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Coloring

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ABSTRACT

The graph coloring problem is to assign colors to vertices of graphs such that no two adjacent vertices receive the same color; such a coloring is referred to as a legal coloring of G. The minimum number of colors required to do a legal coloring is known as the chromatic number of G, and is denoted χ (G). Graph coloring is a fundamental and extensively studied problem.

Now we're going to prove a theorem about chicken tournaments. The result is not very useful, but the proof involves both induction and digraphs.

KEYWORDS :

Vertex Coloring:
Let G = (V, E) be a graph and let C= {c_1, c_2 ..... c_k} ☐ be a finite set of colors (labels). A vertex coloring is a mapping c: V☐→ C with the property that if☐ {v_1, v_2} ☐ ∈ E, then c(v_1)≠c(v_2).

Now we show an example of a graph coloring in bellow Figure 1 in contain graph has three colors.

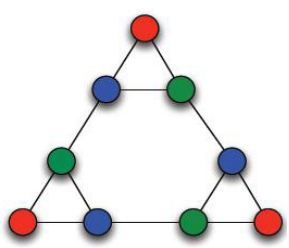


Figure 1 graph of coloring
K-Colorable:

A graph G = (V, E) is a k-colorable if there is a vertex coloring with k colors.

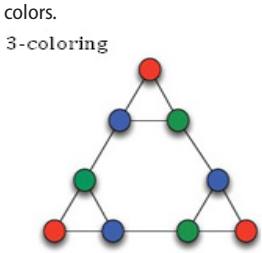


Figure 2 graph of 3-colorable

Clearly, every graph G = (V, E) is |V| colorable, since we can assign a different color to each vertex. We are usually interested in the minimum number of colors we can get away with and still color a graph.

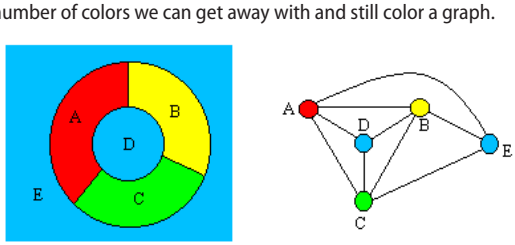


Figure 3: graph of 4-colorable

Chromatic Number:
Let G = (V, E) be a graph. The chromatic number of G, written χ (G) is the minimum integer k such that G is k-colorable.

Bipartite Graphs:
The 2colorable graphs are important enough to merit a special name; they are called bipartite graphs. Suppose that G is bipartite. This means we can color every vertex in G either one color or second color so that adjacent vertices get different colors. Then we can put all the one color vertices in a clump on the left and all the second color vertices in a clump on the right. Since every edge joins differently colored vertices, every edge must run between the two clumps. Therefore, every bipartite graph looks something like this:

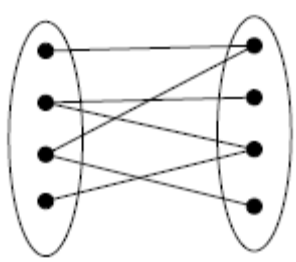


Figure 4 bipartite graph

Lemma: For a finite, loop-free graph, G, χ (G) = 2 if and only if G is a bipartite graph.

Proof: Firstly, suppose that G is a bipartite graph, with parts A and B, then color all vertices in A one color, and all vertices in B second color. Then every edge joins a second color vertex to a one color vertex. Therefore, G is 2-colorable, so χ (G) =<2.

Now, χ (G) > 1, since once you have an edge, you need at least two colors.

Next, assume that χ (G) = 2. Consider any vertex coloring of G with colors one color and second color. Then, by the definition of a vertex coloring, every edge joins a one color vertex to a blue vertex. Therefore, G is bipartite with one color vertices in one part, and second color vertices in the other part. This completes the proof of the lemma.

Theorem 1: A graph with maximum degree at most n is (n + 1) colorable.
Proof: We use induction on the number of vertices in the graph, which we denote by n.

Let P (n) be the proposition that an n-vertex graph with maximum degree at most n is (n + 1) colorable.

A 1vertex graph has maximum degree 0 and is 1colorable, so P (1) is true.

Now assume that P(k) is true, and let G be an (k+ 1)vertex graph with maximum degree at most k.

Remove a vertex v, leaving an n-vertex graph G 0. The maximum degree of G0 is at most k, and so G0 is (k + 1) colorable by our assumption P (k). Now add back vertex -v. We can assign v a color different from all adjacent vertices; since v has degree at most k and k + 1 color are available. Therefore, G is (k + 1) colorable. The theorem follows by induction.

Theorem: If every vertex of G has degree d(v) < k, then G is k-colorable.
Proof: Use induction on n, the number of vertices of the graph G.

If n = 1 or n = 2, the assertion is easily seen to be true.

Suppose n > 2, and assume that the proposition is valid for all graphs with fewer than n vertices.

Choose any vertex v of G and delete it and all the edges incident to v. This leaves a sub graph H of G with n - 1 vertices satisfying the given hypothesis (i.e. that every vertex has degree less than k). By the inductive hypothesis, χ (H) k, i.e. H is k-colorable.

