Volume : 2 | Issue : 3 | March 2013 • ISSN No 2277 - 8160

Research Paper | Mathematics

Electrical Network on F-Circuit and F-Cut Set

D.R.Kirubaharana Scholar in Mathematics,Manonmaniyam Sundarnar University,Thirunelveli, Tamilnadu,India..

Dr G.Nirmala Research Advisor,PG & Research Department of Mathematics,Kunthavai Natchiyar Govt Arts College for women (Auto),Thanjavur.Tamilnadu,India.

ABSTRACT

Like every mathematical theory, we have to begin with a long list of definitions and in the interest of precision these have to be formally defined. Fortunately, many of these terms that we will define have nearly the same intuitive meaning as in everyday English and so very little conscious effort is required to remember them. In order to the monotony of these necessary preliminaries, we will use diagrams to illustrate our points. Electricity serves man in an almost infinite variety of ways. In performing design tasks, we finds that much of the early work in this field can now be tied to graph theory. The basic principles postulated earlier by Kirchoff in the study of electrical flows have been adopted as the basis of the study of network flows in graph theory Like every mathematical theory, we have to begin with a long list of definitions and in the interest of precision these have to be formally defined. Fortunately, many of these terms that we will define have nearly the same intuitive meaning as in everyday English and so very little conscious effort is required to remember them. In order to the monotony of these necessary preliminaries, we will use diagrams to illustrate our points. Electricity serves man in an almost infinite variety of ways. In performing design tasks, we finds that much of the early work in this field can now be tied to graph theory. The basic principles postulated earlier by Kirchoff in the study of electrical flows have been adopted as the basis of the study of network flows in graph theory Like every mathematical theory, we have to begin with a long list of definitions and in the interest of precision these have to be formally defined. Fortunately, many of these terms that we will define have nearly the same intuitive meaning as in everyday English and so very little conscious effort is required to remember them. In order to the monotony of these necessary preliminaries, we will use diagrams to illustrate our points. Electricity serves man in an almost infinite variety of ways. In performing design tasks, we finds that much of the early work in this field can now be tied to graph theory. The basic principles postulated earlier by Kirchoff in the study of electrical flows have been adopted as the basis of the study of network flows in graph theory

KEYWORDS: Punica granatum,Punicaceae, flavonol glycoside, Sexangularetin

INTRODUCTION

In to-day's era of electronics, the transistor and printed circuits have replaced the electronic tube and wiring in most electronic items. In a printed circuit, conductors made by plating a metallic material on a non-conductor have replaced wires.

Electrical signals follow the paths of these plated conductors. Originally, plastic non-conductors were plated on both sides with appropriate connections being made between the two sides by means of connecting wires passing through holes in the non-conducting material. More recently, connections between the sides have been made by plating the insides of the connecting holes in the non-conductors. Because the plated conductors on a given side of the non-conductor are not insolated. It is important that no two of them cross except at a junction. Thus such an electrical network can be plated onto a single plane if it can be described as a planar graph.

This type of circuit cards contain a large number of digital integrated – circuit packages that are flow – soldered to the card.

An electrical circuit can be printed onto one side for a circuit card only, if the circuit can be described as a planar graph.

A planar graph is one that can be drawn on a surface so that none of its edges intersect except at its vertices.

Manufactures today are stacking boards, sandwich style, with layers of insulation between the boards, but the planar design is still required for these.

Suppose, we have a non-planar network, we want to print on two sides of a circuit card. Such a network can be successfully printed if

(i) We can partition the graph into two planar sub graphs.
(ii) Some form of inter connection is permitted between the two sub graphs.

2. LITERATURE REVIEW

W.K.Chen (1976) showed that electric network is in applied graph theory.Kirchoff (1847) made the first comprehensive investigation of the electrical network problems, and proved the existence of a solution to a resistive network. Maxwell (1892) pointed out; that Kirchoff's formulation omitted the concept of potential.

