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Unreliable Batch Arrival Retrial Queue With Delayed Repair, Randomised J Vacations and Orbital Search

Mathematics

KEYWORDS : Retrial, J Vacation, Orbital search, Delayed Repair.

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ABSTRACT

Batch arrival retrial queueing system with vacation and orbital search is considered. Customers arrive in batches according to Poisson process. At the arrival epoch of a batch if the server is idle, then one of the customers enters for service and others join the orbit. Otherwise all the customers join the orbit. The server is subject to active breakdown and delayed repair. Service of the interrupted customer resumes immediately after a repair process. As soon as the system becomes empty the server takes a vacation. If the orbit is empty upon returning from a vacation, the server either remains idle or leaves for another vacation. This pattern continues until the number of vacations reaches J. At the end of Jth vacation, even if the orbit is empty, the server dormants idly in the system for new arrival. If the orbit is nonempty the server searches for customers from the orbit or remains idle. Steady state analysis is performed using supplementary variable technique. Stochastic decomposition law is established. Various performance measures are obtained numerically.

Introduction

Queueing systems with repeated attempts are found suitable for modelling the processes in telephone switching systems, digital cellular mobile networks, packet switching networks, local area networks, stock and flow etc. Review of retrial queueing literature can be found in the survey papers of Yang and Templeton (1987) and Falin (1990), the bibliographies of Artalejo (1999a, 1999b) and the books by Falin and Templeton (1997).

Queues with server subject to breakdowns and repairs are often encountered in many practical applications. Li et al. (2006) provided BMAP/G/1 retrial queue with server breakdowns and repairs considering both from queueing view point and reliability view point. Atencia et al. (2008) studied a batch arrival retrial queue subject to breakdown where the retrial time is exponential and independent of the number of customers applying for service. Zhou et al. (2009) discussed an M/G/1 retrial queue with repairable server and exhaustive vacation and derived the necessary and sufficient condition for system stability and queue indices using supplementary variable technique.

In most of the papers with unreliable server it is assumed that whenever the system breaks down the repair process starts instantaneously. However, in many real life situations, the system has to wait for repair to start. Recent work on unreliable retrial queueing system with delayed repair includes Prakash Rani et al. (2011), Choudhury and Ke (2012), Ebenesar Anna Bagyam and Udaya Chandrika (2012) and Ayyappan and Shyamala (2014).

In the retrial setup, each service is preceded and followed by the server's idle time because of the ignorance of the status of the server and orbital customers by each other. Server's idle time is reduced by the introduction of search of orbital customers immediately after a service completion. Artalejo et al. (2002) and Dudin et al. (2004) considered retrial queueing models with orbital search. Krishnamoorthy et al. (2005) analysed M/G/1 retrial queue with non persistent customers and orbital search using supplementary variable method. Sumitha and Udaya Chandrika (2011) discussed a single server batch arrival retrial queueing system with additional optional service and orbital search and obtained the steady state distributions of the server state and the number of customers in the orbit.

In this paper, the model analysed by Sumitha and Udaya Chandrika (2011) is extended to server breakdown and randomized J vacations.

Model Description

Consider an infinite capacity queueing system with single server. Assume that customers arrive in batches according to Poisson process with rate λ. The batch size is a random variable Y with $P(Y=k)=C_k$. The probability generating function of the sequence $\{C_k\}$ is C(z) with first two moments m_1 and m_2. If an arriving batch finds the server idle then, one of the customers starts its service time and others join the orbit, otherwise, all the arriving customers join the orbit. The retrial time follows an arbitrary distribution with distribution function A(x), Laplace Stieltjes transform $A^*(s)$ and hazard rate function $\eta(x) = \frac{a(x)}{1-A(x)}$. The service time follows a general distribution with distribution function B(x), Laplace Stieltjes transform $B^*(s)$, hazard rate function $\mu(x) = \frac{b(x)}{1-B(x)}$ and first two moments μ_1 and μ_2.

The server is subject to unpredictable breakdown while it is working. The life time of the server is exponentially distributed with rate α. The repair of the failed server starts after a random amount of time called delay time. Delay time is generally distributed with distribution function D(x), Laplace Stieltjes transform $D^*(s)$, hazard rate function $\gamma(x) = \frac{d(x)}{1-D(x)}$ and first two moments γ_1 and γ_2. The repair time is also generally distributed with distribution function R(x), Laplace Stieltjes transform $R^*(s)$, hazard rate function $\beta(x) = \frac{r(x)}{1-R(x)}$ and first

two moments β_1 and β_2. After repair completion the server resumes the service of the interrupted customer.

If the orbit becomes empty, the server leaves for a vacation of random length V. At the vacation completion epoch, if the orbit is still empty the server either remains idle in the system with probability p or takes another vacation with probability $\bar{p}$ (= 1 – p) of same length. This pattern continues until the server finds atleast one customer in the orbit or number of vacations reaches J whichever occurs earlier. At the end of the J^{th} vacation, even if the orbit is empty the server remains in the system. Vacation time is arbitrarily distributed with distribution function V(x), Laplace Stieltjes transform $V^*(s)$, hazard rate function $\nu(x) = \frac{v(x)}{1 - V(x)}$ and the first two moments ν_1 and ν_2. If the orbit is non-empty, the server searches for the customers in the orbit with probability θ or remains idle with probability $\bar{\theta}$ (= 1 – θ).

Stability Condition

Theorem 1

The necessary and sufficient condition for the system to be stable is

$$\lambda m_1 \mu_1 (1 + \alpha \gamma_1 + \alpha \beta_1) + m_1 (1 - A^*(\lambda)) < 1$$

Proof

The theorem can be proved along similar lines as in Gomez-Corral (1999).

Definitions and Notations

Assuming the existence of steady state, define the following probabilities.

$I_n(x)$ is the probability that there are n customers in the system, the server is idle and the elapsed retrial time is x.

