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Unified Integrals Formulas Involving Product of Srivastava's Polynomials and Generalized Bessel-Maitland Function

Mathematics

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D.L. Suthar — Department of Mathematics, Wollo University, Dessie,P.O. Box: 1145, South Wollo, Amhara Region, (Ethiopia).

G.V. Reddy — Department of Mathematics, Wollo University, Dessie, P.O. Box: 1145, South Wollo, Amhara Region, (Ethiopia).

Temesgen Tsegaye — Department of Mathematics, Wollo University, Dessie, P.O. Box: 1145, South Wollo, Amhara Region, (Ethiopia).

ABSTRACT

The aim of this paper is to establish two general finite integral formulas involving product of Srivastava's polynomials and generalized Bessel-Maitland functions $J^{\mu,\gamma}_{\nu,q}(z)$. The result given in terms of generalized (Wright's) hypergeometric functions ${}_p\Psi_q$ and generalized hypergeometric functions ${}_pF_q$. These results are obtained with the help of finite integral due to Lavoie and Trottier. Some interesting special cases involving Bessel-Maitland function, Struve's functions, Bessel functions, generalized Bessel functions, Wright function, generalized Mittag-Leffler functions are deduced.

1. Introduction and Preliminaries

In Applied sciences, many important functions are defined via improper integrals or series (or finite products). The general name of these important functions knows as special functions. In special function, one of the most important function (Bessel function) is widely used in physics and engineering; therefore, they are of interest to physicists and engineers as well as mathematician. In recent year, a remarkably large number of integral formulas involving a variety of special function have been developed by many authors as Agarwal et al. [2, 3], Brychkov[4], Choi et al. [5], Choi and Agarwal [6, 7], Manaria et al. [13, 14], Nisar et al. [16], and Unnima et al. [19]. We aim at presenting two generalized integral formulas involving the Bessel-Maitland function, which are expressed in term of the generalized (Wright's) hypergeometric and generalized hypergeometric functions.

For our purpose, we begin by recalling some known function and earlier works. The Bessel-Maitland function $J^{\mu}_{\nu}(z)$ defined by the following series representation Merichev [12] as follow:

$$J^{\mu}_{\nu}(z)=\sum_{m=0}^{\infty}\frac{(-z)^m}{\Gamma(\nu+\mu m+1)\,m!}, \qquad \text{where } \mu>0;\ z\in\mathbb{C} \tag{1.1}$$

The generalized Bessel function of the form $J^{\mu}_{\nu,\sigma}(z)$,is defined by Jain and Agarwal [10] as follow:

$$J^{\mu}_{\nu,\sigma}(z)=\sum_{m=0}^{\infty}\frac{(-1)^m\left(\frac{z}{2}\right)^{\nu+2\sigma+2m}}{\Gamma(\nu+\sigma+\mu m+1)\Gamma(\sigma+m+1)}, \tag{1.2}$$

where $z\in\mathbb{C}\setminus(-\infty,0];\mu>0,\nu,\sigma\in\mathbb{C}$.

Further, generalization of the generalized Bessel-Maitland function $J^{\mu,\gamma}_{\nu,q}(z)$,defined by Pathak [17] as follow:

$$J^{\mu,\gamma}_{\nu,q}(z)=\sum_{m=0}^{\infty}\frac{(\gamma)_{qm}(-z)^m}{\Gamma(\nu+\mu m+1)\,m!} \tag{1.3}$$

Where $\mu,\nu,\gamma\in\mathbb{C}$ $\Re(\mu)\geq 0$, $\Re(\nu)\geq -1$, $\Re(\gamma)\geq 0$ and $q\in(0,1)\cup\mathbb{N}$,

and $(\gamma)_0=1$, $(\gamma)_{qm}=\frac{\Gamma(\gamma+qm)}{\Gamma(\gamma)}$ is known as generalized Pochhammer symbol defined by Mittag-Leffler [15].

