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An Application of Stochastic Modelling to Ncd System of General Insurance Company

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ABSTRACT

In most of general insurance particularly automobile insurance premiums are determined by use of a No-Claim-Discount (NCD) system. NCD is a system that adjusts the premium paid by a customer according to his/her individual claim history. The fundamental principle of NCD system is that the higher frequency of policyholder the higher the insurance cost that on an average are charged to the policyholder.

In this paper we trace for a stochastic model to represent the NCD system. Here we consider a discrete time parameter Markov Chain, Where the state-space consists of a different level of a premium and the state of a particular insured shift randomly from a year to the next. The randomness of the transition is governed by the transition probability of causing an accident in a given year. For doing this we consider a positive integer valued state and the annual premium is a function of these states.

KEYWORDS: NCD, stochastic process, Markov Chain, transition probability, state space

Introduction:

Insurance other than 'Life Insurance' falls under the category of General Insurance. General insurance comprises of insurance of property against fire, burglary etc., personal insurance such as Accident and health insurance, and liability insurance which covers legal liabilities. They also covers such as Errors and Omissions insurance for professionals, credit insurance etc. No Claim Discount (NCD) system (also called No Claim Bonus (NCB)) sometimes called Bonus Malus System (BMS) are experience rating system which are commonly used in motor insurance. The term Bonus Malus (Latin for good-bad) is used for number of business arrangement which alternately reward (bonus) or penalize (Malus). It is used for example, in call centre and insurance industries. In insurance, an NCD is a system that adjusts the premium paid by a customer according to his/her individual claim history. Discount usually is a bonus in the premium which is given on the renewal of the policy if no claim is made in the previous year. There is no discount in the premium on the renewal of the policy if there is a claim in the previous year.

The NCD Ratings of Indian car Insurance Companies:

NCD schemes (Karm Pul, B.S. Bodla, M. C. Garg 2007,) represent premiums relative to their claim experience. Those who have made few claims in recent years are rewarded with discounts on their initial premium and hence are enticed to stay with the company. Depending on the rules in the scheme new policy holders may be required to pay the full premium initially and then obtain discounts in the future as a result of claim free years.

Indian car insurance companies offer discounts of 0%, 20%, 25%, 35%, 45% and 50% of the own damage premium, a policy holder's status being determined by the following rules as given in the following table:

All Types of Vehicles	Percentage of Discount
All new policy holders	0%
No claim made or pending during the preceding full year of insurance.	20%
No claim made or pending during the preceding two consecutive years of insurance.	25%
No claim made or pending during the preceding three consecutive years of insurance.	35%
No claim made or pending during the preceding four consecutive years of insurance.	45%
No claim made or pending during the preceding five consecutive years of insurance.	50%

The schedule depreciation for arriving at IDV (Insured Declared Value):

Age of The Vehicle	Percentage of Depreciation for Fixing IDV
Not exceeding 6 Months	5%
Exceeding 6 Months not exceeding one year	15%
Exceeding one year not exceeding two years	20%
Exceeding two years not exceeding three years	30%
Exceeding three years not exceeding four years	40%
Exceeding four years not exceeding five years	50%

Objective of this Work:

In automobile insurance, among other general insurance policies, it is quite common to reduce the premium by a factor in case the insured does not make any claim in a given period. This is popularly known as NCD or No Claim Discount(Anand Ganguly ,2007; Karm Pul et al. 2007). Equally popular is the practice of increasing (Known as 'loading') the premium, in case a claim is made. In effect, either system amount to a multi layer premium policy, where any particular policyholder is required to pick the level depending on the history of claim he/she made in immediate past few years.

In this study, we trace for a stochastic model to represent the NCD system. The general insurance actuarial modeling an NCD scheme would frequently use Markov Chain methods to investigate how premiums and movements take place over time.

The basic framework that we consider is that of a discrete time parameter Markov Chain (Norris, James R., 1998), Where the state-space consists of a different level of a premium and the state of a particular insured shift randomly from a year to the next. The randomness of the transition is governed by the transition probability of causing an accident in a given year. Before going to main modeling of NCD system by using Markov Chain, it is important to describe what is stochastic model and how can we find it.