$P_n(x)$ is the probability that there are n customers in the system, the server is busy and the elapsed service time is x.

$F_{1,n}(x, y)$ is the probability that there are n customers in the system, the server is waiting for repair, the elapsed service time is x and the elapsed delay time is y.

$F_{2,n}(x, y)$ is the probability that there are n customers in the system, the server is under repair, the elapsed service time is x and the elapsed repair time is y.

$V_{j,n}(x)$ is the probability that there are n customers in the system, the server is in j^{th} vacation, the elapsed vacation time is x.

I_0 is the steady state probability that the server is idle in the empty system.

Define the probability generating functions

$$I(x, z) = \sum_{n=1}^{\infty} I_n(x) z^n ;$$

$$P(x, z) = \sum_{n=0}^{\infty} P_n(x) z^n ;$$

$$F_i(x, y, z) = \sum_{n=0}^{\infty} F_{i,n}(x, y) z^n, \ i = 1, 2 \quad \text{and}$$

$$V_j(x, z) = \sum_{n=0}^{\infty} V_{j,n}(x) z^n, \ 1 \le j \le J.$$

Equations Governing the System

The steady state governing equations for the model under study are :

$$\lambda I_0 = \int_0^{\infty} V_{J,0}(x) \ \nu(x) \, dx + p \sum_{j=1}^{J-1} \int_0^{\infty} V_{j,0}(x) \ \nu(x) \, dx \quad (1)$$

$$\frac{d}{dx} I_n(x) = -(\lambda + \eta(x)) I_n(x), \quad n \ge 1 \quad (2)$$

$$\frac{d}{dx} P_0(x) = -(\lambda+\alpha+\mu(x)) P_0(x) + \int_0^{\infty} F_{2,0}(x, y) \beta(y) \, dy \quad (3)$$

$$\frac{d}{dx} P_n(x) = -(\lambda+\alpha+\mu(x)) P_n(x) + \int_0^{\infty} F_{2,n}(x, y) \beta(y) \, dy + \lambda \sum_{j=1}^{n} C_k P_{n-k}(x), \ n \ge 1 \quad (4)$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) F_{1,0}(x, y) = -(\lambda + \gamma(y)) F_{1,0}(x, y) \quad (5)$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) F_{1,n}(x, y) = -(\lambda + \gamma(y)) F_{1,n}(x, y) + \lambda \sum_{k=1}^{n} C_k F_{1,n-k}(x, y), \ n \ge 1 \quad (6)$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) F_{2,0}(x, y) = -(\lambda + \beta(y)) F_{2,0}(x, y) \quad (7)$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) F_{2,n}(x, y) = -(\lambda + \beta(y)) F_{2,n}(x, y) + \lambda \sum_{k=1}^{n} C_k \ F_{2,n-k}(x, y), \ n \ge 1 \quad (8)$$

$$\frac{\partial}{\partial x} \ V_{j,n}(x) = -(\lambda + \nu(x)) V_{j,n}(x) + \lambda \sum_{k=1}^{n} C_k V_{j,n-k}(x), \quad n \ge 0 \ ; \ 1 \le j \le J \quad (9)$$

with boundary conditions

$$I_n(0) = \sum_{j=1}^{J} \bar{\theta} \int_0^{\infty} V_{j,n}(x) \nu(x) \, dx + \int_0^{\infty} P_n(x) \mu(x) \, dx, \ n \ge 1 \quad (10)$$

$$P_0(0) = \lambda C_1 I_0 + \int_0^\infty I_1(x)\,\eta(x)\,dx + \theta \sum_{j=1}^{J} \int_0^\infty V_{j,1}(x)\,\nu(x)\,dx \quad (11)$$

$$P_n(0) = \lambda C_{n+1} I_0 + \int_0^\infty I_{n+1}(x)\,\eta(x)dx + \lambda \sum_{k=1}^{n} C_k \int_0^\infty I_{n-k+1}(x)\,dx + \theta \sum_{j=1}^{J} \int_0^\infty V_{j,n+1}(x)\,\nu(x)dx, \quad n\geq 1 \quad (12)$$

$$F_{1,n}(x, 0) = \alpha P_n(x), \quad n \geq 0 \quad (13)$$

$$F_{2,n}(x, 0) = \int_0^\infty F_{1,n}(x, y)\,\gamma(y)\,dy, \quad n \geq 0 \quad (14)$$

$$V_{1,n}(0) = \begin{cases} \int_0^\infty P_0(x)\,\mu(x)\,dx, & n = 0 \\ 0, & n \geq 1 \end{cases} \quad (15)$$

$$V_{j,n}(0) = \begin{cases} \bar{p} \int_0^\infty V_{j-1,n}(x)\,\nu(x)\,dx, & n = 0\,;\, j = 2, 3, \ldots J \\ 0, & n \geq 1\,;\, j = 2, 3, \ldots J \end{cases} \quad (16)$$

The normalising condition is

$$I_0 + \sum_{n=1}^{\infty} \int_0^\infty I_n(x)\,dx + \sum_{n=0}^{\infty} \int_0^\infty P_n(x)\,dx + \sum_{n=0}^{\infty} \int_0^\infty \int_0^\infty F_{1,n}(x, y)\,dx\,dy + \sum_{n=0}^{\infty} \int_0^\infty \int_0^\infty F_{2,n}(x, y)\,dx\,dy + \sum_{j=1}^{J} \sum_{n=0}^{\infty} \int_0^\infty V_{j,n}(x)\,dx = 1 \quad (17)$$

Orbit Size Distribution at Random Epoch

In this section the orbit size distribution due to the system state of the server are investigated.