From the generalization of the generalized Bessel-Maitland function (1.3), it is possible to find some special cases by giving particular values to the parameters μ,ν,γ,q

1) If $q=1,\gamma=1$ and ν is replaced by $\nu+\sigma$ and z is replaced by $\frac{z^2}{4}$ in (1.3), then we obtain

$$J^{\mu,1}_{\nu+\sigma,1}\left(\frac{z^2}{4}\right)=\Gamma(\sigma+m+1)\left(\frac{z}{2}\right)^{-\nu-2\sigma}J^{\mu}_{\nu,\sigma}(z), \tag{1.4}$$

where $J^{\mu}_{\nu,\sigma}(z)$ denotes Bessel-Maitland function defined by Agarwal et al. [3].

2) If we replace μ by 1 and σ by $\frac{1}{2}$ in (1.4), we obtain

$$J^{1,1}_{\nu+\frac{1}{2},1}\left(\frac{z^2}{4}\right)=\Gamma\left(m+\frac{3}{2}\right)\left(\frac{z}{2}\right)^{-\nu-1}H_{\nu}(z), \tag{1.5}$$

where $H_{\nu}(z)$ denotes Struve's function defined by Erdélyi et al. [8].

$$H_{\nu}(z)=\sum_{m=0}^{\infty}\frac{(-1)^m\left(\frac{z}{2}\right)^{\nu+2m+1}}{\Gamma\left(m+\frac{3}{2}\right)\Gamma\left(\nu+m+\frac{3}{2}\right)}. \tag{1.6}$$

3) If q= 0 , then (1.3) reduces to

$$J^{\mu,\gamma}_{\nu,0}(z)=J^{\mu}_{\nu}(z) \tag{1.7}$$

where $J^{\mu}_{\nu}(z)$ is generalized Bessel function defined by Agarwal [1].

4) If q=0, μ=0 and z is replaced by $\frac{z^2}{4}$ then (1.3) reduces to

$$J^{1,\gamma}_{\nu,0}\left(\frac{z^2}{4}\right)=\left(\frac{z}{2}\right)^{-\nu}J_{\nu}(z), \tag{1.8}$$

where $J_{\nu}(z)$ is called Bessel's function of the first kind and of order ν, where ν is any non-negative constant.

If q=0 and ν is replaced by $\nu-1$ and z is replaced by $-z$, then (1.3) reduces to

$$J^{\mu,\gamma}_{\nu-1,0}(-z)=\phi(\mu,\nu;z) \tag{1.9}$$

where $\phi(\mu,\nu;z)$ is known as Wright function, defined by Choi et al. [5].

Another representation of the generalized Bessel–Maitland function $J^{k,\gamma}_{\nu,q}(z)$ defined by Singh et al. [21], if $\mu=k\in\mathbb{N}$ and $q\in\mathbb{N}$, then (1.3) reduces to

$$J^{k,\gamma}_{\nu,q}(z)=\frac{1}{\Gamma(\nu+1)}{}_qF_k\left[\begin{matrix}\Delta(q;\gamma);\\ \Delta(k;\ \nu+1);\end{matrix}-\frac{q^q}{k^k}z\right]. \tag{1.10}$$

where ${}_qF_k()$ is the generalized hypergeometric function and the symbols $\Delta(q;\gamma)$ is a $q-$ tuple $\frac{\gamma}{q},\frac{\gamma+1}{q},\ \frac{\gamma+2}{q},\ldots,\frac{\gamma+q-1}{q}$ and $\Delta(k;\nu+1)$ is a $k-$tuple $\frac{\nu+1}{k},\frac{\nu+2}{k},\ \frac{\nu+3}{k},\ldots,\frac{\nu+k}{k}$. The generalization of the generalized hypergeometric series ${}_pF_q$ is due to Fox [9] and Wright ([26], [27], [28]) who studied the asymptotic expansion of the generalized Wright hypergeometric function defined by (see, also [23]).