3. PRELIMINARIES

Definition 3.1

A digraph with n-vertices, e-edges and no self-loops is an n by n matrix A= [aij], whose rows correspond to vertices and columns correspond to edges, such that

aij = 1, if jth edge is incident out of ith vertex
= -1, if jth edge is incident into ith vertex.
= 0, if jth edge is not incident on ith vertex.

Definition 3.2

Let the number of different circuits in a graph G be q and the number of edges in G be e. Then a circuit matrix B = [bij] of G is a q by e, (0, 1) – matrix defined as follow:

bij = 1, if ith circuit includes jth edge, and
= 0, otherwise.

Definition 3.3

Let G be a digraph with e-edges and q-circuits. An arbitrary orientation (clock wise) is assigned to each of the q-circuits. Then a circuit matrix B = [bij] of the digraph G is a q by e-matrix defined as bij = 1, if ith circuit includes jth edge and the orientations of the edge and circuit coincide.

= -1, if ith circuit includes jth edge and the orientations of the two are opposite.
= 0, if ith circuit does not include the jth edge.

Definition 3.4

A vector V is called a linear combination of m component vectors I1,I2,…Im if V =Σ z(p).i(t), where z(p)'s are any scalars.

Definition 3.5

Every vector in the vector space can be expressed as a linear combina-

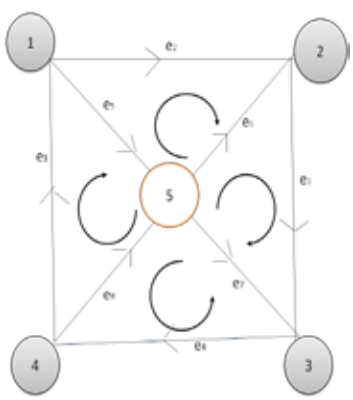
tion of the vectors forming the basis.

Definition 3.6
A minimal set of circuits of a graph from which all other circuits may be obtained by unions of two or more circuits. Such a minimal set is called circuit basis matrix.

Definition 3.7
The number r of the vectors in the basis is called the rank of the vector space (or) The rank r of a graph with v-vertices and c- components are defined as the number r = v - c.

Definition 3.8
The nullity m of a graph with e-edges, v-vertices and c-components is defined as the number m = e – v + c (=e-r).

Definition 3.9
The fundamental circuits or simply f-circuits of a connected directed graph of nullity m with respect to a tree t are the m circuits, each being formed by a chord and the unique tree path connecting the two end points of the chord in t.



(i.e) Bf = [Um B12] = Fundamental circuit matrix.

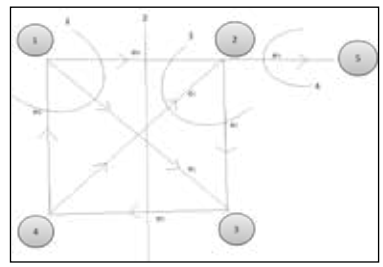
$$B_f = \begin{array}{c|cccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\ \hline 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 2 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 4 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 \end{array}$$

Definition 3.10
A directed (undirected) graph G, a cutset of G is a sub graph consisting of a minimal collection of edges whose removal reduces the rank of G by one.

Definition 3.11
In a circuit matrix, the cut-set matrix is c = [cij] in which the rows correspond to the cut-sets and the columns to the edges of the graph as follows.
cij = 1, if ith cut-set contains jth edge, and
= 0, otherwise.

Definition 3.12
The fundamental cut sets or simply f-cut sets of a connected directed graph with respect to a tree t are the r cut sets in which each cutset includes only one branch of t.

Definition 3.13
A connected directed graph G, the sub matrix of the matrix of G corresponding to a set of r, f-cut sets in G is called a fundamental cutset matrix or simply f- cutset matrix of G and is denoted by Qf . (i.e)



Theorem :3.1
The determinant of every square sub matrix of A, the incidence matrix of a digraph is 1, -1, or 0.

Theorem:3.2
Let C and A be respectively, the circuit matrix and incidence matrix of a self-loop-free digraph such that the columns in C and A are arranged using the same order of edges. Then A.CT = C.AT = 0.

