



An Appropriate Mathematical Model for A Product

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ABSTRACT

An important concern in all business and economic activities is to have some fair idea as to whether the future trends are going to be favorable or unfavorable. It helps the top management take appropriate policy decisions in advance. For example sales may be expected to rise substantially after, say, 5 years. This demand taking measures well in time to build adequate productive capacity so that the future profit potential is not lost to the rival producers. An anticipation of this kind involves long-term planning. On the other hand, product sales may be expected to go up shortly or in the near future. A prudent management would rise to the occasion to take advantage of such opportunities. It may call for making necessary adjustments in the existing production schedule and taking other steps, as the situation calls for. All this effort would be aimed at ensuring that sufficient stocks are kept ready with the given capacity so that the product is easily available when needed. A situation of this kind involves short-term planning. Irrespective of the length of future time period one is interested in being long or short, planners and policy makers at all levels need to know the future trends in business activity relating to several other variables and predict the likely cause-effect relationships among them. This is made possible through business forecasting, which consists of methods of acquiring knowledge about the future. Competition is an essential characteristic of modern business. Many a time, competition tends to be so intense that it poses a constant threat to the very stability of business activity. This factor alone makes fore-knowledge about future business trends extremely important as it enables a business firm to evolve appropriate strategies for holding on to its position or retaining market share already attained. In the absence of any serious impending threat of competition, neither would the need to know about the future be so imperative, nor would the concern for long-term and short-term planning be so impelling.

KEYWORDS :

❖ Research Methodology :

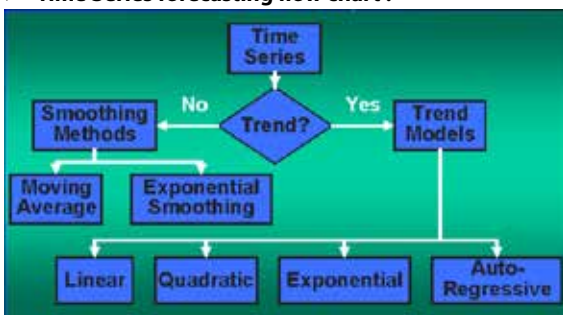
The analysis of time series implies its decomposition into various factors that affect the value of its variable in a given period. It is a quantitative and objective evaluation of the effects of various factors on the activity under consideration.

There are two main objectives of the analysis of any time series data:

- To study the past behavior of data.
- To make forecasts for future.

The study of past behavior is essential because it provides us the knowledge of the effects of various forces. This can facilitate the process of anticipation of future course of events, and, thus, forecasting the value of the variable as well as planning for future.

➤ Time Series forecasting flow chart :



❖ Literature Review :

Forecasting techniques can be categorized in two broad categories: quantitative and qualitative. The techniques in the quantitative category include mathematical models such as moving average, straight-line projection, exponential smoothing, regression, trend-line analysis, simulation, life-cycle analysis, decomposition, expert systems, and neural network. The techniques in the qualitative category include subjective or intuitive models such as jury or executive opinion, sales force composite, and customer expectations (Kress, 1985; Mentzer & Kahn, 1995).

Along with qualitative and quantitative, forecasting models can be categorized as time-series, causal, and judgmental. A time-series model uses past data as the basis for estimating future results. The models that fall into this category include decomposition, moving

average, exponential smoothing, and others. The premise of a causal model is that a particular outcome is directly influenced by some other predictable factor.

These techniques include regression models. Judgmental techniques are often called subjective because they rely on intuition, opinions, and probability to derive the forecast.

These techniques include expert opinion, Delphi, sales force composite, customer expectations (customer surveys), and simulation (Kress, 1985; Wilson & Keating, 1994).

The data presents forecasting techniques categorized by quantitative or qualitative.

Typically, the two forms of forecasting error measures used to judge forecasting performance are mean absolute deviation (MAD) and mean absolute percentage error (MAPE). For both MAD and MAPE, a lower absolute value is preferred to a higher absolute value. MAD is the difference between the actual sales and the forecast sales, absolute values are calculated over a period of time, and the mean is derived from these absolute differences. MAPE is used with large amounts of data, and forecasters may prefer to measure error in percentage (Wilson & Keating, 1994).

