Volume : 2 | Issue : 6 | June 2013 • ISSN No 2277 - 8160

Research Paper
Mathematics

R*- Closed Maps and R*- Open Maps in Topological Spaces

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ABSTRACT *This paper introduces the idea of R*-interior and R*-closure in addition to the concept of R*-open and R*-closed maps. Certain properties of R*-open and R*-closed are also discussed.*

1. SUMMARY

The generalization of closed sets was introduced by N.Levine[7]. This contribution led to many new concepts and results in general topology .Researchers like Arya et al [2] Balachandran et al [3] Arokiarani et al [1] Palaniappan et al [10] have worked on different types of generalized closed sets .Closed mapping in topology was introduced by Malghan [9].Similar studies on wg-closed maps and rwg-closed maps were studied by Nagaveni [8,11] M.Karpagadevi [5], A.Vadivel [12,13],Antony Rex Rodrigo [6]defined RW-closed maps ,rga-closed maps and R-closed maps respectively. In this paper, the notion of R-interior and R*-closure have been defined and their basic properties are studied. For any subset A of X, the complement of R*-interior of A is R*-closure.In addition , this paper includes a new class of closed maps called R*-closed maps. Its relation with similar closed maps is discussed and some properties related to this map are obtained.*

Throughout the paper, X and Y denote the topological spaces (X,) and (Y,) respectively on which no separation axioms are assumed.

KEYWORDS: R*-int(A),R*-cl(A),R*-closed maps,R*-open maps,R*-T1/2

2. Preliminaries

Definition 2.1. A subset A of a topological space (X, τ) is called (1) a regular open[4] if A= int (cl(A)) and regular closed [4] if A = cl (int(A)) The intersection of all regular closed subset of (X, τ) containing A is called the regular closure of A and is denoted by rcl(A).

Definition 2.2. [4] A subset A of a space (X, τ) is called regular semiopen set U such that U $\subset$ A $\subset$ cl(U). The family of all regular semiopen sets of X is denoted by RSO(X).

Lemma 2.3. [4] In a space (X, τ) , the regular closed sets, regular open sets and clopen sets are regular semi open.

Definition 2.4. [4] A subset A of a space (X, τ) is said to be semi regular open if it is both semi open and semi closed.

Definition 2.5. A subset of a topological space (X, τ) is

(1) a regular generalized (briefly rg-closed) [10] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X.
(2) a regular w-closed (briefly rw-closed)[9] if cl(A) $\subset$ U whenever A $\subset$ U and U is regular semiopen in X.
(3) a regular generalized-star closed (briefly rg*-closed)[10] if rcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X.
(4) a R*-closed set [4] if rcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular semi open in X.

The complements of the above mention closed sets are their respectively open sets.

Definition 2.6 A map f: (X,τ) $\longrightarrow$ (Y,σ) is called

(i).R*- continuous [4] if f^{-1} (V) is an R*- closed set in (X,τ) for every closed set V of (Y,σ). (ii).R*- irresolute [4] if f^{-1} (V) is an R*- closed set in (X,τ) for every R*- closed set V of Y,σ). (iii).Strongly R* - closed if f^{-1} (V) is a closed set in (X,τ) for every R*- closed set V of (Y,σ). (iv)Strongly R*-continuous if f^{-1} (V) is an open set in (X,τ) for every R*- open set V of (Y,σ).

Definition 2.7 A topological space X is said to be R*-$T_{1/2}$ space if every R*-closed set is regular closed.

3. R*-interior and R*-closure

We introduce the following definition.

Definition 3.1 Let X be a topological space and let x $\in$ X. A subset N of X is said to be R*-nbhd of x if there exist an R*- open set G such that x $\in$ G $\subset$ N.

Definition 3.2. Let A be a subset of X. A point x $\in$ A is said to be R*-Interior of A if A is a R*-nbhd of x. The set of all R*-Interior points of A is called R*-Interior of A and is denoted by R*-int(A).