Theorem 2

The joint steady state distributions to the server state and orbit size are given by

$$I(z) = \frac{I_0(1 - A^*(\lambda))[z\,T_2(z) - (C(z) + N\,\theta\,V^*(\lambda - \lambda\,C(z)))\,T(z)]}{T_1(z)\,T(z) - z} \quad (18)$$

$$P(z) = \lambda I_0[T_1(z)T_2(z) - C(z) - N\,V^*(\lambda - \lambda C(z))](1 - T(z)) / [(T_1(z)\,T(z) - z)\,(G(\lambda - \lambda\,C(z)))] \quad (19)$$

$$F_1(z) = \alpha\, I_0\,[T_1(z)T_2(z) - C(z) - N\,\theta\,V^*(\lambda - \lambda C(z))]\,(1 - D^*(\lambda - C(z)))(1 - T(z)) / [(T_1(z)T(z) - z)\,(1 - C(z))\,(G(\lambda - \lambda\,C(z)))] \quad (20)$$

$$F_2(z) = \alpha I_0\, D^*(\lambda - \lambda C(z))\,[T_1(z)T_2(z) - C(z) - N\theta\,V^*(\lambda - \lambda C(z))](1 - R^*(\lambda - \lambda C(z)))(1 - T(z))/ [(T_1(z)\,T(z) - z)\,(1 - C(z))\,(G(\lambda - \lambda\,C(z)))] \quad (21)$$

$$V_j(z) = I_0(\bar{p}\,V^*(\lambda))^{J-1}(1 - \bar{p}\,V^*(\lambda))(1 - V^*(\lambda - \lambda C(z))) / [(\bar{p}\,V^*(\lambda))^{J-j}\,V^*(\lambda)[(\bar{p}\,V^*(\lambda))^{J-1}(1 - \bar{p}\,V^*(\lambda)) + p(1 - (\bar{p}\,V^*(\lambda))^{J-1})](1 - C(z))], \quad 1 \leq j \leq J \quad (22)$$

$$I_0 = \frac{1 - \lambda\, m_1\, \mu_1(1 + \alpha\,\gamma_1 + \alpha\,\beta_1) - m_1(1 - A^*(\lambda))}{A^*(\lambda) - (1 - A^*(\lambda))\,N\,\theta + \lambda\,\nu_1\,N} \quad (23)$$

where $G(y) = y + \alpha - \alpha\, D^*(y)\, R^*(y)$

$T(z) = B^*(G(\lambda - \lambda\, C(z)))$

$T_1(z) = C(z) + (1 - C(z))\, A^*(\lambda)$

$T_2(z) = 1 + N - N\,\bar{\theta}\; V^*(\lambda - \lambda\, C(z))$

and

$$N = \frac{1 - (\bar{p}\,V^*(\lambda))^J}{V^*(\lambda)[(\bar{p}\,V^*(\lambda))^{J-1}(1 - \bar{p}\,V^*(\lambda)) + p(1 - (\bar{p}\,V^*(\lambda))^{J-1})]}$$

Proof

Multiplying equations (2) to (14) by z^n and summing over all possible values of n, we get the following differential equations

$$\left(\frac{d}{dx} + \lambda + \eta(x)\right) I(x, z) = 0 \quad (24)$$

$$\left(\frac{d}{dx} + \lambda(1 - C(z)) + \alpha + \mu(x)\right) P(x, z) = \int_0^\infty F_2(x, y, z)\,\beta(y)\,dy \quad (25)$$

$$\left(\frac{d}{dy} + \lambda(1 - C(z)) + \gamma(y)\right) F_1(x, y, z) = 0 \quad (26)$$

$$\left(\frac{d}{dy} + \lambda(1 - C(z)) + \beta(y)\right) F_2(x, y, z) = 0 \quad (27)$$

$$\left(\frac{d}{dx} + \lambda(1 - C(z)) + \nu(x)\right) V_j(x, z) = 0, \; 1 \leq j \leq J \quad (28)$$

$$I(0, z) = \sum_{j=1}^{J} \bar{\theta} \int_0^\infty V_j(x, z)\,\nu(x)\,dx + \int_0^\infty P(x, z)\,\mu(x)\,dx - \lambda\, I_0 - \sum_{j=1}^{J} V_{j,0}(0) \quad (29)$$

$$P(0, z) = \frac{\lambda\, C(z)}{z}\, I_0 + \frac{1}{z} \int_0^\infty I(x, z)\,\eta(x)\,dx + \frac{\lambda}{z}\, C(z) \int_0^\infty I(x, z)\,dx + \frac{\theta}{z} \sum_{j=1}^{J} \int_0^\infty V_j(x, z)\,\nu(x)\,dx \quad (30)$$

$$F_1(x, 0, z) = \alpha\, P(x, z) \quad (31)$$

$$F_2(x, 0, z) = \int_0^\infty F_1(x, y, z)\,\gamma(y)\,dy \quad (32)$$

Solving equation (9) at n = 0 we have

$$V_{j,0}(x) = V_{j,0}(0)\,e^{-\lambda x}(1 - V(x)),\ j = 1, 2, \ldots J \quad (33)$$

Multiplying equation (33) by ν(x) and integrating with respect to x from 0 to ∞, we obtain

$$\int_0^\infty V_{j,0}(x)\,\nu(x)\,dx = V_{j,0}(0)\,V^*(\lambda),\ j = 1, 2, \ldots, J \quad (34)$$

Equation (16) gives

$$V_{j,0}(0) = \bar{p}\int_0^\infty V_{j-1,0}(x)\,\nu(x)\,dx$$

$$= \bar{p}\,V_{j-1,0}(0)V^*(\lambda), j=2,3,...,J \quad (35)$$

From equations (15) and (16) it is clear that

$V_j(0, z) = V_{j,0}(0)$.

