Volume : 2 | Issue : 1 | Jan 2013 • ISSN No 2277 - 8160

Research Paper | Statistics

A Modification over Ratio Estimator using Empirical Data

Nitu Mehta Department of Agricultural Economics & Management, Rajasthan College of Agriculture, MPUAT, Udaipur-31300

Latika Sharma Department of Agricultural Economics & Management, Rajasthan College of Agriculture, MPUAT, Udaipur-31300

ABSTRACT *The present paper deals with a modified ratio estimator for estimation of population mean of the study variable using the linear combination of Median and Co-efficient of Kurtosis of the auxiliary variable, with the intention to improve the efficiency of ratio estimator in simple random sampling. The first order approximation to the bias and mean square error (MSE) of the proposed estimator are obtained. Theoretically, it is shown that the proposed estimator performs better than the existing modified ratio estimators. In order to justify this, existing agricultural data set from censes 2001 was used which shows that the proposed modified ratio estimator is more efficient than the others.*

KEYWORDS: Ratio-type estimators, Simple random sampling, Auxiliary variables, Mean square error

1. Introduction

Use of auxiliary information has been in practice to increase the efficiency of the estimators. When the population mean of an auxiliary variate is known, so many estimators for population parameter(s) of study variate have been discussed in the literature. When correlation between study variate and auxiliary variate is positive (high) ratio method of estimation (Cochran, 1940) is used. On the other hand if correlation is negative, product method of estimation (Robson, 1940; Murty, 1967) is preferred. When the population coefficient of variation of auxiliary variate C_x is known Sisodia and Dwivedi (1981) suggested a modified ratio estimator for population mean of the study variate. Singh and Kakran (1993) developed ratio-type estimator for $\bar{Y}$ when the coefficient of kurtosis of an auxiliary variable $\beta_2(x)$ is known. Later on Upadhyaya and Singh (1999), derived another ratio type estimators using coefficient of variation and coefficient of kurtosis of the auxiliary variate.

The traditional ratio estimator given by Cochran (1940) for the population mean $\bar{Y}$ of the study variable is defined by

$$\bar{y}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \quad (1.1)$$

in which it is assumed that the population mean $\bar{X}$ of the auxiliary variable x is known. Here $\bar{y}$ is the sample mean of the study variable and $\bar{x}$ is the sample mean of the auxiliary variable.

The bias & mean square error (MSE) of $\bar{y}_R$ are given by

$$B(\bar{y}_R) = \theta\bar{Y}(C_x^2 - \rho_{yx}C_yC_x) \quad (1.2)$$

$$MSE(\bar{y}_R) = \theta\bar{Y}^2\left[C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right] \quad (1.3)$$

Where $\theta = \left(\frac{1-f}{n}\right), C_y^2 = \frac{S_y^2}{\bar{Y}^2}, C_x^2 = \frac{S_x^2}{\bar{X}^2},$

$C_{yx} = \frac{Syx}{\bar{X}\bar{Y}} = \rho_{yx}C_yC_x$, here C_y and C_x are

Where.,,,here C_y and C_x are coefficient of variations of y and x respectively and ρ is correlation coefficient.

When the population coefficient of variation of auxiliary variate C_x is known, Sisodia and Dwivedi (1981), given a modified ratio estimator for $\bar{Y}$ as

$$\bar{y}_1 = \bar{y}\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right] \quad (1.4)$$

The bias & mean square error (MSE) of $\bar{y}_1$ are given by

$$B(\bar{y}_1) = \theta\bar{Y}(\alpha^2C_x^2 - \alpha\rho_{yx}C_yC_x) \quad (1.5)$$

$$MSE(\bar{y}_1) = \theta\bar{Y}^2\left[C_y^2 + \alpha^2C_x^2 - 2\alpha\rho_{yx}C_yC_x\right] \quad (1.6)$$

where $\alpha = \frac{\bar{X}}{\bar{X}+C_x}$ and C_y is the population coefficient of variation of variable y

Motivated by Sisodia and Dwivedi (1981), Singh and Kakran (1993) developed ratio-type estimator for $\bar{Y}$ as

$$\bar{y}_2 = \bar{y}\left[\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right] \quad (1.7)$$

where $\beta_2(x)$ is known value of the coefficient of kurtosis of an auxiliary variable.