In most of real life phenomenon enables us to introduce random or chance variables in a model to draw useful conclusions. A model involving a random variable or chance factor is called a stochastic or probability model.

Specification of Stochastic Process and Markov Chain:

Stochastic process (Parzen,E.,1962; *J. Medhi*,1994) is basically a family of random variables indexed by time. It is of paramount important to realize that, in general these random variables are dependent on one another.

A stochastic process is a model for a time dependent random phenom-

enon. So, just as a single random variable describes a static random phenomenon, a stochastic process is a collection of random variables, one for each time in some set J. The process is denoted. The set of values that the random variables are capable of taking is called the state space of the process, S. The stochastic process can be classified in general into the following four types of processes:

- Discrete time discrete state space,
- Discrete time continuous state space,
- Continuous time discrete state space,
- Continuous time continuous state space.

All the four types may be represented by **.** In case of discrete time the parameter generally used is n, i.e. the family is represented by . In case of continuous time both the symbols and (where is a finite or infinite interval) are used.

Markov Process and Markov Chain:

A major simplification occurs if the future development of a process can be predicted from its present state alone, without any reference to its past history. Symbolically we can write:

$$P[(X_t \in A \mid X_{s_1} = x_1, X_{s_2} = x_2, \ldots, X_{s_n} = x_n, X_s = x)] = P(X_t \in A \mid X_s = x) \qquad (1)$$

for all times $s_1 < s_2 < s_3 \ldots < s_n < s < t$, all states and is and all subsets of . This is called the Markov Property.

When a Markov process has a discrete state space and a discrete time set it is called a Markov Chain. i.e. the term Markov Chain refers to those processes in discrete time and with a discrete state space that satisfy the Markov Property.

Let us consider a stochastic process $\{X_n, n = 0, 1, 2, \ldots\}$ that takes on a finite or countable number of possible values. This set of possible values of the process will be denoted by the set of nonnegative integers. $\{0,1,2,\ldots\}$. If $X_n = i$ then the process is said to be in state at time . We suppose that whenever the process is in state , there is a fixed probability that it will next be in state . i.e. we suppose that

$$P\{X_{n+1} = j \mid X_n = i, X_{n-1} = i_{n-1}, \ldots, X_1 = i_1, X_0 = i_0\} = p_{ij} \qquad (2)$$

For all states $i_0, i_1, i_2, \ldots, i_{n-1}, i, j$ and for all $n \geq 0$. Such a stochastic process is known as Markov Chain.

Equation (2) may be interpreted as stating that, for a Markov Chain, the conditional distribution of any future state given the past states and depends only on the present state.

The value *Pij* represents the probability that the process will, when in state *i*, next make a transition into state *j*. Since probabilities are nonnegative and since the process must make a transition into some state, we have that

$$p_{ij} \geq 0, i, j \geq 0; \quad \sum_{j=0}^{\infty} p_{ij} = 1, \quad i = 0, 1, 2, \ldots$$

Let denote the matrix of one state transition probabilities *pij*, so that

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} & \cdots \\ p_{10} & p_{11} & p_{12} & \cdots \\ \vdots & \vdots & \vdots & \cdots \\ p_{i0} & p_{i1} & p_{i2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Chapman-Kolmogorov Equations:

We have already defined the one step transition probabilities . We now define the n step transition probabilities to be the probability that a process in state will be in state after n additional transitions. i.e.,

$$p_{ij}^n = P\{X_{n+k} = j \mid X_k = i\}, \quad n \geq 0, \ i, j \geq 0 \qquad (3)$$

The Chapman Kolmogorov (Papoulis, A., 1984; Sheldon M Ross, 2010) equations provide a method for computing these n-step transition probabilities. These equations are

$$P_{ij}^{n+m} = \sum_{k=0}^{\infty} P_{ik}^n P_{kj}^m, \text{ for all } n, m \geq 0, \text{ all } i, j \qquad (4)$$

and are most easily understood by noting that $P_{ik}^n P_{kj}^m$ represents the probability that
starting in the process will go to state *j* in *n+m* transitions through a path which takes it into state at the th transition. Hence ,summing over all intermediate states yields the probability that the process will be in state after *n+m* transitions. Formally, we have

