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Scattering of Plane Waves in a Micropolar Elastic Half Space

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ABSTRACT *In this work we discusses the scattering of plane thermo-elastic waves at wavy boundary of a micropolar semi-space . Method of small perturbations has been used. The analysi s shows that surface wave breaks into three parts. Rayleigh wave with velocity C scattered waves with velocity of propagation $\frac{CW}{W-rc}$ and $\frac{CW}{W+rC}$. It is also seen that scattered wavevelocity depends on the wave length and also on the wavy nature of the boundary.We consider a micropolar elastic half space-*

$-\infty \prec (x_1,x_3) \prec \infty, x_2 \ge hf(x_1)$ *bounded by a surface* $x^2 = hf(x^1)$ *where* $f(x_1)=\frac{\pi}{r\sin(rx_1)}$ *and h <<1 represent a small perturbation parameter such that and its higher order terms are neglected (i.e the surface is slightly wavy) and we assume that The wavy boundary has a normal traction of the concentrated type , zero shear and zero couple stress.* $\sigma_{nn}=p(\delta s),\sigma_{ns}=0,\mu_{sp}=0 \text{ for } x_2=hf(x_1)$

for Where is the normal stress component for the wavy boundary in the direction of normal to the curve , is the shear stress component for the wavy boundary along the curve , is the couple stress component in the direction of binormal to the curve.The temperature and deformation fields do not depend on the variable .The surface under consideration dissipates according to Newton,s law of cooling

KEYWORDS: Elastic waves, micropolarelastic ,plane waves sinu-soidal surface and wave propagation

INTRODUCTION

The scattering of elastic waves at a rough surface has been discussed by a number of worker (i)Assay, J.R and LipkinJr, J . (ii)Erbay.S. ,Int .J. Enggng .(iii) Toupin, R. A ,;Theories of elasticity with couple – stresses in linear elasticity Arch . .in classical theory of elasticity.Literature survey shows that the corresponding analysis for micropolar elastic solid has not been discussed probably because of much mathematical complexities . In the present analysis we discuss scattering of plane waves in a micropolar elastic half –space bounded by sinu- soidal surface under the following assumption ;

(i) Semi- space is homogrneous , free from any heat source

(ii) The surface is slightly rough i.e the amplitude and curvature of the roughness are sufficiently small. The sinu-soidal model of roughness has been considered . The method of small perturbation is used to investigate the wave propagation .

Mathematical model;

We consider a micropolar elastic half space-

$$-\infty \prec (x_1,x_3) \prec \infty, x_2 \ge hf(x_1)$$

bounded by a surface $f(x_1)=\frac{\pi}{r\sin(rx_1)}$ where $x_2=hf(x_1)$,

and h $\langle\langle 1$ represent a small perturbation parameter such that h^2 and its higher order terms are neglected (i.e the surface is slightly wavy) and we assume that

(i) The wavy boundary has a normal traction of the concentrated type , zero shear and zero couple stress.

$$\sigma_{nn}=p(\delta s),\sigma_{ns}=0,\mu_{sp}=0 \text{ for } x_2=hf(x_1) \quad \text{..................(1.1)}$$

Where σ_{nn} is the normal stress component for the wavy boundary in the direction of normal to the curve , σ_{ns} is the shear stress component for the wavy boundary along the curve , μ_{sp} is the couple stress component in the direction of binormal to the curve.

(ii) The temperature and deformation fields do not depend on the variable x_3 .

