Volume : 2 | Issue : 8 | Aug 2013 • ISSN No 2277 - 8160

Research Paper | Mathematics

On a Traffic Control Problem Using Interval Graph

Niky Baruah — Department of Mathematics, Dibrugarh University, Dibrugarh-786004, Assam

Arun Kumar Baruah — Department of Mathematics, Dibrugarh University, Dibrugarh-786004, Assam

ABSTRACT *The problem of traffic congestion in modern days calls for the design and implementation of efficient control strategies. One of the main uses of traffic control studies is to develop traffic models which can be used for estimation, prediction and control related tasks. In this paper a traffic control problem at an intersection is considered and conflict between traffic participants are prevented by using interval graph. The compatible streams are allowed to overlap in a real line, called an interval graph. An optimal feasible green light assignment and phasing of traffic light is done with the help of interval spanning subgraph of the compatibility graph which is illustrated with an example.*

KEYWORDS: Compatibility Graph, Cycle Time, Cliques, Interval Graph, Traffic Control.

1. Introduction

Modeling of traffic flow has a long history in various fields ranging from engineering to physics and applied mathematics. Traffic related problems are all important to modern society. The solution posed by scientists to alleviate traffic congestion and increase safety starts from improving the management system, major use of IT system, to the automation of vehicles and the road infrastructure [5], [6].

The problem of designing a control scheme for a junction falls naturally into two parts. The streams of traffic and pedestrians movements are first identified and divided into mutually compatible sets which are to receive right-of-way together and an order is determined in which right-of-way is to be granted to these sets. Once this has been done suitable durations are chosen for the periods during which each of the sets receives right-of-way. Optimal traffic control on an isolated intersection is very complex problem, especially because of the combinatorial nature of the problem. The traffic control on a signalised intersection is performed by means of traffic lights of different colours (green, amber and red) that are repeating periodically. Conflict between traffic participants are prevented by diving the cycle time in intervals. The control in one period is defined by one control vector called phase whose components are control variables that control traffic by means of traffic lights. Several traffic flows that are not mutually conflicting can get the right-of-way during the same interval. The development of the methods of combinatorial optimization beside the progress in equipment development was also an unavoidable condition for the solution of such problems. The compatibility approach to the optimal traffic control problem on an isolated intersection was found by Stoffers by introduction of compatibility graph of traffic streams [8], [10]. The traffic streams sets with maximal number of non conflicting traffic streams, which can get the right-of-way simultaneously can be determined by extracting cliques from the compatibility graph [1].

For solving the control problem it is necessary to know relations between traffic streams at an intersection. By traffic streams we mean the following: traffic streams on an intersection are elements of the set of traffic stream, ρ i.e. .

$\rho = \{\sigma_1, \sigma_2, \ldots, \sigma_i, \ldots, \sigma_I\}$; where $i \in \Im$ and $\Im$ is the set of traffic stream indices

$$\Im = \{1, 2, 3, \ldots, i, \ldots, I\}$$

Since the main objective of traffic control by traffic lights is to give the right-of-way to some of traffic streams and to stop others, it is necessary to find in the set of traffic streams of an intersection, the streams that can simultaneously get the right-of-way [7].

2. Interval Graph

The intersection graph of a family of intervals on the real line is called an interval graph. Given a graph *G= (V, E),* we can find whether it is isomorphic to an interval graph. This is equivalent to the assignment of an interval *J(x)* to each vertex x in *V (G)* so that for all u≠v in *V (G).*

$$\{u, v\} \in E(G) \Leftrightarrow J(u) \cap J(v) \neq \phi \qquad (1)$$

To illustrate this, we consider an interval assignment ***J*** satisfying (1) by taking *J(a)* = [1, 5], *J(b)* = [4, 8], *J(c)* = [1, 14], *J(d)*= [7, 11], and *J(e)* = [10, 14].

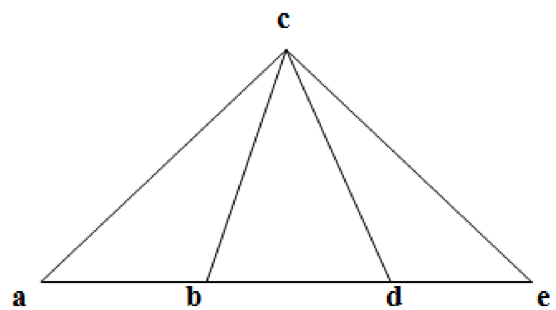

Fig. 1 : Interval graph

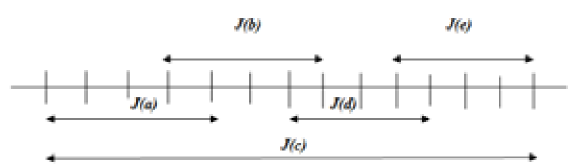

Fig. 2 : Interval representation on real line

Two graphs G = (V, E) and H = (W, F) are isomorphic if there is a one-one onto function f: V W so that

$$\{u, v\} \in E(G) \Leftrightarrow \{f(u), f(v)\} \in E(H)$$

The interval may be open, closed, or half-open. However, without loss of generality, they may all be taken as open or all taken as closed [9].