Theorem:3.4
If G is a graph of n-vertices and G' is it's complement, then (i) if n is less than 8 at least one of the two is planar; (ii) if is greater than 8 at least one of the two is non planar; and (iii) if n=8 one or the other or both may be planar or non planar.

4. DISCUSSION WITH AN EXAMPLE
Digraph-Mathematical Model of an Electrical Network
An electrical network is an assembly of electrical elements such as Resistors (R), Inductors(I), Capacitors (C), Generators etc. with each such element is associated witha pair of real (usually time – dependent) variables: Voltage v(t) and Current i(t). Since these can be take positive or negative values, a reference direction is associated with each: thus i(t) is positive if current flows along the reference direction and v(t) is positive if the voltage drop is in the reference direction. Thus the mathematical model of an electrical network is a digraph with two associated arc functions i(t) and v(t).

The two functions i and v for each element are related by some integro – differential equations. If the network has m elements (edges/arcs), we can represent the collection of these functions by two m-vectors i(t) and v(t). combining all the element equations we can write one vector equations of the form.
v(t) = z(p)i(t) +g(t),

Besides this combined elemental equation, the network variables must also satisfy the following two laws first enunciated by Kirchhoff:

The Current Law (KCL):
The algebraic sum of the currents flowing out of any vertex of the network is zero.

The Voltage Law (KVL):
The algebraic sum of the voltage across the edges around any cycle of the network is zero.

The voltages and currents are no longer independent of other components to connect together in a system. The voltage around every circuit as well as the sum of currents through every cutset is zero. These relations can be expressed as equations in the current and voltage variables.

If the network has m-edges, the two bases together would yield m-equations. Although we can use any cutset basis and circuit basis, we might follow a systematic way of obtaining these bases. First we select a spanning tree. This has n-1 branches where n is the number of vertices. The remaining m-n+1 edges are chords where m is the number of edges. Since every cutset must include at least one branch of the tree,we formulate the cutset basis by taking a branch of the tree and adding to it the requisite chords, but no other branches, to makeup a cutset. We thus have a cutset corresponding to each branch. In this way we obtain n-1 independent cut sets, which form the cutset basis. To formulate the circuit basis, we take a chord and add the requisite number of branches, but no other chords, to make a circuit. This step gives us m-n+1 independent circuits, which form the circuit basis. The computations are further simplified if we can manage to include all the known

variables in one set, either the branches or the chords corresponding to each chord we will have a circuit then the system has:

(i) m component terminal equations.
(ii) m-n+1 circuit equations.
(iii) n-1 cutset equation.

(i.e) 2m equations in all. It is not necessary to set terminal equations for components if either the voltage or current is known. This practice will reduce the total number of equations.

Consider the bridge circuit shown in figure 4.1, we need to determine all of the voltage and current measurements for the circuit. Figure 4.1(a) shows the engineering schematic of the bridge and figure 4.1(b) depicts an equivalent graph. The resistors are indicated by and batteries are.

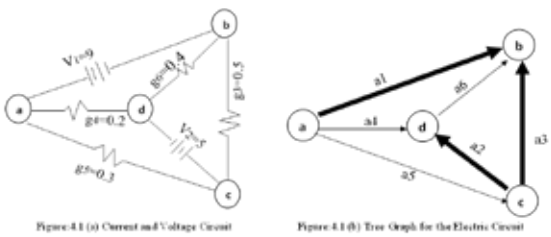


Figure: 4.1 Bridge Circuit
By Ohm's law: ij(t)=gj(t).vj(t) (4.1)