(iii) The surface under consideration dissipates according to Newton,s law of cooling

$\frac{\partial T}{\partial n}+HT=0$ (1.2)

Basic Equation

Nowacki (1969) showed that when displacement $\vec{u}=(u_1,u_2,u_3)$ and rotation $\bar{w}=(w_1,w_2,w_3)$ depend on the variables x_1,x_2 and t ,we face two mutually independent systems of equations ;

$$(\mu+\alpha)\nabla^2u_2+(\mu+\lambda-\alpha)\partial_1 e_1+2\alpha\partial_2\omega_3=\rho\ddot{u}_1+\upsilon\partial_1 T$$
$$(\mu+\alpha)\nabla^2u_2+(\mu+\lambda-\alpha)\partial_2 e_2-2\alpha\partial_1\omega_3=\rho\ddot{u}_2+\upsilon\partial_2 T$$
$$(\gamma+\varepsilon)\nabla^2\omega_3-4\alpha\omega_3+2\alpha(\partial_1u_2-\partial_2u_1)=j\ddot{w}_3 \quad (1.3)$$

And

$$(\gamma+\varepsilon)\nabla^2\omega_1+(\gamma+\beta-\varepsilon)\partial_1\chi_1-4\alpha\omega_1+2\alpha\partial_2u_3=j\ddot{w}_1$$
$$(\gamma+\varepsilon)\nabla^2\omega_2+(\gamma+\beta-\varepsilon)\partial_2\chi_1-4\alpha\omega_2-2\alpha\partial_1u_3=j\ddot{w}_2 \quad (1.4)$$
$$(\mu+\alpha)\nabla^2u_3+2\alpha(\partial_1\omega_2-\partial_2\omega_1)=j\ddot{\rho}_3$$

Where $\lambda,\mu,\alpha,\beta,\gamma,\varepsilon$ are the elastic constants of the micropolar material, ρ is the density, J is the rotational inertia and dots denote the time derivative. The following notation have been used in the equations (1.3) and (1.4)

$\nabla^2=\partial^2{}_1+\partial^2{}_2$, $e_1=\partial_1u_1+\partial_2u_2$, $\chi_1=\partial_1\omega_1+\partial_2\omega_2$

$\upsilon=(3\lambda+2\mu)\alpha_t$ and α_t is the coefficient of linear thermal expansion.

T = Temperature distribution in the material satisfying coupled heat equation in the absence of any heat source .

$(\nabla^2-\frac{1}{\chi}\partial_t)-\eta$ div $\vec{u}$ = 0 (1.5)

Where $\chi=\frac{k}{c_e}$, k denoting the heat conducting coefficient and C_e the specific heat at constant deformation.

$\eta=\frac{\theta_0}{k}$, θ_0 being the absolute temperature of the natural state.

The latter system (1.4) is unperturbed by the thermal field, As such we shall consider types of waves governed by the former system (1.3), with boundary condition given by (1.1) .

Equation in terms of potential

We introduce the elastic potentials $\phi and \psi$ connected with the displacements u_1 and u_2 by

$u_1=\partial_1\phi+\partial_2\psi$

$u_2=\partial_2\phi+\partial_1\psi$ (1.6)

Inserting (1.6) in (1.3) ,we get the following

$$\left(\nabla^2-\frac{1}{K_1^2}\partial_t^2\right)\phi=mT \quad (1.7)$$
$$\left(\nabla^2-\frac{1}{K_2^2}\partial_t^2\right)\psi+\alpha_2\omega_3=0 \quad (1.8)$$
$$(\nabla^2-2\alpha_4-\frac{1}{k_4^2})\omega_3-\alpha_4\nabla^2\psi=0 \quad (1.9)$$

Where

$$\{k_1^2,k_2^2,k_4^2\} = \{k_1^2,k_2^2,k_4^2\} = \left\{\frac{\lambda+2\mu}{\rho},\frac{\mu+\alpha}{\rho},\frac{\gamma+\varepsilon}{j}\right\}$$

$$\{\alpha_2,\alpha_4\} = 2\alpha\left\{\frac{1}{\mu+\alpha},\frac{1}{\gamma+\varepsilon}\right\}$$

m = $\frac{3\lambda+2\mu}{\lambda+2\mu}\alpha_t$

Eliminating Tfrom the equations (!.5) and (1.7) .
We get the wave equation

$$\left(\nabla^2-\frac{1}{k_1^2}\partial_t^2\right)\left(\nabla^2-\frac{1}{\chi}\partial_t\right)\phi-\frac{\varepsilon^{\cdot}}{\chi}\partial_t\nabla^2\phi=0 \quad (1.10)$$