$$P_{ij}^{n+m} = P\{X_{n+m} = j \mid X_0 = i\}$$

$$= \sum_{k=0}^{\infty} P\{X_{n+m} = j, X_n = k \mid X_0 = i\}$$

$$= \sum_{k=0}^{\infty} P\{X_{n+m} = j \mid X_n = k, X_0 = i\} P\{X_n = k \mid X_0 = i\}$$

$$= \sum_{k=0}^{\infty} P_{kj}^m P_{ik}^n$$

If we let $P^{(n)}$ denote the matrix of *n*- step transition probabilities P_{ij}^n, then equation (4) asserts that

$$P^{(n+m)} = P^{(n)} \cdot P^{(m)}$$

where the dot(.) represents matrix multiplication. Hence in particular,

$$P^{(2)} = P^{(1+1)} = P \cdot P = P^2$$

and by induction

$$P^{(n)} = P^{(n-1+1)} = P^{n-1} \cdot P = P^n$$

that is, the -step transition matrix may be obtained by multiplying the matrix *P* by itself *n* times.

Modeling Using Markov Chain:

Using the principle of economy of effort, it is common to start the modeling processed by attempting to fit a simple stochastic model, such as Markov Chain, to a set of observations. If test show that this is inadequate, more sophisticated model can be attempted at the next stage of modeling process.

It is assumed that the model is time homogenous. However, the fitting of a time homogenous model is generally more complicated.

Estimating Transition Probabilities:

The first thing to fix when setting up a Markov Model , the state space which first springs to mind may not be the most suitable and may need some modification before a Marcov Model can be fitted.

Once the state space is determined, however, the Markov model must be fitted to the data by estimating the transition probabilities

Denote by $X_1, X_2, \ldots, X_N$ the available observation and define as the number of times

t($1 \leq t \leq N-1$) such that $x_i = i$; n_{ij} as the number of times t($1 \leq t \leq N-1$) such that $x_t = i$ and $x_{(t+1)} = j$

Thus *nij* is the observed number of transitions from state *i* to *j, ni* is the observed number of transitions from state *i*.

The reason that the definition of only allows to go up to , rather than N, is show that it equals the number of chances for a transition out of state , and not just the number of times it is in state

Then the best estimate of P_{ij} is $\hat{P}_{ij} = \frac{n_{ij}}{n_i}$

If a confidence interval is required for a transition probability, the fact that the conditional distribution of given is Binomialmeans that a confidence interval may be obtained by standard techniques.

The following table specified Indian NCD system of General insurance companies having 12 states:

state	Description	Discount level			State Space
		Own damage premium	Depreciation for fixing IDV	Total	
1	All new policyholder	0%	5%	5%	1
2	New policyholder who make a claim and whose age of vehicle is less than six month	0%	15%	15%	2

3	No claim made during preceding full year	20%	20%	40%	5-
4	A claim made during one consecutive year	0%	20%	20%	3-
5	No claim made during two consecutive years	25%	20%	45%	6
6	A claim made during two consecutive years	0%	20%	20%	3+
7	No claim made during three consecutive years	35%	30%	65%	8
8	A claim made during three consecutive years	0%	30%	30%	4
9	No claim made during four consecutive years	45%	40%	85%	9
10	A claim made during four consecutive years	0%	40%	40%	5+
11	No claim made during five consecutive years and after five years	50%	50%	100%	10
12	A claim made during five consecutive years and after five years	0%	50%	50%	7

Transition between present state to future state:

State	Next state if							
	0 claim				At least 1 claim			
			total	state			total	state
1	20	20	40	5	0	20	20	3
2	20	20	40	5	0	20	20	3
5-	25	20	45	6	0	20	20	3
3-	20	30	50	7	0	30	30	4
6	35	30	65	8	0	30	30	4
3+	20	30	50	7	0	30	30	4
8	45	40	85	9	0	40	40	5+
4	20	40	60	11	0	40	40	5+
9	50	50	100	10	0	50	50	7
5+	20	50	70	12	0	50	50	7
10	50	50	100	10	0	50	50	7
7	20	50	70	12	0	50	50	7
11	25	50	75	13	0	50	50	7
12	25	50	75	13	0	50	50	7
13	35	50	85	9	0	50	50	7

Estimation of Transition Probability Matrix From given Data:

For estimating transition probability matrix first we should convert the data into matrix form: To obtain the matrix (, we consider the different levels of discount as state spaces as given in the above table. The fundamental law of NCD system is that a policyholder get no discount on own damage portion in the next year if he made at least one claim in the present year. Thus, applying this principle, we get the (matrix as follows:

$(n_{ij}) =$

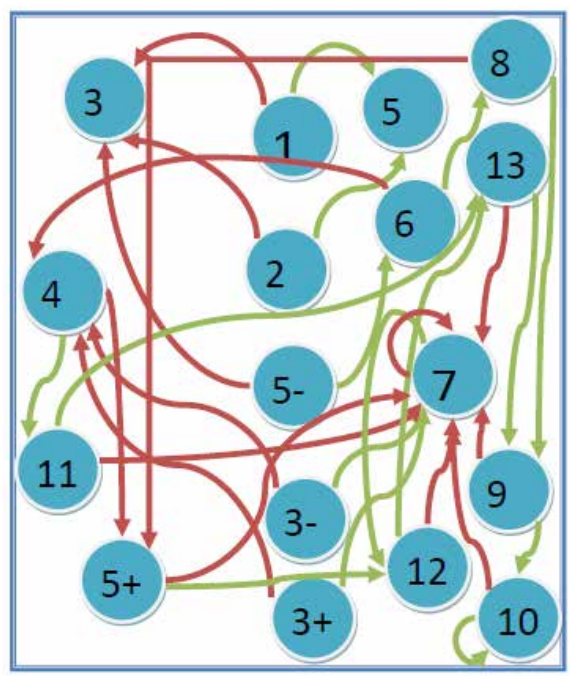
	1	2	5-	3-	6	3+	8	4	9	5+	10	7	11	12	5	13	
1	0	0	0	0	0	13	0	0	0	0	0	0	0	58	0		71
2	0	0	0	0	0	6	0	0	0	0	0	0	0	23	0		29
5-	0	0	0	0	23	9	0	0	0	0	0	0	0	0	0		32
3-	0	0	0	0	0	0	0	14	0	0	0	53	0	0	0		67
6	0	0	0	0	0	0	32	8	0	0	0	0	0	0	0		40
3+	0	0	0	0	0	0	0	7	0	0	0	27	0	0	0		34
8	0	0	0	0	0	0	0	0	31	12	0	0	0	0	0		43
4	0	0	0	0	0	0	0	0	0	15	0	0	34	0	0		49
9	0	0	0	0	0	0	0	0	0	0	21	4	0	0	0		25
5+	0	0	0	0	0	0	0	0	0	0	0	7	0	22	0		29
10	0	0	0	0	0	0	0	0	0	0	21	2	0	0	0		23
7	0	0	0	0	0	0	0	0	0	0	0	9	0	33	0		42
11	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0		47
12	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0		48
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0
13	0	0	0	0	0	0	0	0	37	0	0	12	0	0	0		49

The transition probability matrix is obtained by using maximum likelihood estimate, which is simply obtained by dividing each element of () by corresponding total row sum. Therefore the transition matrix will be:

$P =$

	1	2	5-	3-	6	3+	8	4	9	5+	10	7	11	12	5	13
1	0	0	0	0	0	0.183	0	0	0	0	0	0	0	0	0.817	0
2	0	0	0	0	0	0.206	0	0	0	0	0	0	0	0	0.794	0
5-	0	0	0	0	0.719	0.281	0	0	0	0	0	0	0	0	0	0
3-	0	0	0	0	0	0	0	0.209	0	0	0	0.791	0	0	0	0
6	0	0	0	0	0	0	0.8	0.2	0	0	0	0	0	0	0	0
3+	0	0	0	0	0	0	0	0.206	0	0	0	0.794	0	0	0	0
8	0	0	0	0	0	0	0	0	0.721	0.279	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0.306	0	0	0.694	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0.840	0.160	0	0	0	0
5+	0	0	0	0	0	0	0	0	0	0	0	0.241	0	0.759	0	0
10	0	0	0	0	0	0	0	0	0	0	0.913	0.087	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0.214	0	0.786	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0.064	0	0	0	0.936
12	0	0	0	0	0	0	0	0	0	0	0	0.104	0	0	0	0.896
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0.755	0	0	0.245	0	0	0	0