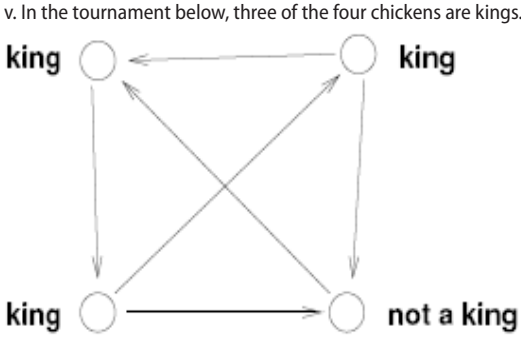
Now, consider any particular k-coloring of H. Since d (v) < k, the vertices of H that were adjacent to v in G are colored with at most k-1 different colors.

Thus, there's at least one color left with which we may color v, so that it is of a different color to each of its neighbors. This gives a coloring of G using the same colors as H. Therefore, G is k-colorable. This completes the proof of the theorem.

The King Chicken Theorem:
There are ***n*** **-**chickens in a farmyard. For each pair of distinct chickens, either the first pecks the second or the second pecks the first, but not both. We say that chicken ***u*** -virtually pecks chicken ***v*** if either:

• Chicken ***u*** pecks chicken ***v***.
• Chicken ***u*** pecks some other chicken ***w*** that in turn pecks chicken ***v***.

A chicken that virtually pecks every other chicken is called a king chicken. We can model this situation with a tournament digraph. The vertices are chickens, and an edge u® v indicates that chicken u pecks chicken v. In the tournament below, three of the four chickens are kings.



Now we're going to prove a theorem about chicken tournaments. The result is not very useful, but the proof involves both induction and digraphs.

Theorem 3: State and Prove King Chicken Theorem.
State: Every nchicken tournament has a king, where n☐≥ 1.

Proof: The proof is by induction on n, the number of chickens in the tournament.

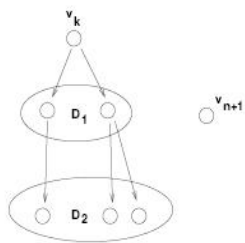
Let P (n) be the proposition that in every nchicken tournament, there is at least one king.

First, we prove P (1). In this case, we can safely say that the lone chicken virtually pecks every other chicken, since there are no others. Therefore, the only chicken in the tournament is a king, and so P (1) is true.

Next, we must show that P (n) implies P (n + 1) whenever n☐≥ 1. Suppose there is a chicken tournament with chickens v1. . . vn+1. If we ignore the last chicken for the

But if a chicken is a king, isn't it male? And if it is male, isn't it a rooster? Oh well. Moments, then we are left with a tournament among the first n chickens. By our induction hypothesis, P (n), this tournament has a king chicken, vk.

Let D1 be the set of chickens pecked by the king, vk. Let D2 be the set of chickens virtually pecked by the king, but not pecked directly. Thus, each chicken in D2 was pecked by some chicken in D1. Since vk is a king, this accounts for all the chickens; that is, ☐{vk}, D1, and D2 form a partition of the set of chickens{v1, . . . , vn}. The situation is represented schematically below.



Now we reintroduce the last chicken, vn+1, and show that the full tournament on n + 1 chickens has a king. There are two cases:

1. Suppose that vk pecks vn+1. Then vk is a king of the full tournament.
2. Otherwise, vn+1 pecks vk.

There are then two sub-cases: (a) If some chicken in D1 pecks vn+1, then vk virtually pecks vn+1 and so vk is again a king of the full tournament. (b) Otherwise, vn+1 pecks every chicken in D1. In this case, vn+1 are a king of the full tournament; he directly pecks vk and all the chickens in D1, and he virtually pecks all the chickens in D2.

In every case, a chicken tournament with $n + 1$ chickens has a king, and

so $P(n + 1)$ holds. Thus, by the principle of induction, the claim is proved.

Conclusion:
This paper information is about coloring of graph. This paper various theorem of coloring of various types of graph. It's helpful to understand coloring graph. But not include planner graph in this paper also include k-colorable graph and bipartite coloring graph. And last, various application of coloring graph but in this paper only one application chicken king involve. So that this paper fantastic about coloring graph.

REFERENCES

Christofides, N. "An Algorithm for the Chromatic Number of a Graph." Computer J. 14, 38-39, 1971. | | Gould, R. (Ed.). Graph Theory. Menlo Park, CA: Benjamin-Cummings, 1988. | | Manvel, B. "Extremely Greedy Coloring Algorithms." In Graphs and Applications (Ed. F. Harary and J. Maybee). New York: Wiley, pp. 257-270, 1985. | | Matula D. W.; Marble, G.; and Isaacson, J. D. "Graph Coloring Algorithms." In Graph Theory and Computing (Ed. R. Read). New York: Academic Press, pp. 109-122, 1972. | | Skiena, S. "Finding a Vertex Coloring." 5.5.3 in Implementing Discrete Mathematics: Combinatorics and Graph Theory withMathematica. Reading, MA: Addison-Wesley, pp. 214-215, 1990. | | Wilf, H. "Backtrack: An Expected Time Algorithm for the Graph Coloring Problem." Info. Proc. Let. 18, 119-121, 1984. | | Lovasz, L., 1975. "Three short proofs in Graph Theory", J. Combinatorial Theory, No., 19, pp.111-113. | | Gibson, A., 1994. Algorithmic Graph Theory, (Cambridge University Press). | | Gross, J. and Yellen, J., 1999. Graph Theory and Its Applications, (CRC Press). | | Guan, D., 1999. "The game chromatic number of outerplanar graphs", Journal of Graph Theory, No. 30, pp. 67-70. |