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Results on Edge Graph

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ABSTRACT

Let G be a (p, q) graph. Construct a Graph with q vertices such that q={e1, e2, e3,…,eq} and e1 and e2 are adjacent if the corresponding edges in G are adjacent and it is denoted by EG(G) called the Edge of the graph G.

In this paper, we proved that Edge graph of P_n is P_n-1 i.e. $EG(P_n) = P_n$, Edge graph of C_n is C_n i.e. $EG(C_n) = C_n$, $EG(k1, n) = k_n$, If G is r – regular then EG(G) is 2(r-1) regular.

KEYWORDS : Edge Graph

1. Introduction

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {uv} of vertices in E is called an edge or a line of G. A graph G is called r-regular if deg(v) = r for each v∈ V(G). If all the vertices in a walk are distinct, then it is called a path and a path of length k is denoted by P_{k+1}. A closed path is called a cycle and a cycle of length k is denoted by C_k.

2. Preliminaries

Let G be a (p, q) graph. Construct a Graph with q vertices such that $q=\{e_1, e_2, e_3, \ldots, e_q\}$ and e_1 and e_2 are adjacent if the corresponding edges in G are adjacent and it is denoted by EG(G) called the Edge of the graph G.

In this paper, we proved that Edge graph of P_n is P_{n-1} i.e. $EG(P_n) = P_n$, Edge graph of C_n is C_n i.e. $EG(C_n) = C_n$, $EG(k_1, n) = k_n$, If G is r – regular then EG(G) is 2(r-1) regular.

3. Main Results

Theorem 3.1
Edge graph of P_n is P_{n-1}. $EG(P_n) = P_n$

Proof:
Let G= P_n, a path of length n-1.

Let $V(P_n) = \{u_1, u_2, u_3, \ldots, u_n\}$ such that $e_i = (u_iu_{i+1})$

Then $V[EG(P_n)] = \{e_1, e_2, e_3, \ldots, e_{n-1}\}$ has n-1 vertices and

$V[EG(P_n)] = \{(e_ie_{i+1}) : 1 \le i \le n-2\}$ has n-2 edges.

Hence $EG(P_n)$ is a graph with n-1 vertices and n-2 edges

$EG(P_n) = P_n$

Theorem 3.2
Edge graph of C_n is C_n . $EG(C_n) = C_n$

Proof:

Let G= C_n, a path of length n.

Let $V(C_n) = \{u_1, u_2, u_3, \ldots, u_n\}$ and $E(C_n) = \{(e_ie_{i+1}) : 1 \le i \le n-1; (e_1e_n)\}$

Each edge (e_ie_{i+1}) is adjacent to the edges $(e_{i+1}e_{i+2})$ and $(e_{i-1}e_i)$.

Hence, $EG(C_n)$ is a cyclic path of length n.

$EG(C_n) = C_n$

Example

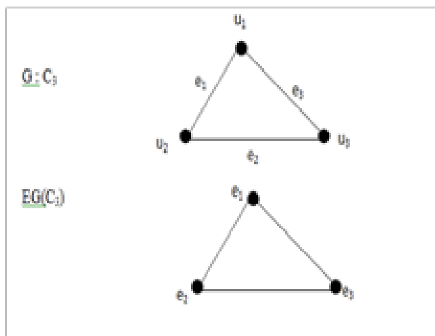

Figure 1

Theorem3.3
$EG(k_1, n) = k_n$

Proof:
Let $V(k_1, n) = \{u, u_i : 1 \le i \le n\}$ and

$E(k_1, n) = \{(uu_i) = e_i : 1 \le i \le n\}$

In the graph k_1, n, each edge e_i is incident at u.

Hence, every edge is adjacent to each other.

Then $V[EG(k_1, n)] = \{e_j : 1 \le i \le n\}$

Considering edges as vertices, each vertex is adjacent to every other vertex.

