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NUCLEI EQUALS THE CENTER IN (-1,1) RING

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ABSTRACT

In a semiprime 2,3-torsion free (-1,1) ring left nucleus equals the center. If the ring is prime and <Alt> is right nilpotent then right nucleus equals to the nucleus. Also if R is without non-zero trivial left ideals then either R is associative or right nucleus is equal to the center.

KEYWORDS :(-1, 1) ring, right nucleus, left nucleus, nucleus, alternative nucleus, alternative ideal.

INTRODUCTION:

Thedy [1] consider rings which are 2-torsion free, prime and right alternative if such ring is without nonzero locally nilpotent ideals particularly if the ring is simple then he showed that right nucleus is equal to nucleus. Nam [8] likewise showed the following cases Hentzel and Smith [7] studied the properties of ideal in (-1, 1) ring. They proved that if J is minimal ideal of the ring R then it is either two sided ideal or ideal it generates is contained in nucleus of R. Kleinfeld and Kleinfeld [3] have shown that if R is prime ring satisfies (x, y, z) + (y, z, x) + (z, x, y) = 0. and the flexible identity then R is either associative or nuclei is equal to center. In this paper using the results of [3, 4, 8] we prove the following results (I) in a semiprime 2,3-torsion free (-1,1) ring, left nucleus is equal to center. (II) if the ring is prime and <alt> is right nilpotent, right nucleus is equal to the nucleus and (III) if R is without nonzero trivial left ideals then either R is associative or right nucleus is equal to the center.

A ring R is said to be (-1, 1) if it satisfies the following two identities:

(x, y, z) + (x, z, y) = 0. … (1)
(x, z, y) + (y, z, x) + (z, x, y) = 0. … (2) For all x,y,z ∈ R.

In a nonassocative ring R we define an associator (x, y, z) = (xy)z – x(yz) and the commutator [x, y] by [x, y] = xy – yx for all x, y, z ∈ R. The nucleus of a ring R is defined as N ={n ∈ R/ (n, R, R) = (R, n, R) = (R, R, n) = 0}, the right nucleus Nr as Nr = {n ∈ R /(R, R, n) = 0}and the left nucleus as Nl = {n ∈ R / (n, R, R) =0}. Thus the nucleus N can be stated as N = Nl ∩Nm ∩Nr . Nβ is said to be an alternative nucleus if Nβ ={v∈ R/(x, x, v) = 0} and the center C as {c∈ N/ [c, R] =0} and the commutative center U = {U ∈ R/ [U, R] =0}.Thus the center C is nothing but N ∩ U . A ring is said to be n-torsion free if there exists a positive integer n such that nx = 0 implies x = 0 for all x∈ R. A ring R is prime whenever A and B are ideals of R such that AB = 0 then either A = 0 or B = 0 and R is semi prime if for any ideal A of R, A2 = 0 implies A = 0. These rings are also required as ring free from trivial ideals.

MAIN SECTION:

In any arbitrary ring the following three identities are satisfied

(wx, y, z) – (w, xy, z) + (w, x, yz) = w(x, y, z) + (w, x, y)z. … (3)
[xy, z] = x[y, z] + [x, z]y + (x, y, z) – (x, z, y) + (z, x, y). …(4)
(x, y, z) + (y, z, x) + (z, x, y) = [xy, z] + [yz, x] + [zx, y]. …(5)

for all w, x, y, z ∈ R.

Equation (3) is known as Teichmuller identity equation (4) is known as semi jacobian identity and equation (5) as Jacobi identity. Now applying equation (2) in equation (5) we obtain

[xy, z] + [yz, x] + [zx, y] = 0. … (6)

The following identities were given by Hentzel [6].

(z, x2, y) = (z, x, xy + yx). … (7)

(z, x, xy) = (z, x, y)x. … (8)
(wx, y, z) + (w, x, [y, z]) = w(x, y, z) + (w, y, z)x. … (9)
([w,x], y, z) – [w, (x, y, z)] + [x, (w, y, z)] = (x, w, [y, z]) – (w, x, [y, z]). … (10)
((a, y, z), b, c) = ((a, b, c), y,z) – (a, b, (c, y, z))– (a, (b, y, z), c) + (a, b, c)[y, z] – (a, b, c[y, z]) + (a, b, [y, z])c. … (11)

Now we derive the following identities:
We shall first note that some facts and identities involving Nr hold in a 2,3- torsion free (-1, 1) ring taking Z∈ Nr in (9) shows [R, Nr] ⊆ Nr . …(12)
and taking b∈Nr in (11) gives 0= (a, (b, y, z), c) = (R, (Nr, R, R), R) = (R, R, (Nr, R, R) from (1)

Thus (Nr, R, R) ⊆ Nr . …(13)
With y ∈ Nr and z = c in (11) and using (1), (12) and we see that