From figure 4.1,

$$v1(t)=9$$
$$v2(t)=5$$
$$\begin{bmatrix} i3(t) \\ i4(t) \\ i5(t) \\ i6(t) \end{bmatrix} = \begin{bmatrix} g3 & 0 & 0 & 0 \\ 0 & g4 & 0 & 0 \\ 0 & 0 & g5 & 0 \\ 0 & 0 & 0 & g6 \end{bmatrix} \begin{bmatrix} v3(t) \\ v4(t) \\ v5(t) \\ v6(t) \end{bmatrix}$$

as per cutset equation, (4.1)

$$\begin{matrix} K_1 \\ K_2 \\ K_3 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \\ i_4(t) \\ i_5(t) \\ i_6(t) \end{bmatrix} = 0$$

and as per fundamental circuit equation (4.2)

$$\begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} \begin{bmatrix} -1 & -1 & 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \\ v_5(t) \\ v_6(t) \end{bmatrix} = 0$$

(4.5)

n equation (4.3) i1, i2, and i3 correspond to the branches and i4, i5 and i6 to the chords. Similarly in equation (4.4) v1, v2 and v3, correspond to the branches and v4, v5 and v6 to the chords. In the circuit basis matrix, the direction of the one and only one chord in the circuit defines the positive direction in the circuit. Now the electrical system has three sets of simultaneous equations (systems equations). From equations (4.2), (4.3), and (4.4) we have ten equations in twelve variables, two of which are known, namely v1(t) and v2(t). To solve the systems equations, we can be substituting the terminal equations in (4.2) into the cutset equations (4.3).

$$\begin{bmatrix} i_1(t) \\ i_2(t) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} i_3(t) \\ i_4(t) \\ i_5(t) \\ i_6(t) \end{bmatrix} = 0. \quad (4.5)$$

Next, equation (4.2) is substituted into equation (4.5).

$$\begin{bmatrix} i_1(t) \\ i_2(t) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} g_3 & 0 & 0 & 0 \\ 0 & g_4 & 0 & 0 \\ 0 & 0 & g_5 & 0 \\ 0 & 0 & 0 & g_6 \end{bmatrix} \begin{bmatrix} v_3(t) \\ v_4(t) \\ v_5(t) \\ v_6(t) \end{bmatrix} = 0. \quad (4.6)$$

The fundamental circuit equations (4.4) and equation (4.6) contain six equations in eight variables where two variables are known. To solve this set of equations is to substitute for the voltage vector in equation (4.6)

$$\begin{bmatrix} v_3,(t) \\ v_4(t) \\ v_5(t) \\ v_6(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} \quad (4.7)$$

The last three equations in (4.7) merely represent a rearranged form of equation (4.4), whereas the first is the identity v3(t) = v3(t). Now, substitute equation (4.7) into equation (4.6) as follows:

$$\begin{bmatrix} i_1(t) \\ i_2(t) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} g_3 & 0 & 0 & 0 \\ 0 & g_4 & 0 & 0 \\ 0 & 0 & g_5 & 0 \\ 0 & 0 & 0 & g_6 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} = 0. \quad (4.8)$$

Matrix operations on equation (4.8) reduce it to:



From the values of figure (4.1)

Now we have only three equations in three variables i1(t), i2(t), and v3 (t). For the only unknown voltage is v3(t), which we can find by solving the last equation in (4.10) and we get

v3(t) = 7.5/1.4 = 5.357

Having established the one unknown tree voltage, we can find all remaining voltages and currents by substituting in equations (4.4) or (4.7). Solving for the remaining voltages we find v4(t) = 8.643, v5 (t) = 3.643, and v6(t) = 0.357. If we substitute in the terminal equations (4.2), the current measurements turn out to be i3(t) = 2.68, i4(t) = 1.73, i5(t) = 1.09, and i6 (t) = 0.14. We find all the unknowns by solving only three simultaneous equations in (4.10) instead of the original ten. Thus we resolve all of the voltages and currents for the bridge circuit.

5. CONCLUSION

For an isolated electrical component the relation between the potential differences (voltage) and the electrical flow (current) can be expressed by the simple equation of Ohm's law. As well as Kirchhoff principles are used in network and we may find out all the unknowns by solving only the simultaneous equations instead of the original equations. Thus we resolve all of the voltages and currents for the bridge circuit.

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