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Construction of two Associate Classes Partially Balanced Incomplete Block Designs With *(S-1)*^{2} Replicates

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ABSTRACT

In this paper, we introduce two associate class partially balanced incomplete block (PBIB) designs with (s-1)2 replicates. Each of the s rows and s columns of square array of order s × s are taken as base for Constructing partially balanced incomplete block (PBIB) designs. Also, illustration of construction of some new PBIB designs along with two associate class association schemes have also been given in detail.

Summary and Discussion:

In this paper, we have established a link between PBIB designs and square array of order s × s in standard order as a result; we get new PBIB designs with two associate classes. Efficiencies of the new designs are also computed for the purpose of comparison. Some designs are new and some newly constructed designs are more efficient, as compared to the existing designs with respective parameters.

KEYWORDS : PBIB designs, square- matrix, association scheme

1. Introduction

Sometimes the number of treatments to be compared is large and we need more experimental units for large number of blocks to accommodate all the treatments. This may increase the cost of experimentation, labour; time etc. and randomized block design (RBD) may not be suitable in such situations because they will require large number of experimental units in blocks to accommodate all the treatments. In such cases when sufficient number of homogeneous experimental units are not available to accommodate all the treatments in a block, then incomplete block designs are used in which each block receives only some and not all the treatments to be compared. Also, it is possible that the blocks that are available can only handle a limited number of treatments due to several reasons. For example, suppose the effect of twenty medicines for a rare disease from different companies is to be tested over patients. These medicines can be treated as treatments. It may be difficult to get sufficient number of patients having the disease to conduct a complete block experiment. In such a case, a possible solution is to have less than twenty patients in each block. Then not all the twenty medicines can be administered in every block. Instead few medicines are administered to the patients in one block and the remaining medicines to the patients in other blocks. The incomplete block designs particularly partially balanced incomplete block (PBIB) designs can be used in this setup. In another example, the pharmaceutical companies and biological experimentalists need animals to conduct their experiments to study the development of any new drug.

The main purpose of our study is to construct efficient partially balanced incomplete block designs (PBIB) designs having m class ($m \geq 2$) association schemes which may be applicable for solving the practical problems of the real world. Various methods of construction of PBIB designs are available in literature, but it may not be possible to get a suitable design in a particular practical situations. Bose et.al (1959) made an exhaustive study on linear associate algebras corresponding to association scheme of partially balanced designs. Bose, R.C., and Nair, K.R. (1939) have established some partially balanced incomplete block designs. Bose et.al (1954) has obtained tables of partially balanced designs with two associate classes. For the literature on PBIB designs, Nair, K.R. (1951) have constructed some two replicate PBIB designs. Recently, Garg and Gurinder (2011) have obtained three and four associate class PBIB designs using method of Duality.

2. Some Definitions

2.1 PBIB Design

Consider a set of symbols S = {1, 2, 3, . . ., v} and an association scheme with m classes, we have a partially balanced incomplete block designs (PBIB) if v symbols are arranged in b blocks of size k(<v) such that

1. Every symbol occurs exactly in r blocks.
2. Every symbol occurs at most once in a block.
3. If two symbols α and β are ith associates, then they occur together λ_i blocks, the number λ_i being independent of the particular pair of the ith associates α and β

The numbers v, b, r, k, λ_i (i = 1,2,3,. . .,m) are called parameters of the first kind, where as the numbers n_i and p^i_{jk} are called parameters of the second kind. If every symbol is taken as 0th associate and of no other symbol, then $n_0 = 1$ and $p^0_{ij} = n_i\,\delta_{ij}$, $p^0_{ij} = \delta_{ik}$, $\lambda_0 = r$, where δ_{ij} is kronecker delta and is defined as $\delta_{ij} = 1$ if i = 1 and $\delta_{ij} = 0$ if $i \neq j$.

2.1.1 Definition: If we delete the ith row and jth column passing through the element say a_{ij} of the matrix A of order s ×s, then order (s-1) × (s-1) of the sub-matrix constitute the blocks of PBIB design.

3. Association scheme of new two associate class PBIB designs:

In this section, we have defined two associate class PBIB designs by using square array of order s×s in standard order i.e s take values 3 and 4 only. The procedure for this association scheme is as follows:

3.1 Association scheme: In this association scheme the number of treatments and blocks as $v = s^2$ and $b = s^2$ respectively and every treatment repeats exactly $r = (s-1)^2$ times in the blocks. Consider those blocks which have exactly one treatment in common namely Θ. On these blocks, we define a two associate class association scheme as follows: Take any two treatments say Θ and Φ**,** then they are first associates of each other if they occur $\lambda_1 = (s-2)^2$ times. If two treatments occur $\boldsymbol{\lambda_2} = (s-1)(s-2)$ times, they are said to be second associate of each other and $n_2 = 2(s-1)$.