Theorem 3.3. If A is a subset of X, then R*-int(A) = $\bigcup$ {G: G is R*-open, G $\subset$ A}. **Proof:**Let A be a subset of X. x $\in$ R*-int(A) $\Longleftrightarrow$ x is a R*-Interior point of A. $\Longleftrightarrow$ A is an R*-nbhd of the point x. $\Longleftrightarrow$ there is a R*-open set G such that x $\in$ G $\subset$ A $\Longleftrightarrow$ x $\in$ $\bigcup$ {G: G is R*-open, G $\subset$ A}.

Theorem 3.4. Let A and B be subsets of X, then (i). R*-int (X) =X and R*-int (ϕ) = ϕ . (ii). R*-int (A) $\subset$ A. (iii). If B is a R*-open set contained in A, then B $\subset$ R*-int (A). (iv). If A $\subset$ B, then R*- int (A) $\subset$ R*- int (B).

Proof:

(i). X and ϕ are open sets and so by Theorem3.3,R*-int(X) = $\bigcup$ {G: G is R*-open, G $\subset$ X} = X $\bigcup$ {all R*-open sets } =X. That is R*-int (X) = X. Since ϕ is the only R*-open set contained in ϕ , R*-int(ϕ)= ϕ .
(ii). Let x $\in$ R*-int (A) $\Longrightarrow$ x is a R*-interior point of A. $\Longrightarrow$ A is a R*-nbhd of x. $\Longrightarrow$ x $\in$ A Thus x $\in$ R*-int (A) $\Longrightarrow$ x $\in$ A. Hence R*-int (A) $\subset$ A.
(iii). Let B be any R*-open set such that B $\subset$ A. Let x $\in$ B, then since B is a R*-open set contained in A, x is a R*-interior point of A. That is x $\in$ R*-int (A). Hence B $\subset$ R*-int (A).
(iv). Let A and B be subsets of X such that A $\subset$ B. let x $\in$ R*-int (A). Then x is a R*-interior point of A and so A is a R*-nbhd of x. Since A $\subset$ B, B is also a R*-nbhd of x. This implies that x $\in$ R*-int (B). Thus x $\in$ R*-int (A) $\Longrightarrow$ x $\in$ R*-int (B).Hence R*-int (A) $\subset$ R*-int (B).

Theorem 3.5. If a subset A of a space X is R*-open, then R*-int (A)=A.

Proof . Let A be R*-open subset of X. We know that R*-int (A) $\subset$ A. Also, A is R*-open set contained in A. From Theorem 3.4 (iii) A $\subset$ R*-int(A). Hence R*-int (A) = A.

Remark 3.6 The converse of the above theorem need not be true as seen in the following example.

Example 3.7 Let X = {a, b, c, d, e} with topology τ = { X, ϕ ,{a},{d},{e},{a, d},{a, e},{d, e},{a, d, e}}. Then R*-O(X) = { X, ϕ

,{a},{b},{c},{d},{e},{b, c},{a, d},{a, e},{d, e},{a, d, e}}. R*-int({a,b}) = {a} $\cup$ {b} $\cup$ ϕ = {a, b}. But {a ,b} is not a R*-open set in X.

Theorem 3.8. If A and B are subsets of X then

(i) R*-int (A) $\cup$ R*-int (B) $\subset$ R*-int(A $\cup$ B).
(ii) R*-int(A $\cap$ B)= R*-int (A) $\cap$ R*-int(B)

Proof.
(i). We know that A $\subset$ A $\cup$ B and B $\subset$ A $\cup$ B. By Theorem 3.4 (iv), R*-int (A) $\subset$ R*-int (A $\cup$ B) and R*-int(B) $\subset$ R*-int(A $\cup$ B). This implies that R*-int (A) $\cup$ R*-int(B) $\subset$ R*-int(A $\cup$ B).

