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Decision Making Under Uncertainty Using Mathematical Optimization: A Survey

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ABSTRACT

The Women Self Help Group (WSHG) represents a unique approach to financial intermediation in India. The system combines access to low-cost financial services with a process of self management and development for the women who are SHG members. The WSHG enable women to grow their savings and to access the credit which banks are increasingly willing to lend. The major challenge of WSHGS are maximizes the expectation of some function of the decisions and the random variables. In this aspect, we provide a survey of mathematical optimization techniques for decision making under uncertainty. That is, this paper analyzes the data related to designing mechanism of WSHG. The concerned data is collected from Tuticorin district, Tamilnadu, India.

KEYWORDS: WSHG, Stochastic programming, Chance constrained programming technique

2. Introduction

After the impact of industrial revolution in India, large scale industries have been established all over India. On the other side, small scale industries have been developed with the assistance of central and state governments and they are controlled by co-operative societies. According to a survey, these schemes have not reached the people who belong to rural area under the poverty line, particularly women citizens. The Government of India has introduced a new scheme, namely, Self Help Group Scheme. Men and Women have formed separate groups.

The concept of WSHG serves the principle, "for the people by the people". As a principle of WSHG, it is expected to extend financial services to the poor and contribute to the alleviation of rural poverty. WSHG members reflect a diverse membership covering different social and economic categories, including the poor. They are either working together or living in the neighbourhood, engaged in similar lines of activity ranging from 12-20 members. The group shall not consist of more than one member from the same family. A person should not be a member of more than one group. Based on the financial aids provided by the Government, WSHGs have utilized the funds in different angles according to the ability of group members and their environment. Some of the groups try to increase the fund level by giving loans to the members themselves. Most of the groups have utilized the fund to produce needy products and earn profits for the groups. Coir, dress materials, beedi, food products, materials from palm and coconut leaves, etc are the products produced by the members individually or jointly.

The analysis of designing mechanism of WSHG is taken into consideration. The study area is Tuticorin district of Tamilnadu in which the members are farmers, designers, small shopkeepers, sales women and etc. The data concern the designers who are doing designing work of dress materials in each unit. Collections of data, which are well referenced in the stochastic programming literature and publicly available, are used. Thus we consider chance-constrained optimization technique when problem data uncertainty is characterized by probabilistic distributions. The sources of random variables may be several, dependency on the nature and the type of problem.

Decision problems of stochastic or probabilistic optimization arise when certain coefficient of an optimization model are not fixed or known but are instead, to some extent, stochastic (or random or probabilistic) quantities. In the late 1950s, stochastic linear programming (SLP) appeared with the further application of linear programming. The study of stochastic programming dates back to Beale (1955) and Dantzig (1955). Stochastic optimization problems deals with chance constraints, which dates back to Charns and Cooper (1959), and is the topic of a series of recent papers (Calafiore and Campi 2003, 2004, 2006; Chen et al. (2007); de Farias and Van Roy (2004); Erdo gan and Iyengar 2004, 2007; Henrion 2007; Lagoa et al. 2005). One issue with stochastic optimization problems is the assumption of full distributional knowledge in each and every of the uncertain data. Because such information may rarely be available in practice, it has rekindled recent interests in robust optimization as an alternative perspective of data uncertainty (Ben-Tal et al. 2004; Bertsimas and Sim 2003, 2004a, b, 2006). A different approach is suggested by Chen et al. (2007) for chance-constrained stochastic programs, which assumes only limited distributional information such as known mean, support, and some deviation measures of the random data.

Stochastic Linear decision variables allow us to derive tractable formulations in a variety of applications such as production planning and scheduling, location, transportation, finance, and engineering design, designing mechanism require that decisions be made in the presence of uncertainty.

2 Stochastic Linear Programming Model

Stochastic programming deals with the theory and methods of incorporating stochastic variations into a mathematical programming model.

A stochastic linear programming problem can be modeled by the following linear program:

Maximize or Minimize $f(X) = c^T X$ (2.1) Subject to

$$A_i^T X = \sum_{j=1}^{n} a_j x_j \geq b_i, \; i=1, 2, \ldots, m \quad (2.2)$$

and $x_j \geq 0$, $j = 1, 2, 3 \ldots n$ (2.3)

Where c_j, a_{ij} and b_i are random variables (the decision variables x_j are assumed to be deterministic for simplicity) with known probability distributions. Several methods are available for solving the problem stated the stochastic linear programming problem. This research addresses chance constrained programming technique will be considered.