Where $\varepsilon^{\cdot}=\eta m\chi$

Also from the coupled equations (!.3) and (1.9) we get two other wave equations.

$$\left[\left(\nabla^2-\frac{1}{k_2^2}\partial_t^2\right)\left(\nabla^2-2\alpha_4-\frac{1}{k_4^2}\partial_t^2\right)+\alpha_2\alpha_4\nabla^2\right]\psi=0 \quad (1.11)$$

$$\left[\left(\nabla^2-\frac{1}{k_2^2}\partial_t^2\right)\left(\nabla^2-2\alpha_4-\frac{1}{k_4^2}\partial_t^2\right)+\alpha_2\alpha_4\nabla^2\right]\omega_3=0 \quad (1.12)$$

It may be noticed that equation (1.10) represent longitutudinal wave where as SOLUTION

Let us introduce a new system of co –ordinates $X_1^{\bullet},X_2^{\bullet},X_3^{\bullet}$ connected with each point of the surface in such a manner that the $X_2^{\bullet}$ – axis is directed along the vector normal to the surface and $X_1^{\bullet}$ axis coincides with the direction of X_3 ,
Tangent and normal stresses and expressed in the new co –ordinate system as follows ;

$$\sigma_{2j}^{\bullet}=\sum_{K=1}^{3}\sum_{i=1}^{3}\sigma_{ik}\cos\left(x^{\bullet}_j,x_1\right)\cos^{-1}\theta\cos\left(x^{\bullet}_j,x_k\right) \quad \text{.......................(1.13)}$$

$$\mu^{\bullet}_{j3}=\sum_{k=1}^{3}\sum_{k=1}^{3}\mu_{ik}\cos\left(x^{\bullet}_j,x_i\right)\cos\left(x^{\bullet}_3,x_k\right), j=1,2,3$$

Boundary condition at $x_2=hf(x_1)$ become

$$\sigma_{nn}=\sigma_{11}\alpha^2x_1+\left(\sigma_{12}+\sigma_{21}\right)n_{x_1}n_{x_2}+\sigma_{22}n^2_{x_2}$$

$$\sigma_{ns}=\left(\sigma_{11}-\sigma_{22}\right)n_{x_1}n_{x_2}-\sigma_{12}n^2_{x_1}+\sigma_{12}n^2_{x_1}+\sigma_{21}n^2_{x_2}$$

$$\mu_{sp}=\mu_{13}n_{x_1}+\mu_{23}n_{x_2} \quad \text{.........................(1.14)}$$

$$\frac{\partial T}{\partial n}+HT=n_{x_1}T_{X_1}+n_{x_2}T_{x_2}+HT=0$$

Here $\sigma_{nn},\sigma_{ns},\mu_{sp}$ $\sigma_{nn},\sigma_{ns},\mu_{sp}$ are the prescribed function at the boundary curve and n_{x_1} and n_{x_2} are the components of linear unit normal and are given by

$$n_{x_1}=hf^{\bullet}\left(1+h^2f^{\bullet 2}\right)^{-\frac{1}{2}}$$

$$n_{x_2}=n_{x_1}=hf^{\bullet}\left(1+h^2f^{\bullet 2}\right)^{-\frac{1}{2}} \quad \text{.............(1.15)}$$

Where prime represents differentiation w.r.t x_1 ,
Expanding (1.15) in power series and retaining terms upto first degree in h and then substituting in (1.14) we get

$$\sigma_{nn}=-hf^{\bullet}\left(\sigma_{12}+\sigma_{21}\right)+\sigma_{22}$$

$$\sigma_{ns}=-hf^{\bullet}\left(\sigma_{11}-\sigma_{22}\right)+\sigma_{21} \quad \text{................................(1.16)}$$

$$\mu_{sp}=-\mu_{13}hf^{\bullet}+\mu_{23}$$

Introducing (1.16) in boundary condition (1.1) we get

h$\left(\sigma_{12}+\sigma_{21}\right)f^{\bullet}+\sigma_{22}=p\delta(x)$

h$\left(\sigma_{11}-\sigma_{22}\right)f^{\bullet}+\sigma_{21}=0$ ....(1.17)

$$-\mu_{13}hf^{\bullet}+\mu_{23}=0$$

And $-h\frac{\partial T}{\partial x_1}f^{\bullet}+\frac{\partial T}{\partial x_2}+HT=0$