The generalized Wright hypergeometric function ${}_p\Psi_q(z)$ (see, for detail, Srivastava and Karlsson [24]), for $z\in\mathbb{C}$ complex, $a_i,\ b_j\in\mathbb{C}$ and $\alpha_i,\ \beta_j\in\mathbb{R}$, where $(\alpha_i,\ \beta_j\neq 0;\ i=1,2,\ldots,p;\ j=1,2,\ldots,q)$, is defined as below:

$$ {}_p\psi_q(z) = {}_p\psi_q\left[\begin{matrix}(a_i,\ \alpha_i)_{1,\ p}\\ (b_j,\ \beta_j)_{1,\ q}\end{matrix}\middle| z\right] = \sum_{k=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma(a_i+\alpha_i k)\,z^k}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j k)\,k!}, \tag{1.11} $$

Introduced by Wright [26], the generalized Wright function and proved several theorems on the asymptotic expansion of ${}_p\psi_q(z)$ for all values of the argument z, under the condition:

$$ 1+\sum_{j=1}^{q}\beta_j-\sum_{i=1}^{p}\alpha_i \geq 0. \tag{1.12} $$

It is noted that the generalized (Wright) hypergeometric function ${}_p\psi_q$ in (1.11) whose asymptotic expansion was investigated by Fox [9] and Wright is an interesting further generalization of the generalized hypergeometric series as follow:

$$ {}_p\Psi_q\left[\begin{matrix}(\alpha_1,1),\ \ldots,\ (\alpha_p,1)\\ (\beta_1,1),\ \ldots,\ (\beta_q,1)\end{matrix} z\right] = \frac{\prod_{j=1}^{p}\Gamma(\alpha_j)}{\prod_{j=1}^{q}\Gamma(\beta_j)}\,{}_pF_q\left[\begin{matrix}\alpha_1,\ldots,\alpha_p;\\ \beta_1,\ldots,\beta_q;\end{matrix} z\right], \tag{1.13} $$

where ${}_pF_q$ is the generalized hypergeometric series defined by (see [23, Section 1.5])

$$ {}_pF_q\left[\begin{matrix}\alpha_1,\ldots,\alpha_p;\\ \beta_1,\ldots,\beta_q;\end{matrix} z\right] = \sum_{n=0}^{\infty}\frac{(\alpha_1)_n\cdots(\alpha_p)_n\,z^n}{(\beta_1)_n\cdots(\beta_q)_n\,n!} = {}_pF_q(\alpha_1,\ldots,\alpha_p;\beta_1,\ldots,\beta_q;z), \tag{1.14} $$

The general class of polynomials is defined by Srivastava [22, p. 1, Eq. (1)] in the following manner:

$$ S_n^m[x] = \sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,x^k, \qquad n = 0, 1, 2, \ldots \tag{1.15} $$

where m is an arbitrary positive integer and the coefficients $A_{n,k}(n, k \geq 0)$ are arbitrary constants, real or complex. The polynomial family $S_n^m[x]$ gives a number of known polynomial as its special cases on suitably specializing the coefficient $A_{n,k}$

For our present investigation, we also need to recall the following Lavoie-Trottier integral formula [11]:

$$ \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\left(1-\frac{x}{3}\right)^{2\alpha-1}\left(1-\frac{x}{4}\right)^{\beta-1}dx = \left(\frac{2}{3}\right)^{2\alpha}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \tag{1.16} $$

provided $\Re(\alpha)>0,\ \Re(\beta)>0$.

2. Main Results

In this section, we established two generalized integral formulas, which are expressed in term of generalized (Wright) hypergeometric functions, by inserting the generalized Bessel-Maitland function (1.3) Srivastava polynomial (1.15) and suitable argument in to the integrand of (1.16).