Hence, $EG(k_1, n) = k_n$

Example:

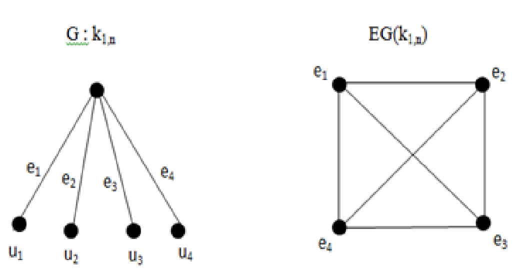

Figure 2

Theorem 3.4

If G is r – regular then EG(G) is 2(r-1) regular.

Proof:

Let G be (p_1, q_1) graph and let G be r – regular

No. of edges incident on each vertex is r.

Considering each edge as vertex in EG(G) the no. of vertex incident on each vertex is r + r – 2 = 2r – 2 = 2(r-1).

Hence, EG(G) is 2(r-1).

Let EG(G) be (p_2,q_2) graph.

Then, $p_2 = q_1 = \frac{r.p_1}{2}$

and $q_2 = \frac{2(r-1).q_1}{2} = \frac{(r-1).r.p_1}{2}$

$$|EG(G)| = \frac{r(r-1)p}{2}$$

where p is the number of vertices of G.

Results:

1) Let G be any graph. Let d^* be the degree of a vertex in EG(G), then The sum of degree of
EG(G) $=\sum d^*(e_i)$
$= \sum d(u_i)\,[d(u_i)-1]$
The no. of edges
$|EG(G)| = \frac{1}{2} \sum d(u_i)\,[d(u_i)-1]$
2) For any cycle C_n, p = q, then EG(G) in (q, p)
G(p, q) <=> EG(G) (q, p).
3) If all the non – pendant vertices are of the same degree k in G then the sum of the degrees of vertices in nk(k-1). Where n is the no. of non – pendant vertices in G.
4) For any graph G except/ other than P_n, EG(G) contains a cycle.
5) EG(C_4) is $k_{2,2}$, the complete bipartite graph.
6) EG(C_n) : n even is a bipartite graph.
7) EG(C_n) : n even is $k_{n/2,n/2}$.
8) EG(G) is connected if and only if G is connected.
9) EG(G) is disconnected if and only if G is disconnected.
10) If e = u_1u_2 in G then for e $\in$ V[EG(G)],
$d^*(e) = d(u_1) + d(u_2) - 2$
11) G is regular =>EG(G) is regular
Proof : Consider G is regular
=> $d(u_i) = k$ for all $u_i \in V(G)$
then for each vertex $e_i \in G(G)$ => $d^*(e_i) = d(u_i) + d(u_j) - 2$
= k+k-2
= 2(k-1) $\forall e_i \in V[EG(G)]$
Therefore, EG(G) is 2(k-1) regular
EG(G) is regular.
12) If e = uv$\in$(G) such that d(u) + d(v) = 2 then the corresponding e$\in$[EG(G)] is an isolated vertex.
13) If e = uv$\in$(G) such that d(u) + d(v) = 3 then the corresponding eV[EG(G)] is an pendant vertex.
14) If deg(v) ≥ and deg(v)=n, v V(G) then there is a subgraphk_n exists in EG(G)
15) The k-star st($\alpha_1, \alpha_2,\ldots, \alpha_k$) is a dis connected graph with k components $k_1, \alpha_1, k_1,\alpha_2,\ldots, k_1,\alpha_k$ then
EG[st($\alpha_1, \alpha_2,\ldots, \alpha_k$)] = ($k_1,\alpha_1, k_1, \alpha_2,\ldots, k_1, \alpha_k$)
16) If G = k_n^c then EG(k_n^c) does not exist.
17) If G is a graph of n non-interesting edges then EG(G) = k_n^c
18) If an edge is a bridge then the corresponding vertex in edge graph EG(G) is a common vertex for the components.

Remarks :

If EG(G) is regular then G need not be regular

Let G : $k_{1,3}$ is not regular but EG($k_{1,3}$) is regular

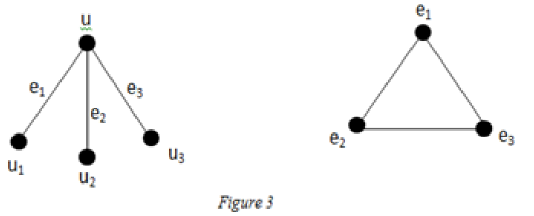

Figure 3

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