In the subsequent considerations we shall discuss only theharmonic vibrations and hence we shall have the following ;

$$\{T,\phi,\psi,\sigma_{\beta},\mu_{\beta},\omega_3\}=\{\overline{T},\ \phi,\psi,\ \overline{\sigma_{\beta}},\ \overline{\mu_{\beta}},\overline{\omega_3}\ \}\ \text{ei}\omega\,\text{t} \quad \text{.................................(1.18)}$$

Where ω is the frequency of the vibrations.Since the subsequent considerations will be restricted to only the vibrations harmonic in time , there fore we shall drop over bars in the symbols of the functions in (1.18).Let us introduce one-dimensional fourier transform of the type

$$\overline{f}\left(\xi\right)=\left(2\pi\right)^{\frac{-1}{2}}\int_{-\infty}^{\infty}f(y)e^{i\xi y}dy \quad \text{......................................(1.19)}$$

f(x)=(2$\pi)^{\frac{-1}{2}}\int_{-\infty}^{\infty}\overline{f}\left(\xi\right)e^{-i\xi x}d\xi$ taking the fourier transformation of (1.10),(1.11),

and (1.12) with respect to the variable x$_1$, we obtain the following transformed ordinary differential equation

$$\left|\left(D^2-\xi^2+P^2_1\right)\left(D^2-\xi^2q\right)-q\upsilon^{\cdot}\left(D^2-\xi^2\right)\right|\overline{\phi}\left(\xi,x_2\right)=0$$

$$\left|\left(D^2-\xi^2+p^2_2\right)\left(D^2-\xi^2-2\alpha_4+p^2_4\right)\right|+\alpha_2\alpha_4\left(D^2-\xi^2\right)\{\overline{\psi}\ \ \left(\xi_1,x_2\right),\overline{\omega_3}\ \ \left(\xi,x_2\right)\ \}$$

…………(1.20)

Where $p_1=\frac{\omega}{k_1},q=\frac{i\omega}{\chi},p_2=\frac{\omega}{k_2},D=\frac{d}{dx},P_4=\frac{\omega}{k_4}$

Solution of equations (1.19) and (1.20) satisfying the condition that stresses and couple- stresses vanish as $x_1^2+x_2^2\to\infty$ is given by

$$\overline{\phi}\ \left(\xi,x_2\right)=Ae^{-\lambda_1x_2}+Be^{-\lambda_2x_2}$$

$$\overline{\psi}\left(\xi,x_2\right)=Ce^{-\chi_1x_2}+De^{-\chi_2x_2} \quad \text{..........................(1.21)}$$

$$\overline{\omega_3}\left(\xi,x_2\right)=Ee^{-\chi_1x_2}+Fe^{-\chi_2x_2}$$

Where $\lambda^2_1,\lambda^2_2=\frac{1}{2}\left[2\xi^2-\left(p^2_1-q_1\right)\pm\sqrt{\left(p^2_1-q_1\right)^2+4p^2_1q}\right]$

$$\chi^2_1,\chi^2_2=\frac{1}{2}\left[2\xi^2-\left(p^2_2+p^2_4+\alpha_2\alpha_4-2\alpha_4\right)\pm\sqrt{\left(p^2_2+p^2_4+\alpha_2\alpha_4-2\alpha_4\right)^2-4p^2\left(p^2_4-2\alpha_4\right)}\right]$$