(ii) We know that A $\cap$ B $\subset$ A and A $\cap$ B $\subset$ B.By Theorem 3.4

(iv) R*-int(A $\cap$ B) $\subset$ R*-int(A) and R*-int(A $\cap$ B) $\subset$ R*-int(B).This implies R*-int(A $\cap$ B) $\subset$ R*-int (A) $\cap$ R*-int(B)…..(a). Again, let x $\in$ R*-int (A) $\cap$ R*-int(B) .Then x $\in$ R*-int (A) and x $\in$ R*-int (B). There exists R*-open sets U andV such that x $\in$ U $\subset$ A and x $\in$ V $\subset$ B.By theorem 3.32 of [4],U $\cap$ V is an R*-open set such that x $\in$ (U $\cap$ V) $\subset$ (A $\cap$ B).Hence x $\in$ R*-int(A $\cap$ B). x $\in$ R*-int (A) $\cap$ R*-int(B) implies x $\in$ R*-int(A $\cap$ B). Therefore R*-int (A) $\cap$ R*-int(B) $\subset$ R*-int(A $\cap$ B) …..(b).From (a) and (b),it follows that R*-int(A $\cap$ B)= R*-int (A) $\cap$ R*-int(B) .

Definition 3.9 Let A be a subset of a space X. We define the R*-closure of A to be the intersection of all R*- closed sets containing A. In symbols R*-cl(A) = $\cap$ {F: A $\subset$ F $\in$ R*-C(X)}.

Theorem 3.10 If A and B are subsets of a space X (i). R*-cl (X) =X and R*-cl(ϕ) = ϕ . (ii). A $\subset$ R*-cl(A) . (iii). If B is R*-closed set containing A, then R*-cl(A) $\subset$ B. (iv). If A $\subset$ B, then R*-cl(A) $\subset$ R*-cl(B). (v).R*-cl(A)=R*cl [R*-cl(A)]

Proof.
(i). By definition of R*-closure, X is the only R*-closed set containing X. Therefore R*-cl(X) = intersection of all R*-closed sets containing X = $\cap$ {X}=X. That is R*-cl(X)=X. Also by definition of R*-closure, R*-cl(ϕ) = intersection of all R*-closed sets containing ϕ = ϕ $\cap$ any R*-closed set containing ϕ = ϕ . That is R*-cl(ϕ)= ϕ .
(ii). By definition of R*-cl(A), it is obvious that A $\subset$ R*-cl(A) .
(iii). Let B be any R*-closed set containing A. Since R*-cl(A) is the intersection of all R*-closed sets containing A, R*-cl(A) is contained in every R*-closed set containing A. In particular R*-cl(A) is contained in B by hypothesis. Hence R*-cl(A) $\subset$ B.
(iv). Let A and B be subsets of X such that A $\subset$ B. By definition of R*-closure, R*-cl(B) = $\cap$ {F: B $\subset$ F $\in$ R*-C(X)}. If B $\subset$ F $\in$ R*-C(X), then by (iii) R*-cl(B) $\subset$ F. Since A $\subset$ B, A $\subset$ B $\subset$ F $\in$ R*-C(X), we have by (iii) R*-cl(A) $\subset$ F. Therefore R*-cl(A) $\subset$ $\cap$ {F: B $\subset$ F $\in$ R*-C(X)} = R*-cl(B). That is R*-cl(A) $\subset$ R*-cl(B).
(v). Let A be any subset of X. By definition of R*-closure, R*-cl(A) = $\cap$ {F: A $\subset$ F $\in$ R*-C(X)}. If A $\subset$ F $\in$ R*-C(X), then R*-cl(A) $\subset$ F. Since F is R*-closed set containing R*-cl(A), by (iii) R*-cl(R*-cl(A)) $\subset$ F. Hence R*-cl(R*-cl(A)) $\subset$ $\cap$ {F: A $\subset$ F $\in$ R*-C(X)} = R*-cl(A). That is R*-cl(A)= R*cl [R*-cl(A)].

Theorem 3.11. If A $\subset$ X is an R*-closed set, then R*-cl(A) =A.

Proof. Let A be R*-closed subset of X. We know that A $\subset$ R*-cl(A) . Also A $\subset$ A and A is R*-closed. By Theorem 3.10 (iii), R*-cl(A) $\subset$ A. Hence R*-cl(A)=A.

The converse of the above theorem need not be true from the following example.

Example 3.12 . Let X ={a,b,c,d,e}with topology

τ ={X, ϕ ,{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e}}

R*-C(X) = {X, ϕ , {c},{b,c},{a,b,c},{b,c,d},{b,c,e},{a,d,e},{a,b,c,d},{a,b,d,e},{a,c,d,e},{b,c,d,e}}.

Now R*-cl({a}) ={a} but {a} is not R*-closed subset of X.