Applying equation (35) repeatedly for j = J, J – 1, … we get

$$V_j(0, z) = \frac{V_{J,0}(0)}{[\bar{p}\,V^*(\lambda)]^{J-j}},\ j = 1, 2, \ldots J - 1 \quad (36)$$

Substituting equation (34) and (36) in equation (1) and after some algebraic manipulations, we get

$$V_{J,0}(0) = \frac{\lambda I_0}{V^*(\lambda)\left[1+\frac{p(1-(\bar{p}\,V^*(\lambda))^{J-1})}{(\bar{p}\,V^*(\lambda))^{J-1}(1-\bar{p}\,V^*(\lambda))}\right]} \quad (37)$$

Solving the partial differential equations (24) to (28) we obtain respectively

$$I(x, z) = I(0, z)\,e^{-\lambda x}(1 - A(x)) \quad (38)$$

$$P(x. z) = P(0, z)\,e^{-G(\lambda-\lambda C(z))x}(1 - B(x)) \quad (39)$$

$$F_1(x, y, z) = F_1(x, 0, z)\,e^{-\lambda(1-C(z))y}(1 - D(y)) \quad (40)$$

$$F_2(x, y, z) = F_2(x, 0, z)\,e^{-\lambda(1-C(z))y}(1 - R(y)) \quad (41)$$

$$V_j(x, z) = V_j(0, z)\,e^{-\lambda(1-C(z))x}(1 - V(x)) \quad (42)$$

Using equations (38) to (42) in equations (29) to (32) and solving we get

$$I(0,z) = \frac{\lambda I_0[(C(z)+N\theta V^*(\lambda-\lambda C(z)))B^*(G(\lambda-\lambda C(z))) + z(N\bar{\theta}\,V^*(\lambda-\lambda C(z))-N-1)]}{z-(C(z)+(1-C(z))A^*(\lambda))B^*(G(\lambda-\lambda C(z)))} \quad (43)$$

$$P(0,z) = \frac{\lambda I_0[C(z)+N\theta V^*(\lambda-\lambda C(z))+(C(z)+(1-C(z))A^*(\lambda))(N\bar{\theta}\,V^*(\lambda-\lambda C(z))-N-1)]}{z-(C(z)+(1-C(z))A^*(\lambda))B^*(G(\lambda-\lambda C(z)))} \quad (44)$$

$$F_1(x,0,z) = \alpha\lambda I_0[C(z)+N\theta V^*(\lambda-\lambda C(z))+(C(z)+(1-C(z))A^*(\lambda))(N\bar{\theta}\,V^*(\lambda-\lambda C(z))-N-1)]\,e^{-G(\lambda-\lambda C(z))x}(1-B(x))/[z-(C(z)+(1-C(z))A^*(\lambda))B^*(G(\lambda-\lambda C(z)))] \quad (45)$$

$$F_2(x,0,z) = \alpha\lambda I_0 D^*(\lambda-\lambda C(z))[C(z)+N\theta V^*(\lambda-\lambda C(z))+(C(z)+(1-C(z))A^*(\lambda))(N\bar{\theta}\,V^*(\lambda-\lambda C(z))-N-1)]e^{-G(\lambda-\lambda C(z))x}(1-B(x))/[z-(C(z)+(1-C(z))A^*(\lambda))B^*(G(\lambda-\lambda C(z)))] \quad (46)$$

Using the expressions of I(0, z), P(0, z) and $V_j(0, z)$ given in equations (43), (44) and (36), the probability generating functions I(x, z), P(x, z) and $V_j(x, z)$ can be expressed in terms of I_0. Integrating the resultant expressions with respect to x from 0 to ∞, we get I(z), P(z) and $V_j(z)$ as in equations (18), (19) and (22).

Substituting expressions of $F_1(x, 0, z)$ and $F_2(x, 0, z)$ in equations (40) and (41) and integrating with respect to x and y from 0 to ∞, we get $F_1(z)$ and $F_2(z)$ as in equations (20) and (21).

The expression of I_0 in equation (23) can be obtained by using normalizing condition.

Result 1

The probability generating function of the number of customers in the orbit is

$$P_q(z) = I_0+I(z)+P(z)+F_1(z)+F_2(z)+\sum_{j=1}^{J} V_j(z)$$

$$= \frac{I_0(1-z)[(1-C(z))(A^*(\lambda)T_2(z)-N\theta V^*(\lambda-\lambda C(z))) + N C(z)(1-V^*(\lambda-\lambda C(z)))]}{[1-C(z)][T_1(z)T(z)-z]} \quad (47)$$

Result 2

The probability generating function of the number of customers in the system is

$$P_S(z) = I_0+I(z)+z[P(z)+F_1(z)+F_2(z)]+\sum_{j=1}^{J} V_j(z)$$

$$= B^*(G(\lambda-\lambda C(z)))\,P_q(z) \quad (48)$$

System Characteristics

Applying limits as Z→1 on I(z), P(z), $F_1(z)$, $F_2(z)$ and V(z) we get respectively the following results.

- The steady state probability that the server is idle in non empty system is

$$I = \frac{I_0(1-A^*(\lambda))[(1+\theta N)(1-\lambda m_1\mu_1(1+\alpha\gamma_1+\alpha\beta_1))-m_1-\lambda m_1 N\nu_1]}{\lambda m_1\mu_1(1+\alpha\gamma_1+\alpha\beta_1)+m_1(1-A^*(\lambda))-1} \quad (49)$$

- The steady state probability that the server is busy is

$$P = \frac{\lambda I_0 m_1\mu_1[\theta N(1-A^*(\lambda))-A^*(\lambda)-N\lambda\nu_1]}{\lambda m_1\mu_1(1+\alpha\gamma_1+\alpha\beta_1)+m_1(1-A^*(\lambda))-1} \quad (50)$$

- The steady state probability that the server is in failure mode is

$$F = \frac{\alpha\lambda I_0 m_1\mu_1(\gamma_1+\beta_1)[\theta N(1-A^*(\lambda))-A^*(\lambda)-N\lambda\nu_1]}{\lambda m_1\mu_1(1+\alpha\gamma_1+\alpha\beta_1)+m_1(1-A^*(\lambda))-1} \quad (51)$$