$q_1=q\left(1+\vartheta^{\cdot}\right)$ and A,B,C,D,E,F, are function of also functions ψ and ω_3 are

connected by the relation

$$\frac{\chi^2_1-\xi^2+p^2_2}{\alpha_2}C,\ F=\frac{\left(\chi^2_2-\xi^2+p^2_2\right)}{\alpha_2}D \quad \text{..............................(1.22)}$$

Now from tress-strain relations we have

$$\sigma_{11}=-2\mu\phi_{22}+2\mu\psi_{21}+\rho\phi^{\bullet\bullet}$$

$$\sigma_{12}=2\mu\phi_{12}+\mu\left(\psi_{22}-\psi_{11}\right)-\alpha\left(\psi_{11}+\psi_{22}+2\omega_3\right) \quad \text{.................(.1.23)}$$

$$\sigma_{22}=-2\mu\phi_{11}-2\mu\psi_{21}+\rho\phi^{\bullet\bullet}$$

$$\mu_{13}=\left(\gamma+\vartheta\right)\omega_{31}\ \sigma_{21}=2\mu\phi_{12}+\mu\left(\psi_{22}-\psi_{11}\right)+\alpha\left(\psi_{11}+\psi_{22}+2\omega_3\right)$$

Substituting (1.24) in (1.16) and applying fourier transformation , we get

$$\overline{\sigma_{nn}}=-h\left(\overline{\sigma_{12}}\right)+\overline{\sigma_{21}}\ )\overline{f}+\overline{\sigma_{22}}$$

$$=-2\mu h\left(\overline{\psi_{22}}-\overline{\psi_{11}}+2\overline{\phi_{12}}\ \right)\overline{f}-2\mu\overline{\phi_{11}}-2\mu\overline{\psi_{21}}+\rho\overline{\phi}^{\bullet\bullet}$$

$$=-2\mu h\left[C\left(\chi^2_1+\xi^2\right)e^{-\chi_1x_2}+D\left(\chi^2_2+\xi^2\ \right)e^{-\chi_2x_2}2i\xi\left(\lambda_1Ae^{-\lambda_1x_2}+\lambda_2Be^{-\lambda_2x_2}\ \right]\overline{f}+2\mu\xi^2\left(Ae^{-\lambda_1x_2}+Be^{-\lambda_2x_2}\ \right)\right.$$ [illegible]

$$\chi_1Ce^{-\chi_1x_2}+\chi_2De^{-\chi_2x_2}\ )-\rho\omega^2\left(Ae^{-\lambda_1x_2}+Be^{-\lambda_2x_2}\ \right)$$

Or

$$\frac{\overline{\sigma_{nn}}}{2\mu}=A\left(-2i\xi h\lambda_1\overline{f}+\frac{\rho\omega^2}{2\mu}\ \right)e^{-\lambda_1x_2}+B\left(-2i\xi h\lambda_2\overline{f}+\xi^2-\frac{\rho\omega^2}{2\mu}\ \right)e^{-\lambda_2x_2}+$$

$$C\left|-h\left(\chi^2_1+\xi^2\ \right)\overline{f}-i\xi\chi_1\ \right|e^{-\chi_1x_2}+D\left|-h\left(\chi^2_2+\xi^2\ \right)\overline{f}-i\xi\chi_2\ \right|e^{-\chi_2x_2}$$

$$=\left[-\left(a_1+a_4\right)h\overline{f}+a_2\right]Ae^{-\lambda_1x_2}+\left[-\left(b_1+b_4\right)h\overline{f}+b_2\right]Be^{-\lambda_2x_2}+\left[-\left(d_1+d_4\right)h\overline{f}+d_2\right]De^{-\chi_2x_2}$$

