Volume : 2 | Issue : 11 | Nov 2013 • ISSN No 2277 - 8160

Research Paper Engineering

Reliability Based Design Optimization of Laminated Composites - A New Approach

P R Krishna Mohan	Aeronautical Engineering /Mechanical Engineering Division, Institute of Aeronautical Engineering, Dundigal Village, Quthbillahpur Mandal, Hyderabad, 500043
N Ananth	Aeronautical Engineering /Mechanical Engineering Division, Institute of Aeronautical Engineering, Dundigal Village, Quthbillahpur Mandal, Hyderabad, 500043

ABSTRACT

The main objective of this work consists of defining a new approach to minimize the Objective function formulated for analyzing composite failure criteria using Reliability based design optimization and also aims at decreasing computational cost. Our approach will mainly focus upon selection of optimum ply thickness of the laminated composites by Monte Carlo Simulation. Numerical simulation and Failure Criteria analysis are exemplified by means of the MATLAB software

KEYWORDS : Laminated Composites, Monte Carlo Simulation, Reliability based design optimization, MATLAB

1. INTRODUCTION

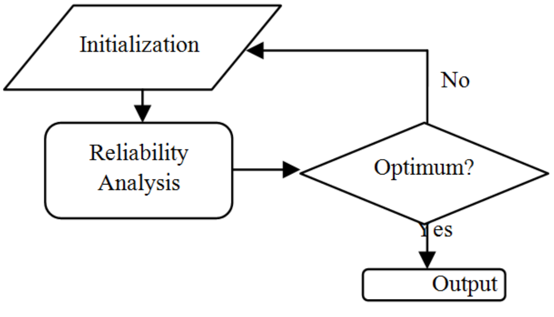
Composite materials are used widely in mechanical, aerospace, shipbuilding, and other industries for their light weight and good strength. To avoid catastrophic failure of composite Structures it is necessary to take the uncertainties into consideration in the stage of design.

Uncertainities in laminated composite structural design is one of the important challenge in the present design engineering. As Reliability Based Design Optimization (RBDO) hand is becoming more reliable approach which suits to the present challenge. The objective of RBDO is to seek a design that achieves a targeted probability of failure and ensures expected optimum performance.

Chen Jianqiao et. al. [1] has proposed a method with target reliability (R_t) of 0.99 and optimization of objective function ($f = h_1 + h_2$) was 9.724 mm, in which computation cost is not effective.

As a result, new approach is developed to solve such problems. Our New approach includes MATLAB based Monte Carlo Simulation (MCS), making it more simplified and optimized which is effective in computational cost.

2. RELIABILITY ANALYSIS FOR COMPOSITES AND RELIABILITY BASED DESIGN PROCEDURE



2.1 Tsai – Wu failure criterion

Tsai–Wu failure criterion [2] is the most reasonable failure criterion for composites. It can be expressed as

$$FI = F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + 2F_{12}\sigma_1\sigma_2 + 2F_{23}\sigma_2\sigma_3 + 2F_{31}\sigma_1\sigma_2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 \le 1 \quad (1)$$

Where

$F_1 = 1/X_T - 1/X_C$, $F_2 = 1/Y_T - 1/Y_C$,

$F_3 = 1/Z_T - 1/Z_C$

$F_{11} = 1/(X_TX_C)$, $F_{22} = 1/(Y_TY_C)$,

$F_{33} = 1/(Z_TZ_C)$

$F_{44} = 1/S^2_{yz}$, $F_{55} = 1/S^2_{zx}$, $F_{66} = 1/S^2_{xy}$

$F_{12} = (-1/2)\sqrt{(F_{11} F_{22})}$,

$F_{23} = (-1/2)\sqrt{(F_{22} F_{33})}$,

$F_{31} = (-1/2)\sqrt{(F_{33} F_{11})}$

where X_T and X_C are tensile and compressive strength in the longitudinal direction, respectively; Y_T, Z_T, Y_C and Z_C are tensile and compressive strength in the transversely isotropic surface, respectively; S_{xy}, S_{yz} and S_{zx} are shear strength in the transversely isotropic surface. Therefore, the limit state function in terms of Tsai–Wu failure criterion can be expressed

$$G = 1 - FI \quad (2)$$

The composite element is in operating state if $G > 0$, in failure state if $G < 0$, and in limit state if $G = 0$. G is the Performance Function.