Interval graph arose from purely mathematical considerations and independently from a problem of genetics. Interval graphs also arise in the measurement of preference and indifference and in seriation in the social sciences, for putting a collection of items or objects in some serial order or sequence. For example in archaeology we are interested in sequence dating a collection of articrafts. In psychology, for putting some traits in a development order, or order individuals according to their opinions. In political Science, to order political candidates from liberal to conservative etc.

3. Characterization of Interval Graphs

Let us consider a graph , where *n* is the number of circuits in the graph. The graph , the circuit of length 3, is an interval graph but that for *n* is not [9].

For example let us consider a graph of circuit of length 4 and label the vertices of as shown in the figure bellow. If there is an interval assignment J satisfying (1), then J(a) and J(b) would have to overlap, since there is an edge between the vertex a to b. Similarly, J(c) overlap J(b) since there is an edge between b and c but not J(a). Finally J(d) must overlap both J(a) and J(c) as shown in the figure but not J(b), which cannot be represented in a line. Hence is not an interval graph.

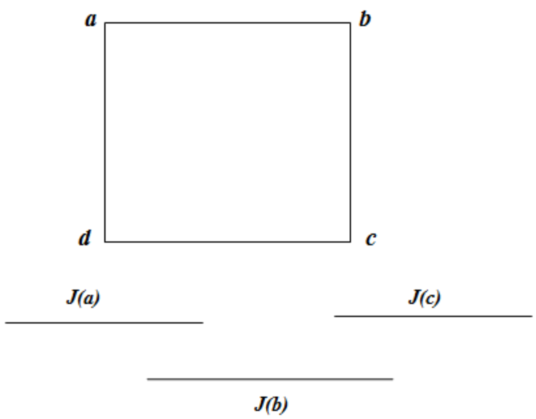

Fig. 3 : is not an interval graph

4. Formulation of the Problem

In this paper we shall use interval graph for phasing of traffic lights at an intersection. The application of interval graph for sequencing of traffic lights at an intersection, for a safer and efficient flow of traffic is of great importance. Let us consider a four legged traffic intersection with six traffic streams as shown in the **Fig. 4**. Approaching the traffic intersection are various traffic streams, patterns or route through the intersection which traffic takes. The compatibility information can be summarized in a graph known as the compatibility graph G [3], [4]. The vertices

of G are the traffic streams and two streams are joined by an edge if and only if they are judged compatible, i.e. the streams do not conflict.

The compatibility graph G obtained from the traffic intersection as shown in the **Fig. 5** is not an interval graph as there is a circuit of length 4 i.e. the subgraph generated by the vertices .

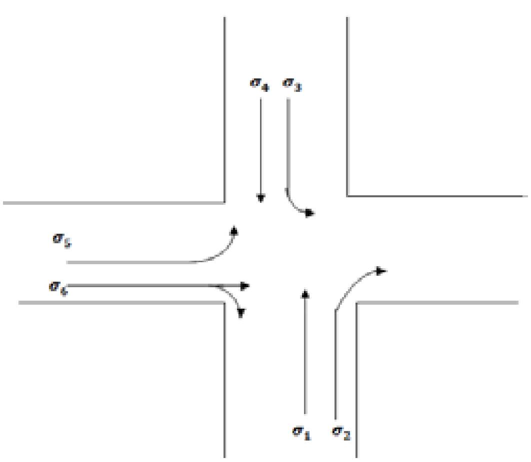

Fig. 4 : Traffic intersection with six traffic streams

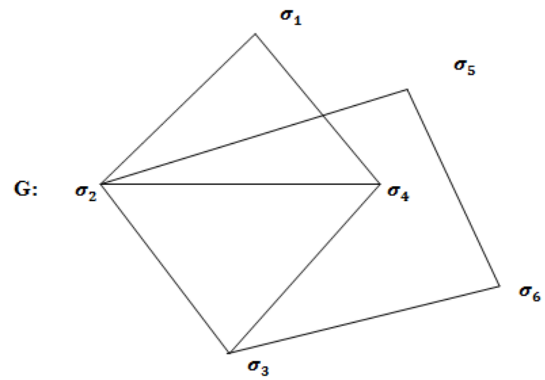

Fig. 5 : Compatibility Graph of the intersection

5. Solution of the Problem

The number of cliques obtained from the compatibility graph represents a solution to the control problem at an intersection. These cliques are nothing but the signal groups can be allowed to move through the intersection [2]. It is obvious from the **Fig. 5** the numbers of cliques formed are five and phasing of traffic lights cannot be done as the numbers of streams are more than the number of variables used to control the flow of traffic i.e. the control vectors.