Theorem 3.13. If A and B are subsets of a space X, then R*-cl (A $\cap$ B) $\subset$ R*-cl(A) $\cap$ R*-cl(B). **Proof.** Let A and B be subsets of X. Clearly A $\cap$ B $\subset$ A and A $\cap$ B $\subset$ B. By Theorem 3.10 (iv), R*-cl(A $\cap$ B) $\subset$ R*-cl(A) and R*-cl (A $\cap$ B) $\subset$ R*-cl(B).Then R*-cl(A $\cap$ B) $\subset$ R*-cl(A) $\cap$ R*-cl(B).

Remark 3.14 The reverse implication of the above theorem is not true.

Example 3.15 In example 3.12 ,let A ={a,b} and B ={c,d,e} then R*-cl(A) $\cap$ R*-cl(B) ={c} and R*-cl(A $\cap$ B) = ϕ
R*-cl(A) $\cap$
R*-cl(B) $\not\subset$ R*-cl(A $\cap$ B).

4 . R*-closed maps and R*-open maps in topological spaces

We introduce the following definitions

Definition 4.1 A map f: (X,τ) →(Y,σ) is said to be R*- closed map if the image of every closed set in (X,τ) is R*-closed in (Y,σ).

Definition 4.2 A map f: (X,τ) →(Y,σ) is said to be R*- open map if the image f(A) of every open set in (X,τ) is R*-open in (Y,σ).

Proposition 4.3 For every bijective function, f: (X,τ) →(Y,σ) the following statements are equivalent. (i) f^{-1} : (Y,σ) → (X,τ) is R*-continuous. (ii) f is an R*-open map. (iii) f is an R*-closed map **Proof** (i) $\Longrightarrow$ (ii) Let U be an open set in (X,τ) By hypothesis $(f^{-1})^{-1}$ (U) is R*-open in (Y,σ) by definition 4.2 . Thus f(U) in R*-open in (Y,σ). Hence f is R*-open.

(ii) $\Longrightarrow$ (iii) Let F be a closed set of (X,τ). Then F^c is open in (X,τ) . By hypothesis f (F^c) is R*-open. Thus f(F) is R*-closed . Thus f is R*-closed.

(iii) $\Longrightarrow$ (i) Let F be a closed set in (X,τ).Then f(F) is R*-closed in (Y,σ). That is $(f^{-1})^{-1}$ (F) is R*-closed in (Y,σ).Thus (f^{-1}) is R*- continuous.

Proposition 4.4: If a mapping f: (X,τ) →(Y,σ) is R*-closed, then R*-cl (f(A)) $\subseteq$ f(rcl(A)) for every subset A of (X,τ). **Proof:** Suppose that f is R*-closed and A $\subseteq$ X . Note that rcl(A) is closed in X and f(rcl(A)) is R*-closed in (Y,σ).We know that, A $\subseteq$ rcl(A) .Thus f (A) $\subseteq$ f(rcl(A)). By theorem 3.10 (iv) R*- cl (f(A)) $\subseteq$ R*- cl (f(rcl(A)))--------(i). Also by theorem 3.11, since f(rcl(A)) is an R*-closed set R*- cl (f(rcl(A))) =(f(rcl(A)))-------(ii). From (i) and (ii), R*- cl ((f(A))) $\subseteq$ f(rcl(A)) for every subset A of (X,τ).

Remark 4.5:

The converse of the proposition 4.4 is not true in general as seen from the following example.

Example 4.6 Let X= Y= { a,b,c}, τ = {X, ϕ , {a}, {b,c}}, σ = { X, ϕ ,{a},{b},{a,b}} Define f: (X,τ) →(Y,σ) by f(x) = x for every x $\in$ X. Then R*-cl (f(A)) $\subseteq$ f (rcl(A)) for every subset of (X,τ). But f is not R*-closed, since f{a} ={a} which is not R*-closed in (Y,σ).

Definition 4.7: Let τ_{R^*} be the topology on X generated by R*-closure in the usual manner. That is τ_{R^*} = { A $\subset$ X : R*cl (A^c)= A^c }

Collorary 4.8: If a mapping f: (X,τ) →(Y,σ) is R*-closed, then the image f(A) of a regular closed set A in (X,τ) is τ_{R^*} – closed in (Y,σ).