2.2 The Limit State Function

In the case considered in this paper, particular attention has been focused on the physical properties of the composite material, as in Table 1. All the combinations of the attainable values define the n-dimensional domain where the joint probability density function (PDF) of the properties distributions is evaluated by Roberto d'Ippolito et al [2]. To assess the probability of failure, a performance function should be chosen; this function acts as a criterion that partitions the stochastic space in two regions: a failure region and a safe region. The boundary between the safe and the failure region is called Limit State Function (LSF). By definition, performance functions have positive values in the safe region and negative values in the failure region.

Distribution	X_T	X_C	Y_T	Y_C	S
Mean	1500	1500	48	246	68
Standard Deviation	150	150	4.8	24.6	6.8

Table 1. Random Variables (Mpa)

2.3 Monte Carlo Simulation

The general idea of this method is to solve mathematical problems by random generation of parameter vectors for the given input distribution and deterministic simulation of each random parameter vector. In this way, the simulation of any process influenced by random factors is possible.

The Monte Carlo simulation is used for probabilistic structural analysis for two main purposes:

(1) numerical validation of analytical methods

(2) solving large, complex systems when analytical approximations are not feasible.

Limit State Function (LSF) is the integral that gives the probability of failure of a particular system cannot be solved analytically. Mathematically, this integral is given by:

$$P_f = P[G(x) \le 0] = \int_\Omega \ldots \int f_x(X)dx \quad (3)$$

where $f_x(X)$ is the joint PDF of the input variables

The Monte Carlo simulation method is used to numerically integrate the multidimensional integral by sampling the stochastic domain Ω randomly and estimate the probability of failure with the simple formula:

$P_f = N_f/N$ (4) Roberto d'Ippolito et al[2]

Where

- N_f is the number of samples that yield a failure (that is $G(X)\le 0$, a negative LSF evaluation) and
- N is the total number of trials.

3. OPTIMIZATION DESIGN OF FRP LAMINATE USING NEW APPROACH

When FRP laminated plates are used in composite structures, the primary concern to designers is how to reduce their weight without compromising their performance. Such issues can usually be summarized as follows: minimize the structural weight with reliability constraints.

$$\min_{\mathbf{d}} h(\mathbf{d}) = \sum_{i=1}^{n} h_i(\mathbf{d})$$

$$\text{s.t. } P\{g \ge 0\} \ge R_t \quad (5)$$

where d is the design variable (e.g.,ply angle etc.), h(d) the total ply thickness, and R_t the target system reliability.

In the present case study, a simply supported symmetric laminated composite plate (20cm X 12.5 cm) under both compression load N_x= 500kN/m and uniform transverse load p= 0.2MPa, as shown in Fig. 1 as considered[1]

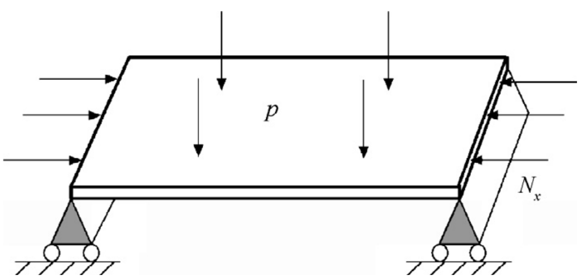


Figure 1. A laminate subjected to axial compression load and uniform lateral load.

The stacking structure is $[0 / +45 / -45 / 90]^s$. The thicknesses of 0° layer and 90° layer are both 0.25mm, and those of +45° layer and -45° layer are h1 x 0.125 mm, h2 x 0.125 mm, respectively. The laminate is made of T300/ 5208 graphite/ epoxy material with E_1 = 181GPa, E_2 = 10.3GPa, G_{12}= 7.17GPa, and μ_{12}= 0.28. The strength parameters are considered as normally distributed random variables of independence and their distribution characteristics are shown in Table 1. The objective function of this problem is $f = h_1 + h_2$ with h_1 and h_2 being the design variables. The target system reliability is R_t=0.99 (P_f = 0.01).