2.1. Chance Constrained Programming Technique

Chance Constrained Programming belongs to the major approaches for dealing random parameters in optimization problems. Typical areas of application are engineering and finance where uncertainties like product demand, meteorological or demographic conditions, current exchange rates etc.

In chance constrained programming, the stochastic linear programming problem is stated as follows:

Maximize or Minimize $f(X) = \sum_{j=1}^{n} c_j x_j$ (2.1.1)

Subject to

$P\left[\sum_{j=1}^{n} a_{ij} x_j \le b_i\right] \ge p_i, i = 1,2,\ldots,m$ (2.1.2)

and $x_j \geq 0$, j =1, 2, 3….n (2.1.3)

Where c_j, a_{ij} and b_i are random variables and p_i are specified probabilities. Notice that equations (2.1.2) indicate that the ith constraint, $\sum_{j=1}^{n} a_{ij} x_j \le b_i$ has to be satisfied with a probability of at least p_i where $0 \le p_i \le 1$. For simplicity, we are assuming that the decision variables x_j are deterministic. We shall first consider special cases where only c_j or a_{ij} or b_i are random variables before considering the general case in which c_j, a_{ij} and b_i are all random variables. We shall further assume that all the random variables are normally distributed with known mean and standard deviations.

When c_j, a_{ij}, b_i are random variables: As the random variables c_j j = 1, 2, ... ,n, appear only in the objective function, the new objective function is

Maximize $F(X) = k_1 \overline{f} + k_2 \sqrt{Var(f)}$ (2.1.4)

Subject to

$$P[h_i \le 0] \ge p_i, i = 1,2,\ldots,m \quad (2.1.5)$$

Where $\overline{f} = \sum_{j=1}^{n} \overline{c_j} x_j$, k_1 and k_2 are nonnegative constants whose values indicate the relative important of $\overline{f}$ and standard deviation of f for minimization or maximization and Where h_i is a new random variable defined as

$h_i = \sum_{j=1}^{n} a_{ij} x_j - b_i = \sum_{k=1}^{n+1} q_k y_k$

where $q_k = a_k$, k=1,2,3……n, $q_{i,n+1} = b_i$, $y_k = x_k$, $k = 1,2,\ldots,n$ and $y_{n+1} = -1$, it will follow normal distribution, the mean and the variance of h_i are given by

$$\overline{h_i} = \sum_{k=1}^{n+1} \overline{q_k} \overline{y_k} = \sum_{j=1}^{n} \overline{a_j} \overline{x_j} - \overline{b} \quad (2.1.6)$$

and Var $(h_i) = Y^T V_i Y$ (2.1.7)

where $Y = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ y_{n+1} \end{Bmatrix}$

and

$$V_i = \begin{bmatrix} Var(q_{i1}) & Cov(q_{i1}, q_{i2}) & . & . & Cov(q_{i1}, q_{i,n+1}) \\ Cov(q_{i2}, q_{i1}) & Var(q_{i2}) & . & . & Cov(q_{i2}, q_{i,n+1}) \\ . & . & . & . & . \\ . & . & . & . & . \\ Cov(q_{i,n+1}, q_{i1}) & Cov(q_{i,n+1}, q_{i2}) & . & . & Var(q_{i,n+1}) \end{bmatrix}$$

This can be written as

$$Var(h_i) = \sum_{k=1}^{n+1} \left[x_k^2 Var(a_k) + 2 \sum_{i=k+1}^{n} x_k x_i Cov(a_k, a_i) \right] + Var(b_i) - 2 \sum_{k=1}^{n} x_k Cov(a_k, b_i) \quad (2.1.8)$$

Thus the constraints in (2.1.5) can be restated as

$$P\left[\frac{h_i - \overline{h_i}}{\sqrt{Var(h_i)}} \le \frac{-h_i}{\sqrt{Var(h_i)}}\right] \ge p_i, \quad i = 1,2,\ldots,m \quad (2.1.9)$$

Where $\frac{h_i - \overline{h_i}}{\sqrt{Var(h_i)}}$ represents a standard normal variable with zero mean and unit variance.