Proof: Let A be a closed set in (X,τ). Since f is R*-closed , by the above proposition 4.4

R*-cl (f(A)) $\subseteq$ f(rcl(A))-----------(i)
Also rcl(A) =A as A is regular closed set and so f(rcl(A))=f(A)-----------(ii). From (i) and (ii) we have R*-cl(A) $\subseteq$ f(A). We know that($f(A))^c$ $\subset$ R*-cl $(f(A))^c$ and so R*-cl $((f(A))^c$ = $(f(A))^c$. Therefore f(A) is τ_{R^*} -closed in (Y,σ).

Theorem 4.9: A mapping f:(X,τ) →(Y,σ) is R*-closed if and only if for each subset S of (Y,σ) and each open set U containing f^{-1} (S), there is an R*-open set V of (Y,σ) such that S $\subset$ V and f^{-1} (V) $\subset$ U.

Proof: Suppose f is R*- closed . Let S $\subset$ Y and U be open set of (X, τ) such that f^{-1} (S) $\subset$ U . Now X-U is a closed set of (X, τ) such that f^{-1} (S) $\subset$ U.Since f is R*-closed, f (X-U) is R*-closed in (Y ,σ).Then V= Y-f (X-U) is an R*-open set in (Y, σ).Note that f^{-1} (S) $\subset$ U implies S $\subset$ V and f^{-1} (V) = X- f^{-1} (f (X-U)) $\subset$ X- (X- U) = U. That is f^{-1} (V) $\subset$ U. Conversely, let

F be a closed set of (X, τ).Then $f^{-1}(f(F^c)) \subset F^c$ and F^c is open in (X,τ). By hypothesis, there exist an R*-open set V in (Y, σ) such that $f(F^c) \subset V$ and $f^{-1}(V) \subset F^c$ and so $F \subset (f^{-1}(V))^c$. Hence $V^c \subset f(F) \subset f(((f^{-1}(V))^c) \subset V^c$, which implies $f(F) = V^c$ .Since V^c is R*-closed ,f (F) is R*-closed in (Y,σ).That is f is R*-closed.

Theorem 4.10: Closed maps and R*-closed maps are independent.

Example 4.11: Let X=Y={a ,b, c, d} τ = {X,{a},{a,b},{a,b,c}} σ ={Y,{a},{b},{a,b}{a,b,c}, ϕ ,} R*-C(Y)={Y, φ ,{a, b},{c, d},{a, b, c},{a ,b, d},{b, c, d},{a, c, d}}.Define a function f: (X, τ) →(Y ,σ) as the identity mapping .f is a closed map but not R*-closed map since f({d})={d} which is not R*-closed.

Example 4.12 Let X=Y={a, b, c,}
τ ={ X, φ , {a},{b, c}}
σ={ X, φ , {a}}

R*-C(Y)={Y, φ ,{a},{b},{c},{a, b},{b, c},{a, c}}.Define a function f: (X, τ) → (Y, σ) as the identity mapping . f is an R*- closed map but not a closed map since, f ({a}) = {a} which is not closed in (Y,σ).

Remark 4.13 The composition of two R*-closed maps need not be R*-closed map in general as shown by the following example.

Example.4.14
Consider X = Y = Z ={ a, b, c},
τ = { X, φ , {a},{b},{a, b}}
σ = { X, φ ,{a},{b, c}}
η={ X, φ ,{b},{a, c}}

Define f: (X, τ) → (Y, σ) and g : (Y, σ) → (Z ,η) as f(a) = c, f(b) = a, f(c) = b and g to be the identity mapping . Here f and g are R*-closed maps but $g \circ f$ is not an R*-closed map. Since $g \circ f$ ({c}) = g(f(c)) = g {b} = {b} which is not R*-closed in (Z,η).

Theorem 4.15 If f : (X, τ)→(Y, σ) is a closed map and g :(Y, σ) → (Z, η) is an R*-closed map , then the composition $g \circ f$:(X ,τ)→(Z, η) is R*-closed in (Z,η).