- The steady state probability that the server is on vacation is

$$V = \lambda I_0 N \nu_1 \quad (52)$$

- The steady state probability of unsuccessful retrials made by the customer is given by

$$V_R = \frac{P+F+V}{1-I_0}$$

$$= \frac{\lambda[(1-m_1(1-A^*(\lambda)))N\nu_1 + m_1\mu_1(1+\alpha\gamma_1+\alpha\beta_1)}{(A^*(\lambda)-\theta N(1-A^*(\lambda)))]} \quad (53)$$

$$(1-A^*(\lambda))(m_1-N\theta-1)+\lambda m_1\mu_1(1+\alpha\gamma_1+\alpha\beta_1)+\lambda N\nu_1$$

- The mean number of customers in the orbit under steady state is

$$L_q = \lim_{z\to 1}\frac{d}{dz}P_q(z)$$

Let Nr(z) and Dr(z) be the numerator and denominator of $P_q(z)$. Since $Nr(1) = Dr(1) = Nr'(1) = Dr'(1) = 0$, applying L Hospital rule we get

$$L_q = \frac{Dr''(1)\ Nr'''(1) - Nr''(1)\ Dr'''(1)}{3Dr''(1)^2} \quad (54)$$

where

$Nr''(1) = 2m_1[A^*(\lambda) + N(\lambda\nu_1 - \theta(1-A^*(\lambda)))]\ I_0$

$Nr'''(1) = 3[m_2 A^*(\lambda) + 2N\bar{\theta}\lambda m_1^2 \nu_1(1-A^*(\lambda)) + N(\lambda m_2\nu_1+\lambda^2 m_1^2\nu_2) - N\theta m_2(1-A^*(\lambda))]\ I_0$

$Dr''(1) = 2m_1[1-m_1(1-A^*(\lambda))-\lambda m_1\mu_1(1+\alpha\gamma_1+\alpha\beta_1)]$

and

$Dr'''(1) = -3[m_1(\mu_2\lambda^2 m_1^2(1+\alpha\gamma_1+\alpha\beta_1)^2 + (1-A^*(\lambda))(2\lambda m_1^2\mu_1(1+\alpha\gamma_1+\alpha\beta_1)+m_2)) + m_2(\lambda m_1\mu_1(1+\alpha\gamma_1+\alpha\beta_1) + m_1(1-A^*(\lambda)) - 1)]$

- The mean number of customers in the system at steady state is

$$L_s = L_q + P + F \quad (55)$$

Reliability Analysis

Let A(t) be the availability of the server at time t, that is, the probability that the server is either busy or idle.

Theorem 3

The steady state availability of the server is

$$A = \frac{(A^*(\lambda)-\theta N(1-A^*(\lambda)))(1-\lambda m_1\mu_1\alpha(\gamma_1+\beta_1)) + N\lambda m_1\nu_1(1+\lambda\mu_1-A^*(\lambda))}{A^*(\lambda)-(1-A^*(\lambda))N\theta+N\lambda\nu_1} \quad (56)$$

Proof

$$A = I_0 + \lim_{z\to 1}\left[\int_0^\infty I(x,z)\,dx + \int_0^\infty P(x,z)\,dx\right]$$

$$= I_0 + \lim_{z\to 1}[I(z)+P(z)]$$

$$= I_0 + I + P \quad (57)$$

Equation (56) is obtained by substituting the expressions of I_0, I and P in equation (57).

Theorem 4

The steady state failure frequency of the server is

$$F = \alpha\lambda m_1\mu_1 \quad (58)$$

Proof

$$F = \alpha\lim_{z\to 1}\int_0^\infty P(x,z)\,dx$$

$$= \alpha\lim_{z\to 1}P(z)$$

$$= \alpha\lambda m_1\mu_1$$

Orbit Size Distribution at Departure Epoch

Theorem 5

The probability generating function $\pi(z)$ of the number of customers in the system at a departure epoch can be decomposed as

$$\pi(z) = \frac{1-C(z)}{m_1(1-z)}P_s(z) \quad (59)$$

where $\frac{1-C(z)}{m_1(1-z)}$ is the probability generating function of the number of customers placed before a tagged customer in a batch in which tagged customer arrives.

Proof

The departing customer will see j customers in the orbit after the departure if and only if j customers are there in the orbit just before departure. Let π_j be the probability that there are j customers in the orbit at the departure point of time. Then

$$\pi_j = B_0\int_0^\infty P_j(x)\,\mu(x)\,dx$$

where B_0 is the normalising constant.

Let $\pi(z)$ be the probability generating function of $\{\pi_j ; j = 0, 1, 2, \ldots\}$ then

$$\pi(z) = \sum_{j=0}^{\infty}\pi_j z^j$$

$$= \frac{B_0\lambda I_0[C(z)+N\theta V^*(\lambda-\lambda C(z))-T_1(z)T_2(z)]T(z)}{z-T_1(z)T(z)} \quad (60)$$

Eliminating the constant B_0 by using normalising condition $\pi(1) = 1$ we get

$$\pi(z) = \frac{I_0}{m_1}\left\{\frac{[C(z)+N\theta V^*(\lambda-\lambda C(z))-T_1(z)T_2(z)]T(z)}{z-T_1(z)T(z)}\right\}$$

From equation (48) we get the result (59).

Stochastic Decomposition

Theorem 6

The number of customers in the system (L_S) can be expressed as the sum of two independent random variables, one of which is the number of

customers in the unreliable batch arrival queue with delayed repair and orbital search (L) and the other is the number of customers in the orbit given that the server is idle or on vacation (L_{IV}).