In the present approach, MCS is combined with MATLAB to perform RBDO. The population size is 40,00,000. The results are listed in Table 2.

Method	h1 mm	h2 mm	f mm	P_f %	N_{itr} No.
PSO-ANSYS [1]	5.013	4.711	9.724	0.01	800
Proposed Approach	4.017	5.447	9.464	0.01	5

Table 2. RBDO Validation

Since performing finite element analysis for composite structure is a time consuming process, the computational cost required by the reliability calculation and optimization is negligibly low, as compared to that by finite element analysis.

Thus, approximately, the efficiency of the two methods is compared by the number of iterations (N_{itr}). It is observed that the method combining MCS and MATLAB obtains a better solution than that obtained by the PSO and ANSYS Roberto d'Ippolito et al [2].

However, the initial solution in the PSO–ANSYS procedure was randomly generated, and the quality of the optimal solution has nothing to do with the selection of the initial solution. Meanwhile, the proposed procedure can reach the optimum regard of randomly generated initial candidate solutions (population). Due to employment of the function approximations, the computational cost required by the optimum PSO and ANSYS is more than that by the proposed method as shown in Table 2.

From the Plot (Figure No. 2) at G(x) = 0.049, f = 9.464 mm, h1 = 4.014 mm, h2 = 5.447 mm are the optimum values for safe working design. Further stress analysis is performed in Ansys, the results are shown in Figure No. 3, which represents a safe design.

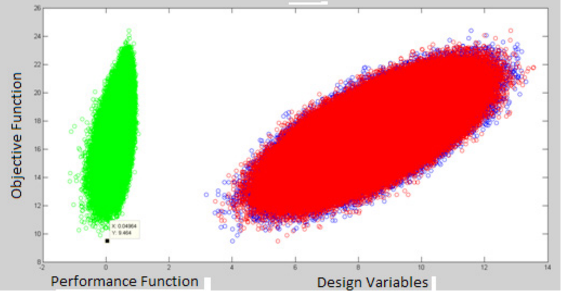


Figure 2. Plot of the optimum result

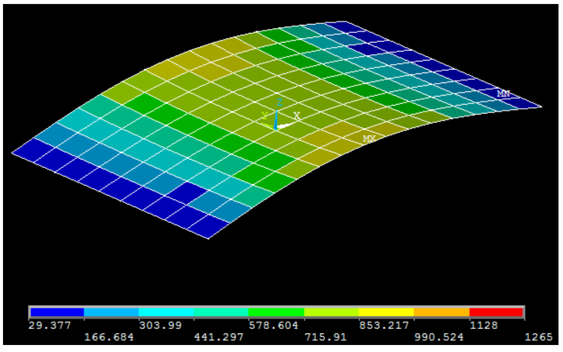


Figure 3. von Mises Stress

4. CONCLUSION

As strength to weight ratio is of main criteria in composite structural design, high reliability is usually required. An attempt is made in the present work, where MATLAB and Monte Carlo Simulation where performed to obtain a optimum design variable values of h1 & h2 which yields a better performance & reliability. This method consists of 3 modules

(1) Reliability Calculation
(2) Optimization
(3) Validation of optimum result in ANSYS for Safe Stress Values

The maximum Stress values obtain with optimum design variable values were found to be within the random variable distribution (Table

1). Obtained results from the proposed approach were tested in ANSYS software and found to be good.

REFERENCES

[1] Chen Jianqiao, Tang Yuanfu, Ge Rui, An Qunli, Guo Xiwei (2013), Reliability design optimization of composite structures based on PSO together with FEA, Chinese Journal of Aeronautics,343–349 | [2] Roberto d'Ippolito, Stijn Donders, Nick Tzannetakis, Joost Van de Peer, Herman Van der Auweraer (2005), Design improvements of composite aerospace structures using reliability analysis, http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.91.7991&rep=rep1&type=pdf | [3] Robert M. Jones (1998), Mechanics of Composite Materials (2nd Edition) , USA, Taylor & Francis |