Theorem 1: *The following integral formula holds true: for* $\rho, j, \mu, \nu, \gamma \in \mathbb{C}$ *with* $\mathrm{Re}(\nu) \geq -1,\ \mathrm{Re}(\gamma)>0,\ \mathrm{Re}(\rho)>0,\ \mathrm{Re}(\rho+j)>0,\ \mathrm{Re}(\rho+k)>0$ *and* $x>0$, *we have*

$$ \int_0^{\infty} x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\alpha+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1} s_n^m\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)J_{\nu,q}^{\mu,\gamma}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx $$
$$ = \left(\frac{2}{3}\right)^{2(\rho+j)}\frac{\Gamma(\alpha+j)}{\Gamma(\gamma)}\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,y^k\,{}_2\Psi_2\left[\begin{matrix}(\gamma,\ q),\ (\rho+k,\ 1)\\ (\nu+1,\ \mu),\ (2\rho+j+k,\ 1);\end{matrix} -y\right]. \tag{2.1} $$

Proof: Now applying (1.3) and (1.15) to the integrand of (2.1) and then interchanging the order of integration and summation, which is verified by uniform convergence of the involved series under the given conditions in Theorem 1, we get

$$ \int_0^{\infty} x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1} s_n^m\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)J_{\nu,q}^{\mu,\gamma}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx $$
$$ = \sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,y^k\sum_{m=0}^{\infty}\frac{(\gamma)_{qm}(-1)^m y^m}{\Gamma(\nu+\mu m+1)\,m!} $$
$$ \times\int_0^{\infty} x^{\rho+j-1}(1-x)^{2(\rho+mk)-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho+mk-1}dx, $$

By considering the condition given in Theorem 1, since $\mathrm{Re}(\nu)\geq -1,\ \mathrm{Re}(\rho+j)>0$ $\mathrm{Re}(\rho+m+k)>0,\ (n,k\geq 0),\ q\in(0,1)\cup\mathbb{N}$ and applying (1.16),

$$ = \sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,y^k\sum_{m=0}^{\infty}\frac{(\gamma)_{qm}(-1)^m y^m}{\Gamma(\nu+\mu m+1)m!}\left(\frac{2}{3}\right)^{2(\rho+j)}\frac{\Gamma(\alpha+j)\,\Gamma(\rho+m+k)}{\Gamma(2\rho+j+m+k)}, $$
$$ = \left(\frac{2}{3}\right)^{2(\rho+j)}\frac{\Gamma(\alpha+j)}{\Gamma(\gamma)}\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,y^k\sum_{m=0}^{\infty}\frac{\Gamma(\gamma+qm)\Gamma(\rho+m+k)(-1)^m y^m}{\Gamma(\nu+\mu m+1)\,\Gamma(2\rho+j+m+k)\,m!}, $$

which upon using the definition (1.11), we get the desired result (2.1).

Theorem 2: *The following integral formula holds true: for* $\rho, j, \mu, \nu, \gamma \in \mathbb{C}$ *with* $\mathrm{Re}(\nu)\geq -1,\ \mathrm{Re}(\gamma)>0,\ \mathrm{Re}(\rho)>0,\ \mathrm{Re}(\rho+k)>0$ *and* $x>0$, *we have*

$$ \int_0^{\infty} x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{\rho+j-1} s_n^m\left(yx\left(1-\frac{x}{3}\right)^2\right)J_{\nu,q}^{\mu,\gamma}\left(yx\left(1-\frac{x}{3}\right)^2\right)dx $$
$$ = \left(\frac{2}{3}\right)^{2\rho}\frac{\Gamma(\rho+j)}{\Gamma(\gamma)}\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\left(\frac{4y}{9}\right)^k{}_2\Psi_2\left[\begin{matrix}(\gamma,\ q),\ (\rho+k,\ 1)\\ (\nu+1,\ \mu),\ (2\rho+k+j,\ 1);\end{matrix} -\frac{4y}{9}\right]. \tag{2.2} $$

By similar manner as in proof of Theorem 1, we can also prove the integral formula (2.2).