The Bias and MSE of this estimator were given by

$$B(\bar{y}_2) = \theta\bar{Y}(\lambda^2C_x^2 - \lambda\rho_{yx}C_yC_x) \quad (1.8)$$

$$MSE(\bar{y}_2) = \theta\bar{Y}^2\left[C_y^2 + \lambda^2C_x^2 - 2\lambda\rho_{yx}C_yC_x)\right] \quad (1.9)$$

where $\lambda = \frac{\bar{X}}{\bar{X}+\beta_2(x)}$.

Upadhyaya and Singh (1999) considered both coefficients of variation and Kurtosis in their ratio-type estimator as

$$\bar{y}_3 = \bar{y}\left[\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}C_x+\beta_2(x)}\right] \quad (1.10)$$

The Bias and MSE of this estimator were given by

$$B(\bar{y}_3) = \theta\bar{Y}(\gamma_1^2C_x^2 - \gamma_1\rho_{yx}C_yC_x) \quad (1.11)$$

$$MSE(\bar{y}_3) = \theta\bar{Y}^2\left[C_y^2 + \gamma_1^2C_x^2 - 2\gamma_1\rho_{yx}C_yC_x)\right] \quad (1.12)$$

where $\gamma_1 = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$.

Upadhyaya and Singh (1999) proposed another estimate by changing the place of coefficient of kurtosis and coefficient of variation as

$$\bar{y}_4 = \bar{y}\left[\frac{\bar{X}\beta_2(x)+C_x}{\bar{x}\beta_2(x)+C_x}\right] \quad (1.13)$$

The Bias and MSE of this estimator were given by

$$B(\bar{y}_4) = \theta\bar{Y}(\gamma_2^{\,2}C_x^2 - \gamma_2\rho_{yx}C_yC_x) \quad (1.14)$$

$$MSE(\bar{y}_4) = \theta\bar{Y}^2\left[C_y^2 + \gamma_2^{\,2}C_x^2 - 2\gamma_2\rho_{yx}C_yC_x\right] \quad (1.15)$$

where $\gamma_2 = \dfrac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x) + C_x}$.

2. The Suggested Estimator

The proposed modified ratio estimator using the linear combination of Median and Co-efficient of Kurtosis of the auxiliary variable is given as

$$\bar{y}_5 = \bar{y}\left[\frac{\bar{X}\beta_2(x) + Md}{\bar{x}\beta_2(x) + Md}\right] \quad (2.1)$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y$, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x$, Md = Median of auxiliary variable and $\beta_2(x)$ = Co-efficient of Kurtosis of the auxiliary variable.

To obtain bias and MSE of $\bar{y}_5$, we put $\bar{y} = \bar{Y}(1+\varepsilon_0)$ and $\bar{x} = \bar{X}(1+\varepsilon_1)$ so that $E(\varepsilon_0) = E(\varepsilon_1) = 0$ and $E(\varepsilon_0^2) = \frac{(1-f)}{n}C_y^2$, $E(\varepsilon_1^2) = \frac{(1-f)}{n}C_x^2$ and $E(\varepsilon_0\varepsilon_1) = \frac{(1-f)}{n}\rho_{yx}C_yC_x$.

Here $\theta = \left(\frac{1-f}{n}\right)$, $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$, $C_x^2 = \frac{S_x^2}{\bar{X}^2}$, $C_{yx} = \frac{Syx}{\bar{X}\bar{Y}} = \rho_{yx}C_yC_x$, C_y and C_x are coefficient of variations of y and x respectively and ρ is correlation coefficient. For the population observation we have use the following quantities

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y, \qquad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X$$

$$S_y^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(Y_i - \bar{Y})^2,$$

$$S_x^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(X_i - \bar{X})^2,$$

$$S_{yx} = \frac{1}{(N-1)}\sum_{i=1}^{N}(Y_i - \bar{Y})(X_i - \bar{X}).$$

The Bias and MSE of $\bar{y}_5$ can be found as follows-

$$B(\bar{y}_5) = E(\bar{y}_5) - \bar{Y}$$

Here $\bar{y}_5 = \bar{Y}(1+\varepsilon_0)(1+\delta\varepsilon_1)^{-1}$

where $\delta = \dfrac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x) + Md}$.