Proof: Let F be any closed set in (X, τ). Since f is a closed map f (F) is closed in (Y ,σ). Since g is R*-closed map g (f(F)) is a R*-closed set in (Z ,η). That is $g \circ f$ (F) = g (f (F)) is R*-closed and hence $g \circ f$ is R*-closed map.

Remark 4.16 If f : (X, τ)→(Y, σ) is an R*- closed map and g :(Y, σ) → (Z, η) is a closed map , then the composition $g \circ f$:(X ,τ)→(Z, η) need not be an R*-closed map in (Z ,η) as seen from the following example.

Example 4.17. Let X=Y=Z={a,b,c,d}τ = { X, φ , {a},{c,d},{a,c,d }} , σ = { Y, φ , {a},{b},{a, b},{a,b,c},{a,b,d}}, η ={ X, φ ,{b},{d},{b,d},{a, b,d},{b,c,d}}, R*-C(Y) = { Y, φ , {d},{a,b},{a, d},{b,d},{c,d},{a,b,c},{a,b.d},{a,c,d},{b,c,d}} . R*-C(Z) ={ Z, φ , {a,c},{b,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}} Define f: (X, τ) → (Y, σ) and g : (Y, σ) → (Z ,η) as follows f(a) = b,f(b) = d,f(c) = a,f(d) = c ; g (a) = b,g (b) = d, g (c) = a, g(d) = c.Here f is a R*-closed map and g is a closed map but $g \circ f$: (X, τ)→ (Z, η) is not an R*- closed map since $g \circ f$ ({b}) = g(f({b}) =g({d}) ={c} which is not R*-closed in (Z,η).

Theorem 4.18 Let (X ,τ) and(Z, η) be topological spaces and (Y, σ) be a topological space where every " R*-closed set is closed" .Then the composition $g \circ f$: (X, τ) → (Z, η) of the two R*-closed maps f: (X, τ) → (Y, σ) and g:(Y, σ) → (Z, η) , is R*-closed .

Proof: Let A be closed set of (X, τ) since f is R*-closed , f(A) is R*-closed in (Y ,σ). Then by hypothesis f(A) is closed. Since g is R*- closed , g(f(A)) is R*-closed in (Z, η) and g(f(A)) = $g \circ f$ (A). Therefore $g \circ f$ is R*-closed .

Theorem 4.19. If f : (X ,τ) → (Y, σ) and g : (Y, σ) → (Z, η) be two mappings such that their composition $g \circ f$: (X ,τ)→(Z ,η) be an R*-closed mapping .Then the following statements are true

(1) If f is continuous and surjective, then g is R*-closed.
(2) If g is R*-irresolute and injective, then f is R*-closed.
(3) If f is R*-continuous, surjective and (X,τ) is R*-$T_{1/2}$ space ,then g is R*-closed.
(4) If g is strongly R*-continuous and injective, then f is closed.

Proof:
(1) Let A be a closed set in (Y,σ).Since f is continuous ,$f^{-1}(A)$ is closed in (X,τ) and since $g \circ f$ is R*-closed $g \circ f(f^{-1}(A))$ is R*-closed in (Z,η). That is g(A) is R*-closed in (Z, η),since f is surjective. Therefore g is R*-closed.
(2) Let B be a closed set in (X, τ).Since g is irresolute,$g^{-1}(g \circ f(B))$ is R*-closed set in (Y,σ).That is f(B) is R*-closed in (Y,σ) since f is injective. Therefore f is R*-closed . (3)Let C be a closed set in (Y,σ). Since f is R*- continuous , $f^{-1}(C)$ is an R*-closed set in (X,τ).Since (X,τ) is R*-$T_{1/2}$ space , $f^{-1}(C)$ is regular closed in (X,τ). By hypothesis $g \circ f$ is R*-closed and hence $g \circ f(f^{-1}(C))$ is R*-closed in (Z,η). That is g(C) is R*-closed in (Z,η), since f is surjective . Therefore g is R*-closed . (4) .Let D be a closed set in (X,τ) . since $g \circ f$ *(D)* is R*-closed in (Z,η). Since g is strongly R*-continuous and injective $g^{-1}(g \circ f)(D)$ is closed in (Y,σ).That is f(D) is a closed set in (Y,σ) . Therefore f is closed .

