
2	1.3341	1.3424	1.3503	1.3577	1.3648

Table 5.5: Effect of ξ_2 in Mean Waiting time

t \ ξ_2	1	1.5	2	2.5	3
0.2	0.799	0.799	0.799	0.799	0.799
0.4	0.9246	0.9246	0.9246	0.9245	0.9245
0.6	1.0375	1.0374	1.0373	1.0373	1.0372
0.8	1.1339	1.1336	1.1334	1.1332	1.133
1	1.2124	1.2117	1.2111	1.2106	1.2102
1.2	1.2734	1.2721	1.2709	1.2698	1.2689
1.4	1.3186	1.316	1.3139	1.3121	1.3106
1.6	1.3498	1.3458	1.3424	1.3396	1.3373
1.8	1.3694	1.3634	1.3586	1.3547	1.3514
2	1.3794	1.3713	1.3648	1.3595	1.3553

Table 5.6: Effect of μ in Mean Waiting time

t \ μ	1.4	1.6	1.8	2	2.2
0.2	0.799	0.799	0.799	0.799	0.799
0.4	0.9246	0.9246	0.9246	0.9246	0.9246
0.6	1.0375	1.0375	1.0374	1.0374	1.0373
0.8	1.1339	1.1338	1.1336	1.1335	1.1333
1	1.2124	1.212	1.2117	1.2113	1.211
1.2	1.2733	1.2725	1.2718	1.2712	1.2705
1.4	1.318	1.3168	1.3156	1.3145	1.3134
1.6	1.3487	1.3468	1.345	1.3432	1.3416
1.8	1.3677	1.3649	1.3623	1.3598	1.3575
2	1.3773	1.3734	1.3698	1.3664	1.3632

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Table 5.7: Effect of α_1 in Mean Waiting time

α_1 / t	1.2	1.4	1.6	1.8	2
0.2	0.799	0.799	0.799	0.799	0.799
0.4	0.9246	0.9246	0.9246	0.9246	0.9246
0.6	1.0373	1.0373	1.0373	1.0373	1.0374
0.8	1.1333	1.1334	1.1334	1.1334	1.1335
1	1.211	1.2111	1.2111	1.2112	1.2113
1.2	1.2705	1.2707	1.2709	1.271	1.2712
1.4	1.3132	1.3136	1.3139	1.3143	1.3146
1.6	1.3412	1.3418	1.3424	1.343	1.3435
1.8	1.3568	1.3577	1.3586	1.3594	1.3602
2	1.3622	1.3635	1.3648	1.3659	1.3671

Table 5.8: Effect of α_2 in Mean Waiting time

α_2 / t	1.5	2	2.5	3	3.5
0.2	0.799	0.799	0.799	0.799	0.799
0.4	0.9246	0.9246	0.9246	0.9246	0.9246
0.6	1.0373	1.0373	1.0373	1.0373	1.0373
0.8	1.1334	1.1334	1.1334	1.1334	1.1333
1	1.2112	1.2111	1.2111	1.2111	1.211
1.2	1.271	1.2709	1.2707	1.2706	1.2705
1.4	1.3142	1.3139	1.3137	1.3135	1.3133
1.6	1.3429	1.3424	1.342	1.3416	1.3413
1.8	1.3595	1.3586	1.3579	1.3573	1.3568
2	1.3661	1.3648	1.3637	1.3628	1.362

Table 5.9: Effect of p in Mean Waiting time

p / t	0.1	0.2	0.3	0.4	0.5
0.2	0.7989	0.7989	0.7989	0.7989	0.799
0.4	0.9233	0.9236	0.9239	0.9242	0.9246
0.6	1.0323	1.0335	1.0348	1.0361	1.0373
0.8	1.1205	1.1237	1.1269	1.1302	1.1334
1	1.1854	1.1918	1.1983	1.2047	1.2111
1.2	1.227	1.238	1.2489	1.2599	1.2709
1.4	1.2468	1.2635	1.2803	1.2971	1.3139
1.6	1.2471	1.2709	1.2947	1.3186	1.3424
1.8	1.231	1.2629	1.2947	1.3267	1.3586
2	1.2016	1.2423	1.283	1.3239	1.3648

From the above table, it is observed that the waiting time increases with the increase in λ, decreases with the increase inθ, increase in β, increases with the increase in ξ_1, decreases with the increase in ξ_2, decreases with the increase in μ, increases with the increase in α1, decreases with the increase in α_2 and increases with the increase in p.

6. Conclusion

We derived performance measures of the queueing system in different states in different time points in the system and also gave numerically evaluated results based on specific parameters.

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