Proof

The probability generating function $\Phi(z)$ of the number of customers in the unreliable batch arrival queue with delayed repair and orbital search is

$$\Phi(z) = \frac{[1-\lambda m_1 \mu_1(1+\alpha\gamma_1+\alpha\beta_1)](1-z)B^*(G(\lambda-\lambda C(z)))}{B^*(G(\lambda-\lambda C(z)))-z} \quad (61)$$

The probability generating function $\psi(z)$ of the number of customers in the orbit given that the server is idle or on vacation is

$$\psi(z) = \frac{I_0 + I(z) + \sum_{j=1}^{J} V_j(z)}{I_0 + I + V}$$

$$= \frac{I_0 (T(z)-z)[(1-C(z))(A^*(\lambda)(1+N-N\bar{\theta}V^*(\lambda-\lambda C(z)))}{-N\theta V^*(\lambda-\lambda C(z)))+N C(z)(1-V^*(\lambda-\lambda C(z)))]}{[1-C(z)][T_1(z)T(z)-z][1-\lambda m_1\mu_1(1+\alpha\gamma_1+\alpha\beta_1)]} \quad (62)$$

From equations (48), (61) and (62) we observe that

$$P_s(z) = \Phi(z)\,\psi(z). \quad (63)$$

and hence we get

$$Ls = L + L_{IV}. \quad (64)$$

Special Cases

Case (i)

If $A^*(\lambda) \to 1$ and $\theta = 0$, then the model reduces to unreliable batch arrival queue with delayed repair and randomised J vacation. In this case

$$I_0 = \frac{1-\lambda m_1 \mu_1(1+\alpha\gamma_1+\alpha\beta_1)}{\lambda\nu_1 N}$$

where

$$N = \frac{1-(\bar{p}(V^*(\lambda)))^J}{V^*(\lambda)[(\bar{p}V^*(\lambda))^{J-1}(1-\bar{p}V^*(\lambda))+p(1-(\bar{p}V^*(\lambda))^{J-1})]}$$

and

$$P_S(z) = \frac{I_0(1-z)[1-C(z)+N-NV^*(\lambda-\lambda C(z))]B^*(G(\lambda-\lambda C(z)))}{(1-C(z))(B^*(G(\lambda-\lambda C(z)))-z)}$$

The above results are consistent with Ke and Huang (2010).

Case (ii)

If $p = 0$, $\alpha = 0$ and $\theta = 0$, then the model reduces to batch arrival retrial queue with modified vacations. In this case we have

$$I_0 = \frac{1-\lambda m_1 \mu_1 - m_1(1-A^*(\lambda))}{A^*(\lambda)+\lambda\nu_1 N}$$

where $N = \frac{1-(V^*(\lambda))^J}{(V^*(\lambda))^J(1-V^*(\lambda))}$ and

$$P_S(z) = \frac{I_0(1-z)B^*(\lambda-\lambda C(z))[(1-C(z))A^*(\lambda) + N(1-V^*(\lambda-\lambda C(z)))(C(z)+(1-C(z))A^*(\lambda))]}{(1-C(z))(A^*(\lambda)+C(z)(1-A^*(\lambda)))(B^*(\lambda-\lambda C(z))-z)}$$

which furnish the results obtained in Chang and Ke (2009).

Case (iii)

If $C(z) \to z$, $J = 1$, $\alpha = 0$ and $\theta = 0$, then the model under study becomes M/G/1 retrial queue with vacation. In this case

$$I_0 = \frac{(A^*(\lambda)-\lambda\mu_1)V^*(\lambda)}{V^*(\lambda)A^*(\lambda)+\lambda\nu_1}$$

$$\frac{I_0}{V^*(\lambda)}[(1-z)A^*(\lambda)V^*(\lambda)+(1-V^*(\lambda-\lambda z))]$$

$$P_S(z) = \frac{(z+(1-z)A^*(\lambda))]B^*(\lambda-\lambda z)}{(z+(1-z)A^*(\lambda))\,B^*(\lambda-\lambda z)-z}$$

which are in accordance with Krishna Kumar and Arivudainambi (2002) without Bernoulli vacation.

Numerical Results

In this section numerical results are presented to illustrate the effect of various parameters on the main performance characteristics. Assume that the retrial time, service time, delay time, repair time and vacation time follow (a) Exponential distribution (b) Erlang-2 distribution and (c) Hyper Exponential distribution with respective rates η, μ, γ, β and ν. We set the default parameters as $\lambda = 0.4$, $\alpha = 2$, $\theta = 0.4$, $p = 0.4$, $J = 5$, $\eta = 30$, $\mu = 20$, $\gamma = 1$, $\beta = 0.8$, $\nu = 0.5$, $C_1 = 0.5$ and $C_2 = 0.5$.

The effect of the parameters (i) vacation probability rate p and the retrial rate η and (ii) orbital search rate θ and repair rate α on the performance measures expected number of customers in the system (L_S), availability (A) and failure frequency (F) of the server corresponding to the selected distributions are given in Tables 1 and 2.

Table 1 depicts that the influence of p and η on L_S, A and F . It is clear that

- increase in p decreases L_S and increases A.
- increase in η decreases L_S and A.
- F is independent of p and η.

From Table 2, it is observed that

- increase in θ decreases L_S and A and has no effect on F.
- increase in α increases L_S and F and decreases A.

Table 3 reveals that

- I_0+I decreases with increase in λ and η and increases with increase in μ.

- P and F increase with increase in λ, decrease with increase in μ and independent of η.
- V increases with increase in λ, μ and η.

Table 4 shows that

- increase in θ and α decreases I_0+I.
- P is independent of θ and α
- F increases with α and independent of θ
- V increases with θ and decreases with α.