Thus if e_i denotes the value of standard normal variable at which

$$\phi(e_i) = p_i \quad (2.1.10)$$

the constraints of equation(2.1.9) can be stated as

$$\phi\left(\frac{-h_i}{\sqrt{Var(h_i)}}\right) \ge \phi(e_i), \text{ i =1, 2,……, m} \quad (2.1.11)$$

The stochastic linear programming problem can be stated as

Minimize $F(X) = k_1 \sum_{j=1}^{n} \overline{c_j} x_j + k_2 \sqrt{X^T K}$,

Subject to

$$\overline{h_i} + e_i \sqrt{Var(h_i)} \le 0, i = 1,2,3,\ldots,m$$

and $x_j \geq 0$, j = 1,2……,n (2.1.12)

3 Analysis

Based on the designing mechanisiam of WSHG consists of two categories. One named as magalir thittam and another is non magalir thittam. These categories designing Embroidery work, Design with glass work and fancy work. The expected designing items are available per week on different categories and the profit on each decorate works are given below. The designing items required on different decorative works for each group are not known precisely (as they vary from worker to worker) but are known to follow normal distribution with mean (μ) and standard deviation (s) as indicated in the following table.

Types of Textiles designing	Working time required per unit (minutes)		Maximum time available per week (minutes)
	category I (Magalier Thittam)	category II (Nonmagalier Thittam)	
	Mean S.D	Mean S.D	Mean S.D
Embroidery	$\overline{a_{11}} = 7.14$ $\sigma_{a_{11}} = 3.18$	$\overline{a_{12}} = 4.16$ $\sigma_{a_{12}} = 2.67$	$\overline{b_1} = 6000$ $\sigma_{b_1} = 600$
Design with glass work	$\overline{a_{21}} = 5.2$ $\sigma_{a_{21}} = 1.9$	$\overline{a_{22}} = 6.4$ $\sigma_{a_{22}} = 2.7$	$\overline{b_2} = 4800$ $\sigma_{b_2} = 400$
Fancy work	$\overline{a_{31}} = 3.2$ $\sigma_{a_{31}} = 1.92$	$\overline{a_{32}} = 3$ $\sigma_{a_{32}} = 1.58$	$\overline{b_3} = 3600$ $\sigma_{b_3} = 200$
Profit per unit (Rs)	c_1=18	c_2=200	

In order to determine the number of category I and category II to be created per week to maximize to profit without exceeding the available the resources working times more than once in 100 weeks.

By denoting the number of WSHG category I and category II manufactured per weeks as x_1 and x_2 respectively, the problem can be stated as follows:

Maximize $f(X) = k_1 (_{180} x_1 + 200 x_{2)} + k_2 \sqrt{400x_1^2 + 2500x_2^2}$

subjects to

Maximize $f(X) = k_1 (_{180} x_1 + 200 x_{2)} + k_2 \sqrt{400x_1^2 + 2500x_2^2}$

subjects to

$$7.14 x_1 + 4.6 x_2 + 2.33 \sqrt{10.11x_1^2 + 7.13x_2^2 + 360000} - 6000 \le 0$$

$$5.2 x_1 + 6.2 x_2 + 2.33 \sqrt{3.61x_1^2 + 7.29x_2^2 + 160000} - 4800 \le 0$$

$$3.2 x_1 + 3 x_2 + 2.33 \sqrt{3.686x_1^2 + 2.49x_2^2 + 40000} - 3600 \le 0$$

$$x_1 \ge 0, \; x_2 \ge 0$$

This problem can be solved by using any of the nonlinear programming methods. So the Kuhn Tucker conditions are

$$\frac{\partial f}{\partial x_1} - \lambda_1 \frac{\partial h}{\partial x_1} = 0$$
$$\frac{\partial f}{\partial x_2} - \lambda_2 \frac{\partial h}{\partial x_2} = 0$$
$$\frac{\partial f}{\partial x_3} - \lambda_3 \frac{\partial h}{\partial x_3} = 0$$
$$\lambda_i h = 0, \; h \le 0, \; \lambda_i \ge 0$$

$where \lambda$ is the Lagragian multiplier

By introducing the slack variables defined by $h^i + S_i^2 = 0$ (i=1, 2, 3) the inequality constraints are converted into the equality ones.

Let $S = (S_1, S_2, S_3)$ construct the Lagrangian function

$$L(x, S, \lambda) = f(x) - \sum_{i=1}^{m} \lambda_i \left[h^i + S_i^2 = 0 \right]$$

In this way, the solution of the stochastic linear programming problem is $x_1 = 128$, $x_2 = 173$, maximize profit about the categories of magalier thittam & non magalier thittam are 96182.61

4 Conclusion

With regard to survey, we have focused on linear stochastic programming model involving discrete random variables. As a general rule, solution methods based on statistical approximations are applicable for both continuous and discrete distributions. In this paper the chance constraint technique, using SLPP, quickly reaches real solutions that is close to optimum.

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