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A Suggestion Branch-and-Bound Technique for MOPS

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ABSTRACT *Outer planar graph are a widely studied graph class with application in graph drawing [1, 2] and with intersecting theoretical properties [3, 4, 5].*

The maximum outer planar sub graph problem (MOPS) is NP-hard. The problem of finding a MPS has important application in circuit layout ,automated graph drawing, and facility layout .the maximum planar sub graph problem(MPS) is too NP-hard. In this article we present branch – and- bound technique for finding MPS and MOP of graph.

KEYWORDS: . MOPS, MPS ,,Branch – and- Bound

1. Introduction

A drawing of a graph is plane it no two distinct edge intersect apart from their end points.

A graph is outer planer if it admits a plane embedding where all its vertices surround the same region and no two distinct edge intersect , otherwise the graph is non outer planar the problem of finding a MPS has important application in the layout of printed circuits boards, for the case of non isulated wires, overlapping wires between electrical components may cause short circuits and thus are prohibited. hence it is desirable to find layouts which the maximize the number of connections while avoiding wire overlaps .similar considerations also hold for the design of VLSI circuits.

The problem of finding a MOPS are a widely studied graph with application in graph drawing and with intersecting theorectical properties.

2. Characterization of outer planar graphs.

Def(2.1).if a graph G=(V,E') is a outer planar of G=(V,E) Such that every graph G=(V,E) obtained from G"=(V,E") by adding on edge from E\E' is outer planar , then G'=(V,E') is called a maximal outer planar sub graph of G.(MOPS)

Def (2.2).let G'= (V, E') be a maximal outer planar sub graph of G"= (V, E") of G with |E"|>|E'|, then G' is a maximum outer planar sub graph.

Def (2.3) .A graph H is said to be homeomorphic from G if either ifG or it is isomorphic to a subdivision of G.

Theorem (2.1). [7] A graph is outer planar if and only if it has sub graph homeomorphic to k_4 or $K_{2',3}$.

Theorem (2.2) a graph is outer planar if and only if G +K_1 is planar.

Theorem (2.3). [6] Let G'= (V, E") be a maximum outer planar sub graph of a graph G=(V,E) then |E| ≤2|V|-3.

Theorem (2.4). [7] Let G'= (V, E') be a maximum outer planar sub graph of a graph G=(V, E) which does not contains any triangle. Then |E| ≤3/2|v|-2.

Corollary (2.1) .if m<6 then G is outer planar.

Theorem (2.5). [9] The maximum outer planer sub graph of a contain $3\times2^{n-1}-2$ edge.

Def (2.4) performance ratio R_A (p) of an approximation algorithm A for a maximization problem P is the minimum ratio of obtained solutions to the cost of optimal solution:

$$RA=\begin{cases}\text{Min } \frac{A(G)}{opt(G)}, \text{ if opt }(G) \neq 0\\ 1, \text{ if opt }(G)=0\end{cases}$$

Since a tree is outer planar, Mops can also approximated by constructing a spanning tree. Since a spanning tree contain n-1 edges, and a maximum outer planar sub graph could (outgain at most 2n-3 edge (by theorem (2.3)), the performance ruction of this method is 1/2.

$$\lim_{n\to\infty}\frac{n-1}{2n-3}=\frac{1}{2}$$

And since a maximal outer planar sub graph of any graph G can have no more than (by theorem (2.4)), and any spanning tree of G ,which as bipartite ,has n-1 edges , the performance ratio of this method is 2/3.

$$\lim_{n\to\infty}\frac{n-1}{\frac{3n}{2}-2}=\frac{2}{3}$$

3. Characterization of planar graphs.

Def(3.1).if a graph G=(V,E') is a planar of G=(V,E) Such that every graph G=(V,E) obtained from G"=(V,E") by adding on edge from E\E' is planar , then G'=(V,E') is called a maximal planar sub graph of G.(MPS)

Def (3.2).let G'= (V, E') be a maximal planar sub graph of G"= (V, E") of G with |E"|>|E'|, them G' is a maximum planar sub graph.

Theorem (3.1). [7] A graph is planar if and only if it has sub graph homeomorphic to $k_5$or $K_{3',3}$.

Theorem (3.2). [11] Let G'= (V, E") be a maximum planar sub graph of a graph G=(V,E) then |E| ≤2|V|-3.

Theorem (3.3). [11] Let G'= (V, E') be a maximum planar sub graph of a graph G=(V, E) which does not contains any triangle. Then |E| ≤3/2|v|-2.

Corollary (3.1) .if m<9 then G is planar.

Similarly by theorem (3.2) and theorem (3.3)

$$\lim_{n\to\infty}\frac{n-1}{3n-6}=\frac{1}{3}$$

If G has not triangle then

$$\lim_{n\to\infty}\frac{n-1}{2n-4}=\frac{1}{2}$$

4. Branch – and – Bound algorithm

One method to solve a discrete and finite optimization problem is to generate all possible solution and then choose the best one of them. For NP-complete problems this exhaustive search method fails since the number of solutions is exponential in the size of the input. For example, given a graph G= (V, E), there exist $2^{|E|}$ different sub graph containing all |V| vertices.

If the problem in question asks a sub graph of the given graph with some specific property, it is possible that all sub graphs.

need to be checked before the right one is found. Only small instance can be solved in this way.

Branch – and – bound is a method that can be used in the exhaustive search by recognizing partial solutions that can not lead to an optional solution. For example, suppose that we know that the optimal solution for a maximization problem is at least k (we found a solution with cost k).

If during the generation of the other solution candidates we recognize that the current solution can not be augmented to any solution with cost≥ k, we can stop generating these solution vindicates and continue searching form more promising solution.

The efficiency of the branch- and- bound techniques depends highly on the problem in question and the order in which the solution candidates are found.

If the lower bound for the optimal solution is bad, there are little possibilities to reject any solutions. The worst case during time of branch - and- bound is still exponentially, but often it decreases the computation time remarkably. [8]

(4.1)branch –and-bound for MPS[10]

Since a planar graph can have no more than 3n-6 edge , and since any graph with fewer than 9 edges is trivially outer planar ,it suffices to generate and test only sub graphs satisfying these size constraints and hence at most

$$\sum_{i=6}^{3n-9} \binom{m}{3n-i}$$

Is number. Secondly, since a non –planar graph may have less than 3n-6 edges, say, 3n-k where **k > 6,** one need test at most

$$\sum_{i=k}^{3n-9} \binom{m}{3n-i}$$

Sub graphs, which is still exponentially large. Similarly by paper [10] we present branch – and- bound technique for finding MOPS of graph..

(4.2) branch –and-bound for MOPS

Since a outer planar graph can have no more than 2n-3 edge , and since any graph with fewer than 6 edges is trivially outer planar ,it suffices to generate and test only sub graphs satisfying these size constraints and hence at most

$$\sum_{i=3}^{2n-6} \binom{m}{2n-i}$$

Is number. Secondly, since a non – outer planar graph may have less than 2n-3 edges, say, 2n-k where **k >3**, one need test at most

$$\sum_{i=k}^{2n-6} \binom{m}{2n-i}$$

Sub graphs, which is still exponentially large.

5.Conclusion

In this paper we show that some results have concerning the planar and outer planar of a graph. In particular, the branch-and-bound technique for MOPS of graph are given.

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