The Transition Diagram of the Transition Probability Matrix of NCD system is given in the following:



Confidence Interval for Transition Probabilities:

A confidence interval provides an 'interval estimate' of an unknown parameter. It is design to contain the parameters value with some stated probability. The width of the interval provides a measure of the precision accuracy of the estimator involved. Generally if sample size is large, the confidence interval for estimate is approximately normally distributed about the population proportion with standard deviation √(pq/n). For the true but unknown standard deviation√(pq/n) , we substitute the sample estimate √((pq)/n). Hence the probability is approximately 0.95 that lies between the limits

$$p - 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} \text{ and } p + 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

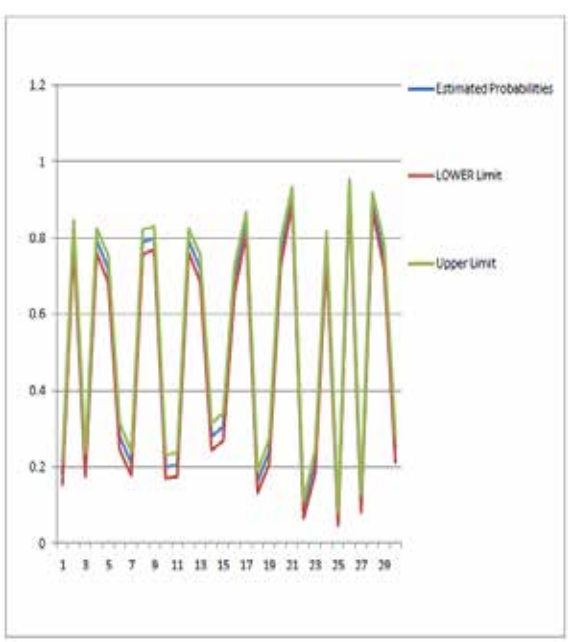
but this statement is equivalent to saying that p lies between

$$\hat{p} - 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} \text{ and } \hat{p} + 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

The 95% confidence intervals for transition probabilities are given in the following table:

Estimated Probabilities	Lower Limit	Upper Limit
0.183	0.152757839	0.213242161
0.817	0.786757839	0.847242161
0.206	0.174368476	0.237631524
0.794	0.762368476	0.825631524
0.719	0.683844453	0.754155547
0.281	0.245844453	0.316155547
0.209	0.17719923	0.24080077
0.791	0.75919923	0.82280077
0.8	0.768714994	0.831285006
0.2	0.168714994	0.231285006
0.206	0.174368476	0.237631524
0.794	0.762368476	0.825631524
0.721	0.685921099	0.756078901
0.279	0.243921099	0.314078901
0.306	0.269957351	0.342042649
0.694	0.657957351	0.730042649
0.84	0.811326818	0.868673182
0.16	0.131326818	0.188673182
0.241	0.207549265	0.274450735
0.759	0.725549265	0.792450735
0.913	0.890956971	0.935043029
0.087	0.064956971	0.109043029
0.214	0.181922951	0.246077049
0.786	0.753922951	0.818077049
0.064	0.044857257	0.083142743
0.936	0.916857257	0.955142743
0.104	0.080124805	0.127875195
0.896	0.872124805	0.919875195
0.755	0.721361799	0.788638201
0.245	0.211361799	0.278638201

The plot of Lower Limit, Upper Limit, and Estimated probabilities are given in the figure in the next page:



Transition probability matrix of higher order determining stationary distribution:

$$P^{(34)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.65195 & 0 & 0.629468 & 0.122613 & 0 & 0.09674 & 0 & 0.08635 \end{bmatrix}$$

We see the matrix , all the rows are identical and the higher order matrices are same. So the stationary distribution of the transition probabilities is:

$V' = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.65195\ 0\ 0.629468\ 0.122613\ 0\ 0.09674\ 0\ 0.08635]$

Conclusion:

In the present work, the main intention was building a model of NCD system of general insurance policy by using Markov Chain analysis. The matrix of transition probabilities has been estimated and it is observed that the fitting is quite well. The stationary distribution of the probabilities has also been obtained. This study demonstrates the use of Markov Chain in analyzing N Claim Discount system of general insurance company.

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