Theorem 4.20. If a map f : (X ,τ) → (Y, σ) is R*-open , then f (rint A) ⊂ R*-int(f(A)). **Proof:**Let f : (X ,τ) → (Y, σ) be an open map and A be any subset of (X,τ). Then rint(A) is open in (X,τ) and so f(rint(A)) is an R*-open set in (Y,σ) . We know that f (rint A) ⊂ f(A) .Therefore by theorem 3.5 , f (rint A) ⊂ R*-int(f(A)).

Remark 4.21. The converse of the above theorem need not be true in general as seen in the following example.

Example 4.22. Let X =Y = {a,b,c,d } τ = {X, ϕ ,{a},{a,d,},{a,b,d},{a,c,d}} σ = {Y, ϕ ,{a},{a,b,},{a,b,c}}Define f : (X ,τ) → (Y, σ) as follows f(a) = b,f(b) = c,f(c) = d, f(d) = a ; Here f (rint A) ⊂ R*-int(f(A)) but f is not R*- open since f ({b,c,d}) ={a,c,d} which is not R*-open.

Theorem 4.23 If a map f : (X ,τ) → (Y, σ) is R*-open then for each nbhd U of x in (X,τ) there exist R*-nbhd W of f(x) in (Y,σ) as W ⊂ f(U). **Proof** Let f : (X ,τ) → (Y, σ) be an R*-open map , let x ∈ X and U be an arbitrary nbhd of x in (X,τ). Then there exist an open set G of (X,τ) such that x ∈ G ⊂ U. Now f(x) ∈ f(G) ⊂ f(U) and f(G) is R*-open in (Y,σ), as f is R*-open map. By theorem 3.3, f(G) is R*-neighbourhood of each of its points. Taking f (G)= W,W is a R*-nbhd of f(x) in (Y ,σ) , such that W ⊂ f(U).

Theorem 4.24. A map f : (X ,τ) → (Y, σ) is R*-open if and only if for any subset S of (Y,σ) and any closed set F containing $f^{-1}(S)$ there exist an R*-closed set K of (Y,σ) containing S , such that $f^{-1}(K) \subset F$.

Proof: Suppose f is R*-open map, let S ⊂ Y and F be a closed set of (X,τ) such that $f^{-1}(S) \subset F$. Now X-F is open set in (X,τ).Since f is R*-open map, f(X-F) is R*-open set in (Y,σ). Then K = Y-f(X-F) is an R*-closed set in(Y,σ) . Note that $f^{-1}(S) \subset F$ implies S ⊂ K and $f^{-1}(K)= X-f^{-1}(f(X-F))=$ X-(X-F)=F. That is $f^{-1}(K) \subset F$.

For the converse , let U be open set in (X,τ).Then $f^{-1}(f(U^c)) \subset U^c$ and U^c is closed set in (X,τ).By hypothesis there exist an R*-closed set K of (Y,σ) such that $f(U^c) \subset K$ and $f^{-1}(K) \subset U^c$ and so $U \subset (f^{-1}(K))^c$. Hence $K^c \subset f(U) \subset f((f^{-1}(K))^c = K^c$, which implies $f(U) \subset K^c$. Since K^c is R*-open , f(U) is R*-open in (Y,σ) and therefore f is an R*-open map.

Theorem 4.25. Every R*-open map is rg-open but not conversely.

Proof : Follows from the definition and the fact that every R*-closed is an rg-closed set .

Remark 4.26. The converse of the above theorem need not be true as seen in the following example.

Example 4.27 Consider X=Y= {a, b, c, d} τ = {X, ϕ ,{b},{a, b},{b ,c ,d}} σ ={Y, ϕ , {d},{c, d},{b ,c ,d},{a, c, d}}. Let f: X→Y be defined by f(a) = d, f(b) = c, f(c) = a, f(d) = b. This function is rg-closed but not R*-closed as the image of f({a})={d} is not R*-closed. .

Theorem 4.28 Every R*-closed map is rw-closed but not conversely. **Proof:** Follows from the definition and the fact that every R*-closed set is rw-closed .

Example 4.29 In example 4.26 f({a})={d} is rw-closed but not R*-closed .

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