Where we have used the notation

$$a_1=a_4=i\xi\lambda_1$$

$$b_1=b_4=i\xi\lambda_2$$

$$a_2=b_2=\xi^2-\frac{\rho\omega^2}{2\mu}$$

$$c_2=-i\xi\chi_1,d_2=-i\xi\chi_2$$

$$c_1=\frac{\left(\mu-\alpha\right)\chi^2_1+\left(\mu+\alpha\right)\xi^2}{2\mu}$$

$$c_4=\frac{\left(\mu+\alpha\right)\chi_1+\left(\mu-\alpha\right)\xi^2}{2\mu}$$

$$d_1=\frac{\left(\mu-\alpha\right)\chi^2_2+\left(\mu+\alpha\right)\xi^2}{2\mu}$$

$$a_4 = \frac{(\mu+\alpha)\chi^2_1 + (\mu-\alpha)\xi^2}{2\alpha}$$

$$\frac{\overline{\sigma_{ns}}}{2\mu} = \left[\left(\lambda_1^2 + \xi^2\right)hf' + i\xi\lambda_1\right]Ae^{-\lambda_1 x_2} + \left[\left(\lambda^2_2 + \xi^2\right)hf' + i\xi\lambda_2\right]Be^{-\lambda_2 x_2}$$
$$+\left[\frac{1}{2}\left(\chi^2_1 + \xi^2\right) + \frac{\alpha}{2\mu}\left(\chi^2_1 - \xi^2\right) - 2i\xi\chi_1 hf'\right]Ce^{-\chi_1 x_2} + \left[\frac{1}{2}\left(\chi^2_1 + \xi^2\right) + \frac{\alpha}{2\mu}\left(\chi^2_2 - \xi^2\right) - 2i\xi\chi_2 hf'\right]De^{-\chi_2 x_2} + \frac{\infty}{}$$
$$+\frac{\alpha}{\mu}Ee^{-\chi_1 x_2} + \frac{\alpha}{\mu}Fe^{-\chi_2 x_2}$$

$$\left[-(a_1 - a_2)hf' + a_4\right]Ae^{-\lambda_1 x_2} + \left[-(b_1 - b_2)hf' + b_4\right]Be^{-\lambda_2 x_2} + \left[-(c_1 - c_2)hf' + c_4\right]Ce^{-\chi_1 x_2} +$$
$$+E(e_1 \quad)e^{-\chi_1 x_2} + F(e_2)e^{-\chi_2 x_2} \, where$$

$$a_1 = -\lambda^2_1 - \frac{\rho\omega^2}{2\mu}, b_1 = -\lambda^2_2 - \frac{\rho\omega^2}{2\mu}$$ qq

$$c_1 = i\xi\chi_1, d_1 = i\xi\chi_2, e_1 = -\frac{\alpha}{\mu}, e_2 = \frac{\alpha}{\mu}$$

Boundary condition (1.16) becomes

$$\overline{\mu_{sp}} = -\overline{\mu_{13}}hf' + \overline{\mu_{23}}$$

$$\overline{\mu_{13}} = (\gamma + \vartheta)\delta_1\omega_3 = -(\gamma + \vartheta)i\xi\left[Ee^{-\chi_1 x_2} + Fe^{-\chi_2 x_2}\right]$$
$$\overline{\mu_{23}} = (\gamma + \vartheta)\delta_2\overline{\omega 3} = (\gamma + \vartheta)\left[-E\chi_1 e^{-\chi_1 x_2} - F\chi_2 e^{-\chi_2 x_2}\right]$$

Hence

$$\frac{\overline{\mu_{13}}}{\gamma + \vartheta} = \left(e_3 hf' - e_4\right)Ee^{-\chi_1 x_2} + \left(f_3 hf' - f_4\right)Fe^{-\chi_2 x_2} \ldots\ldots\ldots\ldots\ldots\ldots(1.26)$$

since the boundary is not uniform A,B,C,D,E,F are also functions of h(perturbation parameter).considering approximation only upto the first order of h ,.

CONCLUSION

It is seen that surface wave breaks into three parts ; (a) Rayleigh wave with velocity C (b) scattered wavws with velocity of propagation $\frac{c\omega}{\omega - rc}$ and $\frac{c\omega}{\omega + rc}$

(c) The scattered waves depend on the wave length as well as on the wavy nature of the boundary

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