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On Signed Graph in the Formation of a Government

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ABSTRACT

In this paper, it is shown how using graph theory we can study the formation of a government among political parties and its stability or unstability, where nodes are used to represent different political parties and edges are used to represent the relation between the political parties with a signed associated with it. The edges with positive (+) sign represent like and the edges with negative (-) sign dislike each other. In this paper, we shall use the signed graph as a graph theoretic tool and balanced concept of signed is used to study the stability of a government. In this context, both the case of balanced and unbalanced situations of a political problem are presented.

KEYWORDS: Balanced, signed graph, stability, unbalanced

INTRODUCTION

Graph theory is that branch of mathematics, which deals with the objects and the relationship between them. Graph theory was born in 1736 with Euler's paper in which he solved Konigsberg bridge problem. Graph theory can be applied in different fields including computer science, science, operation research, business, engineering etc. In 1847, G.R Kirchhoff (1824-1887) developed the theory of trees for their applications in electrical networks. In 1857 A. Cayley (1821-1895) discovered trees while he was trying to enumerate the isomers of saturated hydrocarbons. Graph theory also used in traffic control problems.

In 2002, Riedel has developed a traffic model and he has presented the control algorithm for the chosen model using a combination procedure of 'Dynamic Programming' and 'Branch and Bound' type. In the process, Riedel has used Graph Theory as a tool for visualisation of the algorithm. In the first article on signed graphs - graphs edges are labeled positive or negative (Harary, F., [4]) to treat a question in social psychology [3] giving a simple characterization of those in which the product of signs around every cycle is positive (such graph is called balanced). Later Beineke and Harary [2] included signed vertices and asked the analogus question of characterizing the vertex - signed graphs around every cycle is positive. Also preliminary results may found by Acharya [1] and Rao [6], Hoede [5]. Signed graph and balanced theory are most important part of the graph theory. Graph have also been used extensively in the social science to represent interpersonal relationships. The vertices of the graph correspond to individuals in a group or society, and the edges joint pairs of vertices of individuals who are related in some way. When we used the signed graph and balanced theory on that graph then we know whether the graph is stable or unstable.

SIGNED GRAPH

A graph G is defined usually by a finite set of vertices (or nodes) V and the set E of edges denoted by G = (V, E). In general, the nodes represent objects or entities while the edges represent relationship between the nodes. A signed graph is simply a graph where each edge between the nodes is labelled as either positive (+) or negative (–). It is useful for representing a symmetric binary relation between any two quantities. The concept of signed graph is used if the relationship between each pair of nodes is symmetric. With the help of signed graph we may represent the interpersonal relationships between groups of individuals. The simplest approach to study such a group of individuals is to draw a graph in which the individuals are the nodes and in which there is an edge for 'x' to node 'y' if x is in some relation to y. This relationship may be like or dislike, associate with or avoids and so on. We can include two different relationship in a graph by using two different signs i.e. positive (+) and negative (–) to distinguish them. Then the presence of an edge means that there is a relationship between the nodes and the indication of a (+) sign represents a positive relation such as like, agree etc. and the (–) sign represents the other relation such as dislike, disagree, hate etc. which lead to the existence of signed graph.

Signed graph has been extensively used in Social Sciences to represent the interpersonal relationship among the group of individuals, and is also used in studying international relations between countries or nations, where the nodes corresponds to nation or group of nations and the edges join pairs of nations which are allied with, agrees on a particular strategy etc. Signed graph has its application in political Science or Politics where the Politicians form the nodes and the edges represent whether a Political group allied with other groups in order to form a government.

BALANCED AND UNBALANCED GRAPHS

A relevant concept in describing the structures of signed graph is the concept of structural balance [5]. According to the principle of balanced theory, a structure is said to be balanced (or stable) if all the cycles of the signed graph are positive. Similarly, a structure is said to unbalanced if there exists at least one cycle which is negative. Here it means that for a negative cycle, there is an odd number of negative (–) edges in the cycle while a positive cycle has an even number of negative (–) edges in the cycle [7]. Suppose four person namely a, b, c, d work together but problem is that 'a' likes 'c' but not 'b', 'c' likes 'a' and 'd' but not 'b', 'd' likes 'b' and 'c' and 'b' likes 'd' but not 'a' or 'c'; note that 'a' and 'd' have no strong felling about each other. If we draw the figure we have