Suppose $|\delta\varepsilon_1| < 1$ so that $(1+\delta\varepsilon_1)^{-1}$ is expandable.

$$\bar{y}_5 = \bar{Y}(1+\varepsilon_0)\{1 - \delta\varepsilon_1 + \delta^2\varepsilon_1^2 + O(\delta\varepsilon_1)\}$$

(Using Taylor series expansion, where $O(\varepsilon_1)$ with power more than 2 are neglected for large power of ε_1.)

$$B(\bar{y}_5) = \bar{Y}\left[\delta^2E(\varepsilon_1^2) - \delta E(\varepsilon_0\varepsilon_1)\right],$$

because $E(\varepsilon_0) = E(\varepsilon_1) = 0$

$$\Rightarrow B(\bar{y}_5) = \theta\bar{Y}\left[\delta^2C_x^2 - \delta\rho_{yx}C_yC_x\right] \quad (2.2)$$

The MSE of the estimator $\bar{y}_5$ to the first degree of approximation, is given by

$$MSE(\bar{y}_5) = E(\bar{y}_5 - \bar{Y})^2$$

$$= \bar{Y}^2E\left[\varepsilon_0 - \delta\varepsilon_1 + \delta^2\varepsilon_1^2 - 2\delta\varepsilon_0\varepsilon_1\right]^2$$

$$= \bar{Y}^2E\left[\varepsilon_0^2 + \delta^2\varepsilon_1^2 - 2\delta\varepsilon_0\varepsilon_1\right]$$

$$MSE(\bar{y}_5) = \theta\bar{Y}^2\left[C_y^2 + \delta^2C_x^2 - 2\delta\rho_{yx}C_yC_x\right] \quad (2.3)$$

where $\delta = \dfrac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x) + Md}$.

When we compare equation (2.3) with equations (1.3) , (1.6), (1.9), (1.12) ,(1.15) respectively and we observe that if MSE of the proposed ratio estimator given in (2.3) is smaller than traditional ratio estimator and all the existing modified ratio estimators then we can say that the proposed modified ratio estimator is more efficient than the others. In order to verify this, agricultural data set from censes 2001 was used to show that the proposed ratio estimator given in (2.1) is more efficient than the other ratio estimators given in (1.1), (1.4), (1.7), (1.10) and (1.13).

Empirical Study

To observe the relative performance of different estimators of the population mean $\bar{Y}$ of the study variable , a population data sets from censes 2001 is being considered.

Population (Source: Directorate of Census Operations, Rajasthan)

y = Working population of all districts in Rajasthan State (in thousands).

x = Total population of all districts in Rajasthan State (in thousands).

The required population parameters are

$\bar{Y} = 742.71$, $\bar{X} = 1765$, $N = 32$, $n = 17$, $\rho_{yx} = 0.97$, $\beta_2(x) = 6.61$, $C_x = 0.5$, $C_y = 0.45$, $Md = 1621.40$, $\alpha = 0.9997$,

$\lambda = 0.9962$, $\gamma_1 = 0.9925$, $\gamma_2 = 0.9999$ and $\delta = 0.878$.

Table : MSE values of ratio estimators

Estimators	MSE values
Traditional($\bar{y}_R$)	202.99
Sisodia-Dwivedi($\bar{y}_1$)	201.73
Singh-Kakran($\bar{y}_2$)	200.46
Upadhyaya-Singh($\bar{y}_3$)	196.65
Upadhyaya-Singh($\bar{y}_4$)	202.99
Proposed($\bar{y}_5$)	152.25

From the results of empirical study, we conclude that all the proposed estimators are more efficient than the traditional ratio estimators and proposed estimator given in (2.1) is the most efficient estimator for this data set. Thus, if Coefficient of variation, Coefficient of kurtosis and Median are known of auxiliary variable x , these proposed estimators are recommended for use in practice.

REFERENCES

1. Cochran, W.G. (1940). Some properties of estimators based on sampling scheme with varying probabilities. Austral. J. Statist., 17, 22-28. | 2. Singh, H.P. and Kakran M.S. (1993). A Modified Ratio Estimator Using known Coefficient of Kurtosis of an Auxiliary Character, unpublished. | 3. Sisodia, B.V.S. and Dwivedi, V.K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable. Journal of Indian Society of Agricultural Statistics, 33,13-18. | 4. Upadhyaya, L.N. and Singh, H.P. (1999). Use of transformed auxiliary variable in estimating the finite population mean. Biometrical journal, 41, 627-636. | 5. Murthy, M.N. (1967). Sampling Theory and Methods, Statistical Publishing Society, Calcultta. | 6. Robson, D.S. (1957). Application of multivariate polykays to the theory of unbiased ratio-type estimation. Journal of the American Statistical Association, 52, 511-522. |