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ON THE HOMOGENEOUS BIQUADRATIC EQUATION WITH FOUR UNKNOWNS $x^4+y^4+z^4=98w^4$

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ABSTRACT The bi-quadratic equation with 4 unknowns given by $x^4+y^4+z^4=98w^4$ is analysed for its patterns of non-zero distinct integral solutions. Six different patterns of integer solutions to the above bi-quadratic equation are presented. A few interesting relations between the solutions and special numbers, namely, polygonal number and pyramidal number are exhibited.

KEYWORDS : Homogeneous bi-quadratic, bi-quadratic equation with four unknowns, integer solutions, polygonal and pyramidal numbers.

1. Introduction:

There is a great interest for Mathematicians since uncertainty in homogeneous and non-homogeneous Bi-quadratic Diophantine Equations [1-3]. In this context, one may refer [4-15] for varieties of problems on the bi-quadratic Diophantine equations with three and four variables. In this paper, bi-quadratic equations with four variables given by $x^4+y^4+z^4=98w^4$ is analyzed for its non-zero distinct integer solutions. A few interesting relations between the solutions and special polygonal numbers, pyramidal numbers are exhibited.

NOTATIONS:

p_n^m - Pyramidal number of rank n with size m.
cp_n^m – Centered pyramidal number of rank n with size m.
j_n – Jacobsthal number of rank n.
G_n – Gnomonic number of rank n.
Pr_n – Pronic number of rank n.

2. Method of Analysis:

The Diophantine equation representing the bi-quadratic equation with four unknowns under consideration is given by

(9,6,3,3) (4,2,6,2), (-6,4,2,-2) (1,2,3,1) (-1,-2,-3,-1), (3,2,1,1) (2,1,3,1) (3,1,2,1) (2,3,1,1) (6,9,3,3) (3,9,6,3) (6,2,4,2) (6,4,2,2) .....

$x^4+y^4+z^4=98w^4$ ----------(1)

Note that (1) is satisfied by the following non-zero integer quadruples.

However, we have other solution for (1), which are illustrated below.
Introducing the transformations
x=u+v; y=u-v; z=2u----------(2)

in (1), it simplifies to
$v^2+3u^2=7w^2$----------(3)

The above equation (3) is solved through different methods and thus, one obtains distinct patterns of integer solutions of (1).

Pattern I:

Assume $w=w(a,b)=a^2+3b^2$ ----------(4)

Where a and b are non-zero distinct integers.
Write 7 as $7=(2+\sqrt{3}\,i)(2-\sqrt{3}\,i)$----------(5)

Substituting (4) and (5) in (3) and using the method of factorization, define
$v+i\sqrt{3}\,u=(2+\sqrt{3}\,I)(a+i\sqrt{3}\,b)^2$

Equating real and imaginary parts, we have
$v=2a^2-6ab-6b^2$
$u=a^2+4ab-3b^2$

Hence in view of (2) and (4), the non-zero distinct integral solutions of (1) are given by
$x=x(a,b)=3a^2-9b^2-2ab$
$y=y(a,b)=-a^2+10ab+3b^2$---(6)
$z=z(a,b)=2^2+8ab-6b^2$
$w=w(a,b)=a^2+3b^2$

Properties:

1. $y(a^2,a^2)+z(a^2,a^2)$ is a bi-quadratic integer.
2. $x(a,a^2+1)-\omega(a,a^2+1)+144FN_a^4+2CP_a^6-17T_{6,a}\equiv 12(mod\ 19)$
3. $2y(a^2,a+1)+z(a^2,a+1)-56P_a^5\equiv 0$
4. $z(a,a+1)-2\omega(a,a+1)+4pr_a+6Gno_a+18j_n\equiv 0$
5. $x(a^2,a^2+1)-3\omega(a^2,a^2+1)+240FN_a^4+58T_{4,a}+18j_n\equiv 0$

Pattern II:

Introducing the linear transformations
$u=X+2T$, $v=2X-3T$----------(7)

in (3), it leads to
$X^2+3T^2=w^2$----------(8)

Which is satisfied by
$X=3m^2-n^2$
$T=2mn$----------(A)
$w=3m^2+n^2$

Substituting the above values in (7), we get,
$u=u(m,n)=3m^2-n^2+4mn$
$v=v(m,n)=6m^2-2n^2-6mn$

In view of (2), we have
$x=x(m,n)=9m^2-3n^2-2mn$
$y=y(m,n)=-3m^2+n^2+10mn$---(9)
$z=z(m,n)=6m^2-2n^2+8mn$
$w=w(m,n)=3m^2+n^2$

Properties:

1. $x(a^2,2a^2-1)-3\omega(a^2,2a^2-1)+28Bi_{a}\equiv -6(mod\ 26)$
2. $x(a,1)+y(a,1)-z(a,1)\equiv 0$
3. $z(a^2,a^2)+\omega(a^2,a^2)$ is a biquadratic integer.
4. $2y(a,a^2+1)+z(a,a^2+1)-36cp_a^2\equiv 0$
5. The expression $y(a+1,a+1)+\omega(a+1,a+1)-12ct_{2,a}-Gno_a$ represents a perfect number.

NOTE:

Equation (A) can be written as
$X=m^2-3n^2$
$T=2mn$
$W=m^2+3n^2$
In view of (7) in (2), we have

$x=x(m,n)=3m^2-9n^2-2mn$
$y=y(m,n)=-m^2+3n^2+10mn$
$z=z(m,n)=2m^2-6n^2+8mn$
$w=w(m,n)=m^2+3n^2$

Pattern III:

Equation (3) can be written as

$v^2+3u^2=7w^2*1$----------(10)

Write 7 as $7 = \frac{(4+2\sqrt{3}i)(4-2\sqrt{3}i)}{4}$

And also 1 as $1 = \frac{(1+\sqrt{3}i)(1-\sqrt{3}i)}{4}$

----------(11)

Assume that $w=w(a,b)=a^2+3b^2$----------(12)

Substituting (11) & (12) in (10) and using the method of factorization, define

$$v+i\sqrt{3}u=\frac{1}{4}\{(4+2\sqrt{3}i)(1+\sqrt{3}i)(a+i\sqrt{3}b)^2\}$$ ----------(13)

Equating real and imaginary parts, we get

$$v=\frac{1}{4}\{-2a^2-36ab+6b^2\}$$

$$u=\frac{1}{4}\{6a^2-4ab-18b^2\}$$ ----------(14)

Again substituting (12) and (14) in (2), we get,

$x=x(a,b)=a^2-10ab-3b^2$
$y=y(a,b)=2a^2+8ab-6b^2$
$z=z(a,b)=3a^2-2ab-9b^2$
$w=w(a,b)=a^2+3b^2$

Properties:

1. $x(2a,a^2)-\omega(2a,a^2)+6Biq_a-20cp_a^6\equiv 0$
2. The expression $y(a^2,a^2+1)+z(a^2,a^2+1)+48FN_a^4-28t_{4,a}$ represent a trapezoidal numbers.
3. $z(3a^2,2a+1)+3\omega(3a^2,2a+1)-27FN_a^4+24p_a^5-42T_{4,a}\equiv 0$
4. $x(2a-1,a)+\omega(2a-1,a)+3T_{8,a}+3T_{4,a}\equiv 2(mod\ 4)$

3. Conclusion:

In this paper, we have presented infinitely many non-zero integral solutions for the non-homogeneous bi-quadratic equation with four unknowns $x^4+y^4+z^4=98w^4$. As bi-quadratic equation are rich in variety, one may consider the other forms of bi-quadratic equations with variables greater than or equal to 4 and search for their corresponding integer solutions.

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