So we construct the interval graph from the compatibility graph G, i.e. we are to obtain a spanning subgraph of the compatibility graph G. This spanning subgraph is achieved by deleting the edges of the compatibility graph to a stage where there should not be any isolated vertex. This interval graph obtained from the compatibility graph is the spanning subgraph of the compatibility graph. The phasing of traffic lights is done using the interval graph **(Fig. 6)** of the traffic intersection.

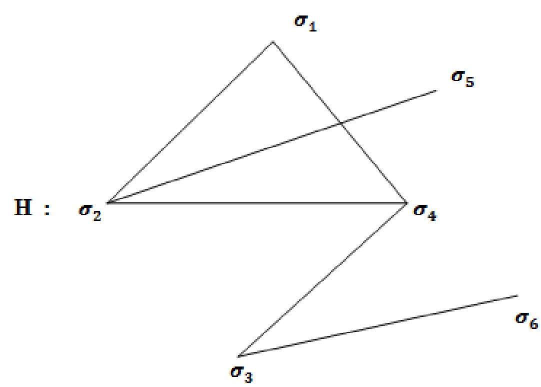

Fig. 6 : Interval Graph of the intersection

The cliques of the interval graph H are :

$K_1 = \{\sigma_1, \sigma_2, \sigma_4\}$

$K_2 = \{\sigma_2, \sigma_5\}$

$K_3 = \{\sigma_3, \sigma_4\}$

$K_4 = \{\sigma_3, \sigma_6\}$

Sequencing of traffic lights can be done by dividing the cliques into the four phases which is repeated periodically according to the cycle time, red, amber, green, red amber **(Fig. 7).**

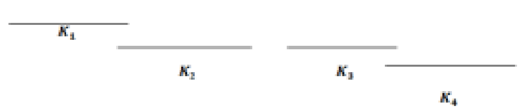

Fig. 7 : Phasing of traffic lights

Thus to find an optimal feasible green light assignment we must identify the interval subgraph *H* of the compatibility graph *G*, which is a spanning subgraph. The cliques obtained from the interval spanning subgraph *H* is used for feasible green light assignment of the traffic intersection, hence providing a solution to the control problem at an intersection.

6. Conclusion

In this paper we have used interval graph as a graph theoretic tool to study traffic control problem at an intersection. The interval graph is the intersection graph represented on a real line in which overlapping of the interval exists. Phasing of traffic lights into the four phases is done using interval graph, i.e. the spanning subgraph of the compatibility graph. The solution obtained in this paper offers an efficient tool to control the flow of traffic at an intersection.

7. Acknowledgement

This Research is funded by grants from the UGC, New Delhi, India as a Major Research Project awarded to Arun Kumar Baruah. Niky Baruah is associated with the project as a Project Fellow.

REFERENCES

[1] Augustson, J .G., Minker, J., 1970, An analysis of some graph theoretical cluster techniques, Journal of ACM, Vol. 17, No. 4, pp. 571-588. | | [2] Baruah, A. K., Baruah, N., 2012, Signal Group of Compatible Graph in Traffic control Problems, Int. J of Advance Networking and Application, Vol. 04, Issue : 01, pp. 1473 - 1480. | | [3] Chartrand, G., 1977, Introductory Graph Theory, Dover Publishers, Inc., New York. | | [4] Deo, N., 2002, Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India, New Delhi. | | [5] Gazis, D. C., 1970, Traffic Science, John Wiley & Sons, New York. | | [6] Gazis, D. C., 2002, Traffic Theory, Kluwer Academic Publisher, London. | | [7] Guberinic, S., Senborn, G., Lazic, B., 2008, Optimal Traffic Control : Urban Intersections, CRC Press, Taylor and Francis Group, New York. | | [8] Mitten, L. G., 1970, Branch and Bound Methods: General Formulation and Properties, Opns. Res., Vol. 18, pp. 24-34. | | [9] Roberts, F. S., Graph Theory and Its Application to Social Science, 1978, Regional Conference Series in Applied Mathematics, SIAM, Philadelphia, USA.. | [10] Stoffers, E. K., 1968, Scheduling of Traffic Lights – A New Approach, Transportation Research, 2, pp. 199. |