(a,b,c)[y,z] = 0. …(14)

where y∈Nr
Also assuming R is prime and not associative we define
= { n Nr / nR ⊆ Nr}. …(15)

Substitute y=z=n in equation (3) we obtain (w, x, n2) = 0 …(16)
From (6) we with z=x=y=n ∈ we get Nr[n2, n] + [n2,n] + [n2, n] = 0 we obtain

Thus [Nr, Nr] + [Nr, Nr] + [Nr, Nr] = 0 n2 ∈ Nr from (16).
Thus 3[Nr, Nr] =0. Since the ring is 3-torsion free we have [Nr, Nr] = 0 . …(17)
We now derive [n,x][n,y] = 0. …(18)

For from (3) with z ∈ Nr we obtain (w, x, yz) = (w, x, y)z.

Now let w = t, x = u, y = v, z = [n,x]2 ∈ Nr in the above equation.then

(w, x, y, [n,x]2) = (w, x, y[n,x].[n,x])= (w,x, y [n,x])[n,x] = 0 as [n,x] ⊆ Nr .

Thus from above (w, x, y[n,x]2) = 0.Therefore [n,x]2 = from (15)

Now linearizing [n,x]2 = 0 on x we obtain [n,x] [n,y] + [n,y][n,x] =0.Since Nr is commutative we obtain 2[n,x][n,y] =0 and by 2- torsion free we have [n,x][n,y] = 0.

We next prove that [n,x][[n,x],y] = 0 From equation (3) we obtain
-(w, z, y)[x, n]) = (w,y,[x, n] z) …(A) Again from (3) we obtain (w, y, z[x,n]) =
- (w, z, y)[x, n] ...(B)

Subtracting (A) - (B) we obtain (w, y, [[x, n], z]) = 0

Thus [[R,Nr], R] $\subseteq$ Nr . ...(19)

from (4), (12), (1) and (19) weget

0 = [[n,x], y] = [n,x][[n,x],y] + [[n,x],y][n,x] + 2([n,x],[n,x],y] + (y, [n,x-],[n,x])
= [n,x][[n,x],y] + [[n,x],y][n,x] = 2[n,x][[n,x],y] = [n,x] [n,x],y] as the ring is 2- torsion free (-1,1) .

THEOREM 1: Let R be a 2,3- torsion free semi prime (-1, 1) ring then N = C.

PROOF: We know that a 3- torsion free (-1, 1) ring satisfies
(w, [x,y],z) = ([x,y],z,w) =(z, w,[x,y]). …(20)
From (2) in (12) we obtain (w, [x,y], z) = ([x,y],z,w) = (z, w, [x,y]) = 0.
i.e.,[R, R] = N. …(21)

Four applications of (3) leads to (wx, y,z) – (w, xy, z) + (w, x, yz) = w(x, y,z) + (w, x, y)z,-(xy, z, w) + (x, yz, w) – (x, y, zw) = -x (y, z, w) – (x, y, z) w,(yz, w, x) + (y, zw, x) + (y, z, wx) = y (z, w, x) + (y, z, w)x,-(zw, x, y) + (z, wx, y) – (z, w, xy) = -z(w,x,y) – (z, w, x)y.Add all the left sides and apply (2) repeatedly we see [w, (x,y,z)]- [x,(y,z,w)] + [y, (z,w,x)] – [z, (w, x, y)]= 0 substituting w = n $\in$ N in (22) we get [n, (x, y, z)] = 0 i.e., [N, (R, R, R)] = 0.

Substituting y= n$\in$N in (20) we get[R, N] $\subseteq$ N i.e., [x, n] $\in$ (N$\cap$U) =C.again Substituting x = y = z =[x,y] in (2) we see that 3[x,y]3 =0 and since the ring is 3-torsion free we obtain [x,y]3 =0 that is [x,n]2 $\subseteq$ C generates a trivial ideal in R, and since R being semi prime gives [x,n]2=0. But then [x,n] = 0. Thus we have N $\subseteq$ (N $\cap$ U) =C from (10) N= C..

LEMMA 1: If R is 2- torsion free (-1, 1) ring then I = R (Nr, R, R) + R [(Nr, R, R)] is an ideal of R.

PROOF: Using (13) and (12) we have R(R(Nr, R, R)) = R2(Nr, R, R) $\subseteq$ I
But from (1) (R, Nr, R) = - (R, R, Nr) = 0

And we also have (R, (Nr, R, R))R = R ((Nr, R , R), R) $\subseteq$ R {(Nr,R, R), R} + R (Nr, R, R)} $\subseteq$ .Finally using (4) we see that([(Nr, R, R), R], R) R = ([(Nr, R, R), R] R) R

$\subseteq$ {[(Nr, R, R), R2] + [(Nr, R, R), R] +

((Nr, R, R), R, R) + 2 ((R, R, (Nr, R, R)}R

$\subseteq$ R {[(Nr, R, R), R] + R [(Nr, R, R),R] +(Nr, R, R)}$\subseteq$ I which proves the lemma ♦

LEMMA 2 : If R is 2-torsion free (-1, 1) ring then J = {x $\in$ R/ x(Nr, R, R) = x[(Nr, R, R), R] =0} is an ideal of R such that JI = 0.