The P- matrices of the association scheme are

$$P_1 = \begin{bmatrix} (s-2)^2 & s(s-1)/3 \\ s(s-1)/3 & 2 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} (s-1)(s-2) & s-1 \\ (s-1) & (s-2) \end{bmatrix}$$

4. Construction Methodology:
In this section we have constructed PBIB designs by taking square matrix A of order s×s with $r = (s-1)^2$ replicates. The procedure of construction is as follows:

Let us take $v = s^2$ (s = 3, 4) treatments which are arranged in 's' rows and 's' columns in a square matrix 'A' as given below:

$$A = \begin{bmatrix} 1 & 2 & 3 & . & . & . & s \\ s+1 & s+2 & s+3 & . & . & . & 2s \\ 2s+1 & 2s+2 & 2s+3 & . & . & . & 3s \\ . & . & . & & & & . \\ . & . & . & & & & . \\ . & . & . & & & & . \\ (s-1)s+1 & (s-1)s+2 & (s-1)s+3 & & & & s^2 \end{bmatrix}$$

Form these 'v' symbols arranged in's' rows and s columns, where s is a positive integers such that (s take values 3 and 4 only). In this, we delete i^{th} row and j^{th} column (i, j =1,2,3,...,s) passing through the element say (a_{ij}) of the matrix A of order s ×s, then all possible sub-matrix of order (s-1) ×(s-1) constitute the blocks. Therefore there are s^2 numbers of blocks each of size $k = (s-1)^2$. By using the parametric relationship vr = bk, we see that each treatment replicated 'r' times in the blocks. We proceed which we have explained above, then we obtain the following series of PBIB designs with the following parameters.

$v = s^2$, $b = s^2$, $r = (s-1)^2 = k$, $\lambda_1 = (s-2)^2$, $\lambda_2 = (s-1)(s-2)$, $n_1 = (s-1)^2$, $n_2 = 2(s-1)$, . . . , (a)

4.1 Illustration of construction: For s = 3, in the parameters of equation no. (a) the PBIB designs becomes

$v = 9$, $b = 9$, $k = 4$, $r = 4$, $\lambda_1 = 1$, $\lambda_2 = 2$, $n_1 = 4$, $n_2 = 4$

The square matrix 'A' as given below:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Form all symbols arranged in '3' rows and '3' columns. From these '9' symbols we delete i^{th} row and j^{th} column(i, j = 1,2,3) passing through the element say (a_{ij}) of the matrix A of order 3 ×3, then all possible order 2 × 2 of the sub-matrix constitute the blocks. Therefore there are '9' blocks each of size = 4. By using the parametric relationship vr = bk, we see that each treatment replicated 'r' times in a design. The following blocks are as:

1. (5, 6, 8, 9)	4. (2, 3, 8, 9)	7. (2, 3, 5, 6)
2. (4, 6, 7, 9)	5. (1, 3, 7, 9)	8. (1, 3, 4, 6)
3. (4, 5, 7, 8)	6. (1, 2, 7, 8)	9. (1, 2, 4, 5)

The following table explains the association scheme with first and second associates of treatment 1, 2, 3, 4, 5, 6, 7, 8, 9:

Symbols	I^{st} associates	2^{nd} associates
1.	5, 6, 8, 9	2, 3, 4, 7
2.	4, 6, 7, 9	1, 3, 5, 8
3.	4, 5, 7, 8	1, 2, 6, 9
4.	2, 3, 8, 9	1, 5, 6, 7
5.	1, 3, 7, 9	2, 4, 6, 8
6.	1, 2, 7, 8	3, 4, 5, 9
7.	2, 3, 5, 6	1, 4, 8, 9
8.	1, 3, 4, 6	2, 5, 7, 9
10.	1, 2, 4, 5	3, 6, 7, 8

P-matrices of the new association scheme are given by

$$P_1 = \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix}$$

$$P_2 = \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix}$$

4.1.1 Illustration: For s = 4, in the parameters of equation no. (a) the PBIB designs becomes $v = 16$, $b = 16$, $k = 9$, $r = 9$, $\lambda_1 = 4$, $\lambda_2 = 6$, $n_1 = 9$, $n_2 = 6$

The square matrix 'A' as given below:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

From these '16' symbols arranged in '4' rows and '4' columns, we delete i^{th} row and j^{th} column(i, j = 1,2,3,4) passing through the element say (a_{ij}) of the matrix A of order 4 ×4, then all possible order 3 × 3 of the sub-matrix constitute the blocks. By using the parametric relationship vr = bk, we see that each treatment replicated 'r' times in a design.

The following blocks are as:

1. (6, 7, 8, 10, 11, 12, 14, 15, 16)
2. (5, 7, 8, 9, 11, 12, 13, 15, 16)
3. (5, 6, 8, 9, 10, 12, 13, 14, 16)
4. (5, 6, 7, 9, 10, 11, 13, 14, 15)
5. (2, 3, 4, 10, 11, 12, 14, 15,16)
6. (1, 3, 4, 9, 11, 12, 13, 15, 16)
7. (1, 2, 4, 9, 10, 12, 13, 14, 16)
8. (1, 2, 3, 9, 10, 11, 13, 14, 15)
9. (2, 3, 4, 6, 7, 8, 14, 15, 16)
10. (1, 3, 4, 5, 7, 8, 13, 15, 16)
11. (1, 2, 4, 5, 6, 8, 13, 14, 16)
12. (1, 2, 3, 5, 6, 7, 13, 14, 15)

13. (2, 3, 4, 6, 7, 8, 10, 11, 12)
14. (1, 3, 4, 5, 7, 8, 9, 11, 12)
15. (1, 2, 4, 5, 6, 8, 9, 10, 12)
16. (1, 2, 3, 5, 6, 7, 9, 10, 11)

P-matrices of the new association scheme are given by

$$P_1 = \begin{bmatrix} 4 & 4 \\ 4 & 2 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 6 & 3 \\ 3 & 2 \end{bmatrix}$$

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