Table 1 L_S, A and F versus p and η

p	η	L_S			A			F		
		Exponential	Erlang 2	Hyper Exponential	Exponential	Erlang 2	Hyper Exponential	Exponential	Erlang 2	Hyper Exponential
0.1	2.50	1.5224	45.2992	2.4684	0.3195	0.445	0.3350	0.0600	0.1200	0.0030
	2.75	1.4878	16.5166	2.4618	0.3093	0.4177	0.3322	0.0600	0.1200	0.0030
	3.00	1.4591	11.0970	2.4578	0.3006	0.3940	0.3306	0.0600	0.1200	0.0030
	3.25	1.4349	8.7975	2.4556	0.2932	0.3736	0.3297	0.0600	0.1200	0.0030
	3.50	1.4144	7.5219	2.4546	0.2869	0.3558	0.3293	0.0600	0.1200	0.0030
0.3	2.50	1.4576	45.2684	2.4529	0.3503	0.4457	0.3393	0.0600	0.1200	0.0030
	2.75	1.4235	16.4857	2.4463	0.3408	0.4184	0.3366	0.0600	0.1200	0.0030
	3.00	1.3954	11.0660	2.4423	0.3327	0.3951	0.3350	0.0600	0.1200	0.0030
	3.25	1.3717	8.7665	2.4401	0.3259	0.3750	0.3341	0.0600	0.1200	0.0030
	3.50	1.3515	7.4908	2.4391	0.3200	0.3576	0.3337	0.0600	0.1200	0.0030
0.5	2.50	1.3757	45.2251	2.3744	0.3891	0.4460	0.3615	0.0600	0.1200	0.0030
	2.75	1.3424	16.4423	2.3678	0.3805	0.4193	0.3589	0.0600	0.1200	0.0030
	3.00	1.3150	11.0225	2.3638	0.3733	0.3966	0.3574	0.0600	0.1200	0.0030
	3.25	1.2919	8.7229	2.3616	0.3671	0.3770	0.3565	0.0600	0.1200	0.0030
	3.50	1.2723	7.4472	2.3606	0.3618	0.3599	0.3561	0.0600	0.1200	0.0030
0.7	2.50	1.3084	45.1985	2.3020	0.4210	0.4462	0.3819	0.0600	0.1200	0.0030
	2.75	1.2759	16.4156	2.2953	0.4132	0.4199	0.3794	0.0600	0.1200	0.0030
	3.00	1.2491	10.9959	2.2914	0.4065	0.3975	0.3780	0.0600	0.1200	0.0030
	3.25	1.2265	8.6963	2.2892	0.4009	0.3782	0.3772	0.0600	0.1200	0.0030
	3.50	1.2073	7.4206	2.2882	0.3960	0.3614	0.3768	0.0600	0.1200	0.0030
0.9	2.50	1.2583	45.1850	2.2466	0.4448	0.4463	0.3975	0.0600	0.1200	0.0030
	2.75	1.2263	16.4021	2.2400	0.4375	0.4202	0.3952	0.0600	0.1200	0.0030
	3.00	1.2000	10.9823	2.2360	0.4313	0.3979	0.3937	0.0600	0.1200	0.0030
	3.25	1.1779	8.6827	2.2338	0.4260	0.3788	0.3930	0.0600	0.1200	0.0030
	3.50	1.1590	7.4070	2.2328	0.4215	0.3621	0.3926	0.0600	0.1200	0.0030

Table 2 L_S, A and F versus θ and α

θ	α	L_S			A			F		
		Exponential	Erlang 2	Hyper Exponential	Exponential	Erlang 2	Hyper Exponential	Exponential	Erlang 2	Hyper Exponential
0.1	0.5	0.9557	1.8691	2.4489	0.3047	0.1938	0.3863	0.0150	0.0300	0.0008
	1.0	0.9969	2.1106	2.4520	0.2953	0.1767	0.3860	0.0300	0.0600	0.0015
	1.5	1.0413	2.4651	2.4551	0.2860	0.1601	0.3858	0.0450	0.0900	0.0023
	2.0	1.0894	3.0597	2.4583	0.2766	0.1435	0.3855	0.0600	0.1200	0.0030
	2.5	1.14147	4.3249	2.4615	0.2672	0.1269	0.3852	0.0750	0.1500	0.0037
0.3	0.5	0.9549	1.8669	2.4469	0.3032	0.1913	0.3731	0.0150	0.0300	0.0008
	1.0	0.9961	2.1083	2.4500	0.2938	0.1751	0.3729	0.0300	0.0600	0.0015
	1.5	1.0405	2.4628	2.4531	0.2845	0.1589	0.3727	0.0450	0.0900	0.0023
	2.0	1.0886	3.0574	2.4563	0.2752	0.1426	0.3725	0.0600	0.1200	0.0030
	2.5	1.1409	4.3226	2.4595	0.2658	0.1264	0.3723	0.0750	0.1500	0.0037
0.5	0.5	0.9542	1.8646	2.4448	0.3016	0.1894	0.3595	0.0150	0.0300	0.0008
	1.0	0.9953	2.1060	2.4479	0.2923	0.1735	0.3593	0.0300	0.0600	0.0015
	1.5	1.0398	2.4605	2.4511	0.2830	0.1576	0.3591	0.0450	0.0900	0.0023
	2.0	1.0879	3.0551	2.4542	0.2738	0.1417	0.3589	0.0600	0.1200	0.0030
	2.5	1.1402	4.3203	2.4574	0.2645	0.1258	0.3587	0.0750	0.1500	0.0037
0.7	0.5	0.9534	1.8623	2.4426	0.3000	0.1875	0.3451	0.0150	0.0300	0.0008
	1.0	0.9945	2.1037	2.4457	0.2908	0.1719	0.3450	0.0300	0.0600	0.0015
	1.5	1.0390	2.4582	2.4489	0.2816	0.1563	0.3449	0.0450	0.0900	0.0023
	2.0	1.0871	3.0528	2.4520	0.2723	0.1408	0.3447	0.0600	0.1200	0.0030
	2.5	1.1394	4.3180	2.4552	0.2631	0.1252	0.3446	0.0750	0.1500	0.0037
0.9	0.5	0.9526	1.8600	2.4403	0.2984	0.1855	0.3302	0.0150	0.0300	0.0008
	1.0	0.9938	2.1014	2.4435	0.2892	0.1703	0.3301	0.0300	0.0600	0.0015
	1.5	1.0382	2.4559	2.4466	0.2801	0.1551	0.3300	0.0450	0.0900	0.0023
	2.0	1.0863	3.0505	2.4497	0.2709	0.1399	0.3299	0.0600	0.1200	0.0030
	2.5	1.1386	4.3157	2.4529	0.2618	0.1247	0.3298	0.0750	0.1500	0.0037