PROOF: Let x $\in$ J thus using (13) and (12) we have
0 = (Rx) (Nr, R, R) = R(x (Nr, R, R)) 0 = (Rx) [(Nr, R, R), R] = R(x [(Nr, R, R),R]).

Since (R, Nr, R) = 0.We also have (xR)(Nr , R, R) = x (R,(Nr,R,R)) $\subseteq$ x{[(Nr, R, R)] + (Nr, R, R)R} = (x(Nr, R, R))R = 0 Finally (4) we use (xR) [(Nr,R, R), R] = x (R[(Nr, R, R),R] $\subseteq$ x{[(Nr, R, R), R2] + [(Nr, R, R), R] R +2 (R, R, (Nr, R, R)) + ((Nr, R, R), R, R)

$\subseteq$([(Nr, R, R), R]) R + x[Nr, R, R] = 0. which shows J is an ideal.

Now JI = J { R (Nr, R, R) + R [(Nr, R, R), R]} (JR) {(Nr, R, R) + R [(Nr, R, R), R]}

$\subseteq$ J {(Nr, R, R) + [(Nr, R, R), R]} = 0 which completes the proof. ♦

For a 2- torsion free (-1, 1) ring. Let M be the submodule of R generated by all associators of the form (x, x, y). By lemma (11) in [10] <alt>= M+ MR. is an ideal which is commonly known as the alternator ideal of R. We also note that letting R[1] = R and defining inductively R[k]= R[k-1] R, a ring R is said be right nilpotent of R[n]= 0 for some integer ≥ 1. Left nilpotent of R is defined analogously.

THEOREM 2: Let R be a 2- Torsion free prime (-1,1) ring .If <Alt> is right nilpotent, then Nr = N.

Proof: If R is associative then clearly Nr = N .Thus we may assume that R is not associative and so by (15) Nr = {n $\in$ Nr / Rn $\subseteq$ Nr} = 0. Now since R is prime by Lemma (1) and (2) we either have I = R(Nr,R, R) + R[(Nr, R, R), R] = 0. or J ={ x$\in$R/ x(Nr,R,R) = x[(Nr, R, R), R] = 0} = 0. Suppose first that I = 0. Then (Nr, R, R) $\subseteq$ Nr =0 and so Nr $\subseteq$ (Nr $\cap$ Nl) = N, i.e., Nr = N. On the other hand, suppose J =0 since R/ <Alt> is associative, it follows that (Nr, R, R) +[(Nr, R, R), R] $\subseteq$ <Alt> [n] = 0 then <Alt>[n-1]((Nr, R, R) + [(Nr, R, R), R]) $\subseteq$ <Alt>[n] = 0
i.e., <Alt>[n-1] $\subseteq$J = 0. Since iteration of this argument gives <Alt> = 0 we see that <Alt> = right nilpotent implies R is a (γ, δ) but in any (γ, δ) ring Nr = N and so this completes the proof of the theorem ♦

THEOREM 3: Let R be a prime 2- torsion free (-1, 1) ring, if R is without nonzero trivial left ideals, then either R is associative or Nr = C.

PROOF: We suppose that R is nonassociative, and let n$\in$Nr. It is immediate from (13) that L= R[n,x] is left ideal of R. Also from (19), (R, Nr,, R) = 0, from (18) we see that [n,x](y,[n,x]) = [n, x] +{[y, [n, x]] + [n, x]y} = [n, x]([n, x]y) = [n, x]2y = 0. Thus L2 = (R[n,x]) (R[n,x]) = R ([n,x](R[n,x])) = 0. Since R is without nonzero trivial left ideals, this means L=R[n,x] = 0.Thus we have each [n, x] $\in$ Nr = 0 by (14). But, then Nr $\subseteq$ (Nr $\cap$ U)= C and so Nr = C.

COROLLARY 1: Let R be a prime 2- torsion free (-1, 1) ring satisfies the minimum condition on trivial left ideals, then either R is associative or Nr = C.

PROOF: Suppose R is not associative, and that L ≠ 0 is a trivial left ideals of R. Then by the minimum condition on trivial left ideals, there is a nonzero minimal left ideals $\subseteq$ L. But by the corollary to Theorem 1 in [3] is an ideal of R and $\subseteq$ 0. Since R is prime, this means = 0, which is contradiction. Thus R must be without nonzero left trivial left ideals, and so Nr = C.

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