Next, we consider other variation of Theorem 1 and Theorem 2. In fact, we establish some integral formulas for the generalized Bessel-Maitland function expressed in terms of the generalized hypergeometric function. To do this, we recall the well known Legendre duplication formula (see, [23]) as

$$ (\lambda)_{2m} = 2^{2m}\left(\frac{\lambda}{2}\right)_m\left(\frac{\lambda+1}{2}\right)_m, \quad m\in\mathbb{N}_0 \tag{2.3} $$

Corollary 1: *Let the condition of theorem 1 be satisfied and replacing* μ *by* δ *in the generalized Bessel-Maitland function* $J_{\nu,q}^{\mu,\gamma}$ *in (2.1) and using (1.13). Then the following integral formula holds true:*

$$ \int_0^{\infty} x^{\alpha+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1} s_n^m\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)J_{\nu,q}^{\delta,\gamma}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx $$
$$ = \left(\frac{2}{3}\right)^{2(\rho+j)}\frac{\Gamma(\alpha+j)}{\Gamma(\nu+1)}\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,y^k\frac{\Gamma(\rho+k)}{\Gamma(2\rho+j+k)} $$
$$ \times\,{}_{q+1}F_{\delta+1}\left[\begin{matrix}\Delta(q;\ \gamma),\ (\rho+k)\\ \Delta(\delta;\ \nu+1),\ (2\rho+j+k);\end{matrix} \frac{yq^q}{\delta^\delta}\right]. \tag{2.4} $$

where $\Re(\rho+j)>0,\ \Re(\rho+k)>0$, $\Delta(q;\gamma)$ is a $q-$tuple $\frac{\gamma}{q}, \frac{\gamma+1}{q}, \frac{\gamma+2}{q}, \ldots, \frac{\gamma+q-1}{q}$ and $\Delta(\delta;\nu+1)$ is a $\delta-$tuple $\frac{\nu+1}{\delta}, \frac{\nu+2}{\delta}, \frac{\nu+3}{\delta}, \ldots, \frac{\nu+\delta}{\delta}$.

Corollary 2: *Let the condition of Theorem 2 be satisfied and replacing* μ *by* δ *in the generalized Bessel-Maitland function* $J_{\nu,q}^{\mu,\gamma}(z)$ *in (2.2) and using (1.13). Then the following integral formula holds true:*

$$ \int_0^{\infty} x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{\rho+j-1} s_n^m\left(yx\left(1-\frac{x}{3}\right)^2\right)J_{\nu,q}^{\delta,\gamma}\left(yx\left(1-\frac{x}{3}\right)^2\right)dx $$
$$ = \left(\frac{2}{3}\right)^{2\rho}\frac{\Gamma(\rho+j)}{\Gamma(\nu+1)}\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\frac{\Gamma(\rho+k)}{\Gamma(2\rho+j+k)}\left(\frac{4y}{9}\right)^k $$
$$ \times\,{}_{q+1}F_{\delta+1}\left[\begin{matrix}\Delta(q;\ \gamma),\ (\rho+k)\\ \Delta(\delta;\ \nu+1),\ (2\rho+j+k);\end{matrix} \frac{4yq^q}{9\delta^\delta}\right]. \tag{2.5} $$

Proof: By writing the right hand side of (2.1) in the original summation formula, after a little simplification, we find that, when the last resulting summation is expressed in the term of hypergeometric in the relation (1.14), this completes the proof of corollary1. Similarly, it is easy to see that a similar argument as in proof of corollary 1 will established the integral formula (2.4). Therefore we omit the details of the proof of the corollary 2.

3. Special Cases

In this section, we represent certain cases of generalized form of Bessel-Maitland function (1.3).