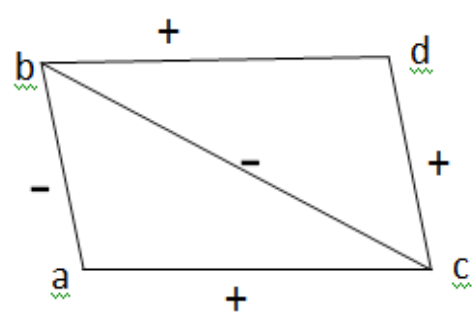
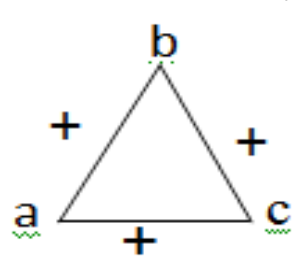
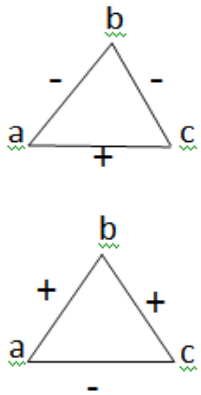


Fig-1

Now we consider of following diagrams which illustrate some of the situations that can occur when three people work together.





From this figure, we have to see that in first case there is no tension because they work together, In second case 'a' likes 'c' but 'a' and 'c' both dislikes 'b' that is 'a' and 'c' do not wish to work together with 'b' so 'b' work on his on. And 'a' and 'c' work together so the graph is balanced because there is even number of negative (-) so there is no tension. In third case, 'b' likes both 'a' and 'c' and would like to work together but 'a' and 'c' dislike each other and do not wish to work together in this case there is a tension that is unbalanced. We express this by saying that the first two case are called balanced where as the third case is unbalanced.

Definition (1) : A graph G = (V, E) is called a signed graph if a sign either positive (+) or negative (–) is associated with each of its edges.

Definition (2) : A relationships is called symmetric if x likes y if and only if y likes x where x and y are groups, societies etc.

Theorem (1) : A complete signed graph G is balanced if and only if its vertex set V is partition into two disjoint subsets X and Y, one of which may be empty, such that all lines between the points of the same subset are positive and all lines between points of two different subsets are negative [7].

FORMULATION OF THE PROBLEM

Here we consider a political party problem for symmetric case of six political parties where, a, b, c, d, e, f are the political parties. Here we shall show two cases, one for the balanced situation and another for the unbalanced situation as shown below :

Case – I : Representing a balance of political party problem by using signed graph. We consider the relationship of the political parties as shown in the following figure :

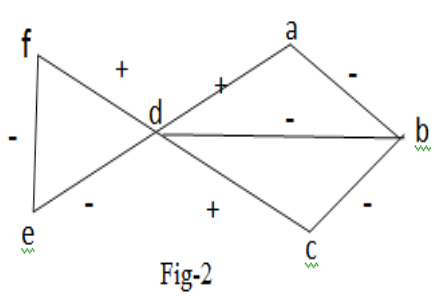


Fig-2

Here, vertices represent the political parties and the edges joining the vertices represent a relationship between the vertices (political parties). The positive (+) sign indicates that the political parties likes each other, and the negative (-) sign represent that the political parties dislike each other. Therefore from the Theorems (1), we found that the system represented by Fig-2 is a balanced. As we can partition the vertex set into two disjoint sub-sets and edges joining the vertices of the same sub set are positive and the edges joining the vertices of two different sub-sets are negative. Here the vertices a, c, d, f, can be put in one set as {a, c, d, f}, and the vertices b, c, can be put another set as {b, c}, such that every edge between the vertices of {a, c, d, f} is positive and the edges from the vertex set {b, c} to any vertex of {a, c, d, f} are negative and hence it is balanced.

Case – II : Representing an unbalanced of political party problem by using signed graph. We consider the relationship of the political parties as shown in the figure below:

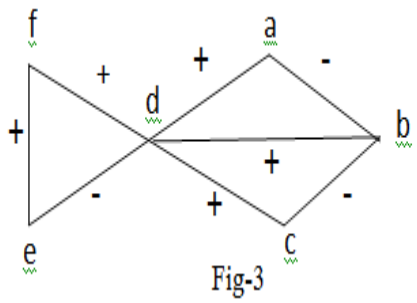


Fig-3

The above signed graph is unbalanced. Here the set of vertex (i.c political party) can not be partitioned into two disjoint subsets, satisfying the conditions of the Theorem (1). Also, there are two negative cycles which mean that the graph representing the above political party problem is unbalanced. From the above discussion it is clear that the political party problem at a balanced situation is smooth, efficient that is it is easier in the formation of governments. On the other hand the political party problem at an unbalanced situation is less efficient as compared to the balanced situation in the formation of governments.

CONCLUSION

Here for our illustration, we have considered two examples, one for the balanced case and the other for the unbalanced case of a political party problem. The examples considered here contain less number of political parties in the formation of governments, however the procedure can be generalised and can be applied to problems consisting of large number of political parties, to check whether formation of a government is possible or not.

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