Table 3 Performance Measures versus λ, μ and η

λ	μ	η	L_0+I	P	F	V
0.15	20	5	0.5290	0.0113	0.0506	0.4091
		15	0.5253	0.0113	0.0506	0.4129
		25	0.5245	0.0113	0.0506	0.4136
		35	0.5242	0.0113	0.0506	0.4139
	30	5	0.5402	0.0075	0.0337	0.4185
		15	0.5367	0.0075	0.0337	0.4221
		25	0.5359	0.0075	0.0337	0.4228
		35	0.5356	0.0075	0.0337	0.4231
	40	5	0.5458	0.0056	0.0253	0.4233
		15	0.5424	0.0056	0.0253	0.4267
		25	0.4517	0.0056	0.0253	0.4274
		35	0.5414	0.0056	0.0253	0.4277
	50	5	0.5492	0.0045	0.0202	0.4261
		15	0.5458	0.0045	0.0202	0.4295
		25	0.5451	0.0045	0.0202	0.4301
		35	0.5448	0.0045	0.0202	0.4304
0.30	20	5	0.3457	0.0225	0.1013	0.5305
		15	0.3253	0.0225	0.1013	0.5510
		25	0.3212	0.0225	0.1013	0.5551
		35	0.3194	0.0225	0.1013	0.5569
	30	5	0.3593	0.0150	0.0675	0.5582
		15	0.3397	0.0150	0.0675	0.5578
		25	0.3357	0.0150	0.0675	0.5818
		35	0.3340	0.0150	0.0675	0.5835
	40	5	0.3661	0.0113	0.0506	0.5720
		15	0.3469	0.0113	0.0506	0.5912
		25	0.3430	0.0113	0.0506	0.5951
		35	0.3414	0.0113	0.0506	0.5968
	50	5	0.3702	0.0090	0.0405	0.5803
		15	0.3512	0.0090	0.405	0.5993
		25	0.3474	0.0090	0.0405	0.6031
		35	0.3457	0.0090	0.0405	0.6048
0.45	20	5	0.2719	0.0338	0.1519	0.5425
		15	0.2281	0.0338	0.1519	0.5863
		25	0.2191	0.0338	0.1519	0.5953
		35	0.2152	0.0338	0.1519	0.5992
	30	5	0.2851	0.0225	0.1012	0.5911
		15	0.2429	0.0225	0.1012	0.6333
		25	0.2342	0.0225	0.1012	0.6420
		35	0.2305	0.0225	0.1012	0.6458
	40	5	0.2917	0.0169	0.0759	0.6154
		15	0.2503	0.0169	0.0759	0.6569

		25	0.2418	0.0169	0.0759	0.6654
		35	0.2381	0.0169	0.0759	0.6691
	50	5	0.2957	0.0135	0.0608	0.6300
		15	0.2548	0.0135	0.0608	0.6710
		25	0.2463	0.0135	0.0608	0.6794
		35	0.2427	0.0135	0.0608	0.6831
0.60	20	5	0.2468	0.0450	0.2025	0.5057
		15	0.1778	0.0450	0.2025	0.5747
		25	0.1633	0.0450	0.2025	0.5892
		35	0.1570	0.0450	0.2025	0.5955
	30	5	0.2588	0.0300	0.1350	0.5762
		15	0.1921	0.0300	0.1350	0.6429
		25	0.1780	0.0300	0.1350	0.6570
		35	0.1720	0.0300	0.1350	0.6630
	40	5	0.2648	0.0225	0.1013	0.6114
		15	0.1992	0.0225	0.1013	0.6770
		25	0.1854	0.0225	0.1013	0.6908
		35	0.1794	0.0225	0.1013	0.6968
	50	5	0.2684	0.0180	0.0810	0.6326
		15	0.2035	0.0180	0.0810	0.6975
		25	0.1898	0.0180	0.0810	0.7112
		35	0.1839	0.0180	0.0810	0.7171

Table 4 Performance Measures versus θ and α

θ	α	I_0+I	P	F	V
0.1	0.5	0.2747	0.0300	0.0338	0.6615
	1.0	0.2653	0.0300	0.0675	0.6372
	1.5	0.2560	0.0300	0.1013	0.6128
	2.0	0.2465	0.0300	0.1350	0.5884
	2.5	0.2372	0.0300	0.1688	0.5641
0.3	0.5	0.2731	0.0300	0.0338	0.6631
	1.0	0.2638	0.0300	0.0675	0.6387
	1.5	0.2545	0.0300	0.1013	0.6143
	2.0	0.2452	0.0300	0.1350	0.5898
	2.5	0.2358	0.0300	0.1688	0.5654
0.5	0.5	0.2715	0.0300	0.0338	0.6647
	1.0	0.2623	0.0300	0.0675	0.6402
	1.5	0.2530	0.0300	0.1013	0.6157
	2.0	0.2437	0.0300	0.1350	0.5912
	2.5	0.2345	0.0300	0.1688	0.5668
0.7	0.5	0.2700	0.0300	0.0338	0.6663
	1.0	0.2607	0.0300	0.0675	0.6417
	1.5	0.2515	0.0300	0.1013	0.6172
	2.0	0.2424	0.0300	0.1350	0.5927
	2.5	0.2332	0.0300	0.1688	0.5681
0.9	0.5	0.2684	0.0300	0.0338	0.6679
	1.0	0.2592	0.0300	0.0675	0.6433
	1.5	0.2501	0.0300	0.1013	0.6187
	2.0	0.2409	0.0300	0.1350	0.5941
	2.5	0.2318	0.0300	0.1688	0.5695

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