On setting $\gamma=1,\ \nu=\nu+\sigma,\ q=1$ and z is replaced by $(z^2/4)$, in theorem 1 and theorem 2 and making use of the relation (1.4), then the generalized Bessel-Maitland function will have following relation with Bessel-Maitland function as follows:

Corollary 3. *Let the condition of for* $\rho, j, \mu, \nu, \sigma \in \mathbb{C}$ *with* $\mathrm{Re}(\rho)>0,$ $\mathrm{Re}(\nu)\geq -1,\ \mathrm{Re}(\rho+j)>0,\ \mathrm{Re}(\rho+k+\nu+2\sigma)>0$ *be satisfied, then the following integral formula holds true:*

$$ \int_0^{\infty} x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\alpha+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1} s_n^m\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)J_{\nu,\sigma}^{\mu}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx $$
$$ = \left(\frac{2}{3}\right)^{2(\rho+j)}\left(\frac{y}{2}\right)^{\nu+2\sigma}\Gamma(\rho+j)\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,y^k $$
$$ \times\,{}_2\Psi_3\left[\begin{matrix}(\rho+k+\nu+2\sigma,\ 2),\ (1,\ 1)\\ (\nu+\sigma+1,\ \mu),\ (1+\sigma,\ 1),\ (2\rho+j+k+\nu+2\sigma,\ 2);\end{matrix} -\frac{y^2}{4}\right]. \tag{3.1} $$

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Corollary 4. *Let the condition of* $\rho, j, \mu, v, \sigma \in C$ *with* $\mathrm{Re}(\rho)>0,\ \mathrm{Re}(\rho+k+v+2\sigma)>0$

$$\int_0^\infty x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{\rho+j-1} s_n^m\left(yx\left(1-\frac{x}{3}\right)^2\right) J^{\mu}_{v,\sigma}\left(yx\left(1-\frac{x}{3}\right)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2\rho}\left(\frac{2y}{9}\right)^{v+2\sigma}\Gamma(\rho+j)\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\left(\frac{4y}{9}\right)^k$$

$$\times\ {}_2\Psi_3\left[\begin{matrix}(\rho+k+v+2\sigma,\ 2),\ (1,\ 1)\\ (v+\sigma+1,\ \mu),\ (1+\sigma,\ 1),\ (2\rho+j+k+v+2\sigma,\ 2);\end{matrix}\ ;\ -\frac{4y^2}{81}\right]. \qquad (3.2)$$

On setting $\mu=1,\ \sigma=1/2$ in (3.1) and (3.2) and using the relation (1.5), then we get the integral formulas involving the Struve's function $H_v(z)$ as follows:

Corollary 5. *Let the condition of* $\mathrm{Re}(\rho)>0,\ \mathrm{Re}(\rho+k+v+1)>0$ *be satisfied, then the following integral formula holds true:*

$$\int_0^\infty x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\alpha+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1} s_n^m\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right) H_v\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2(\alpha+j)}\left(\frac{y}{2}\right)^{v+1}\Gamma(\rho+j)\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,y^k$$

$$\times\ {}_2\Psi_3\left[\begin{matrix}(\rho+k+v+1,\ 2),\ (1,\ 1);\\ (2\rho+k+j+v+1,\ 2),\ \left(v+\frac{3}{2},\ 1\right),\ \left(\frac{3}{2},\ 1\right);\end{matrix}\ -\frac{y^2}{4}\right]. \qquad (3.3)$$

Corollary 6. *Let the condition of* $\mathrm{Re}(\rho)>0,\ \mathrm{Re}(\rho+k+v+1)>0$ *be satisfied, then the following integral formula holds true:*

$$\int_0^\infty x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{\rho+j-1} s_n^m\left(yx\left(1-\frac{x}{3}\right)^2\right) H_v\left(yx\left(1-\frac{x}{3}\right)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2(\rho+v)}\left(\frac{y}{2}\right)^{v+1}\Gamma(\rho+j)\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\left(\frac{4y}{9}\right)^k$$

$$\times\ {}_2\Psi_3\left[\begin{matrix}(\rho+k+v+1,\ 2),\ (1,\ 1);\\ (2\rho+k+j+v+1,\ 2),\ \left(v+\frac{3}{2},\ 1\right),\ \left(\frac{3}{2},\ 1\right);\end{matrix}\ -\frac{4y^2}{81}\right]. \qquad (3.4)$$

On setting q= 0 in theorem 1 and theorem 2 and making use of the relation (1.7), then the generalized Bessel–Maitland function $J^{\mu,\gamma}_{v,q}(z)$ will have the following relation with Bessel-Maitland function $J^{\mu}_{v}(z)$ follows:

Corollary 7. *Let the condition of* $\mathrm{Re}(\rho)>0,\ \mathrm{Re}(\rho+k+v+1)>0$ *be satisfied, then the following integral formula holds true:*

$$\int_0^\infty x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\alpha+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1} s_n^m\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right) J^{\mu}_{v}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2(\rho+j)}\Gamma(\rho+j)\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,y^k\,{}_1\Psi_2\left[\begin{matrix}(\rho+k,\ 1)\\ (v+1,\ \mu),\ (2\rho+k+j,\ 1);\end{matrix}\ -y\right]. \qquad (3.5)$$

Corollary 8. *Let the condition of* $\mathrm{Re}(\rho)>0,\ \mathrm{Re}(\rho+k+v+1)>0$ *be satisfied, then the following integral formula holds true:*

$$\int_0^\infty x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{\rho+j-1} s_n^m\left(yx\left(1-\frac{x}{3}\right)^2\right) J^{\mu}_{v}\left(xy\left(1-\frac{x}{3}\right)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2\rho}\Gamma(\rho+j)\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\left(\frac{4y}{9}\right)^k\,{}_1\Psi_2\left[\begin{matrix}(\rho+k,1)\\ (v+1,\ \mu),(2\rho+j+k,\ 1);\end{matrix}\ -\frac{4y}{9}\right]. \qquad (3.6)$$

Remark 1. On setting $q=0,\ \mu=1$ and z is replaced by $(z^2/4)$ in theorem 1 and theorem 2 and making use of the relation (1.8), integral formulas involving the ordinary Bessel function, we arrive at the known result given by Menaria et al. [13, p. 103, eq.(17) and (18)].

On setting q=0and v is replaced by 1–v and z is replaced byz–, in theorem 1 and theorem 2 and making use of the relation (1.9), we obtain the following integral formulas involving the Weight function as follows:

Corollary 9. *Let the condition of* $\mathrm{Re}(\rho)>0,\ \mathrm{Re}(\rho+k+v+1)>0$ *be satisfied, then the following integral formula holds true:*

$$\int_0^\infty x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\alpha+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1} s_n^m\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)\phi\left(\mu,v,y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2(\rho+j)}\Gamma(\rho+j)\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,y^k\,{}_1\Psi_2\left[\begin{matrix}(\rho+k,\ 1)\\ (v,\ \mu),\ (2\rho+j+k,\ 1);\end{matrix}\ y\right]. \qquad (3.7)$$

Corollary 10. *Let the condition of* $\mathrm{Re}(\rho)>0,\ \mathrm{Re}(\rho+k+v+1)>0$ *be satisfied, then the following integral formula holds true:*

$$\int_0^\infty x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{\rho+j-1} s_n^m\left(yx\left(1-\frac{x}{3}\right)^2\right)\phi\left(\mu,v;yx\left(1-\frac{x}{3}\right)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2\rho}\Gamma(\rho+j)\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\left(\frac{4y}{9}\right)^k\,{}_1\Psi_2\left[\begin{matrix}(\rho+k,\ 1)\\ (v,\ \mu),\ (2\rho+j+k,\ 1);\end{matrix}\ \frac{4y}{9}\right]. \qquad (3.8)$$

4. Concluding remarks

In the present paper, we investigate new integrals involving the generalized Bessel-Maitland function $J^{\mu,\gamma}_{v,q}(z)$ in terms of generalized (Wright) hypergeometric functions and generalized hypergeometric function ${}_qF_{}$ Certain special cases of integrals involving generalized Bessel Maitland function have been investigated in the literature by a number of authors ([1-7, 13, 14, 16, 19]) with different arguments. Therefore the results presented in this paper are easily converted in terms of a similar type of new interesting integrals with different arguments after some suitable parametric replacements. In this sequel, one can obtain integral representation of more generalized special function, which has much application in physics and engineering Science.

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