❖ Quantitative and Qualitative Techniques of Forecasting :

• Quantitative Forecasting Techniques:

- Regression Analysis: statically relates sales to one or more explanatory (independent) variables. Explanatory variables may be marketing decisions (price changes, for instance), competitive information, economic data, or any other variable related to sales.
- Exponential smoothing makes an exponentially smoothed weighted average of past sales, trends, and seasonality to derive a forecast.
- Moving average takes an average of a specified number of past observations to make a forecast. As new observations become available, they are used in the forecast and the oldest observations are dropped.
- Box-Jenkins uses the auto correlative structure of sales data to develop an autoregressive moving average forecast from past sales and forecast errors.
- Trend line analysis fits a line to the sales data by minimizing the squared error between the line and actual past sales values. This line is then projected into the future as the forecast.

- Decomposition breaks the sales data into seasonal, cyclical, trend and noise components and projects each into the future.
- Straight-line projection is a visual extrapolation of the past data, which is projected into the future as the forecast.
- Life-cycle analysis bases the forecast upon whether the product is judged to be in the introduction, growth, maturity, or decline stage of the life cycle.
- Simulation uses the computer to model the forces, which affect sales: customers, marketing plans, competitors, flow of goods, etc. The simulation model is a mathematical replication of the actual corporation.
- Expert systems use the knowledge of one or more forecasting experts to develop decision rules to arrive at a forecast.
- Neural networks look for patterns in previous history of sales and explanatory data to uncover relationships. These relationships are used to produce the forecast.

• **Qualitative Forecasting Techniques**

- Jury of executive opinion consists of combining top executives' views concerning future sales.
- Sales force composite combines the individual forecasts of salespeople.
- Customer expectations (customer surveys) use customers' expectations as the basis for the forecast. The data are typically gathered by a customer survey by the sales force.
- Delphi model is similar to jury of executive opinion in taking advantage of the wisdom of experts. However, it has the additional advantage of anonymity Among participants.
- Naïve model assumes that the next period will be identical to the present. The forecast is based on the most recent observation of data.

❖ **Data Presentation and Data Analysis :**

Measures of Forecast Accuracy

There are various parameters for measuring forecast accuracy. They can be divided as follow:

- Mean Squared Error (MSE)
- Mean Absolute Deviation (MAD)

4.1.1 Mean Squared Error (MSE)

The average of the squared forecast errors for the historical data is calculated. The forecasting method or parameter(s) which minimize this mean squared error is then selected.

4.1.2 Mean Absolute Deviation (MAD)

The mean of the absolute values of all forecast errors is calculated, and the forecasting method or parameter(s) which minimize this measure is selected. The mean absolute deviation measure is less sensitive to individual large forecast errors than the mean squared error measure.

One may choose either of the above criteria for evaluating the accuracy of a method (or parameter).

❖ **Conclusions :**

Methods of least square

The trend project method fits a trend line to a series of historical data points and then projects the line into the future for medium-to-long range forecasts. Several mathematical trend equations can be developed (such as exponential and quadratic), depending upon movement of time-series data.

Reasons to study trend: A few reasons to study trends are as follows:

1. The study of trend allows us to describe a historical pattern so that we may evaluate the success of previous policy.
2. The study allows us to use trends as an aid in making intermediate and long-range forecasting projections in the future.
3. The study of trends helps us to isolate and then eliminate its influencing effects on the time-series model as a guide to short-run (one year or less) forecasting of general business cycle conditions.

Linear Trend Model

If we decide to develop a linear trend line by a precise statistical

method, we can apply the least squares method. A least squares line is described in terms of its y-intercept (the height at which it intercepts the y-axis) and its slope (the angle of the line).

If we can compute the y- intercept and slope, we can express the line with the following equation

$$\hat{y} = a + bx$$

where \hat{y} = predicted value of the dependent variable

a = y-axis intercept

b = slope of the regression line

x = independent variable (which is time in this case)

Least squares is one of the most widely used methods of fitting trends to data because it yields what is mathematically described as a 'line of best fit'. This trend line has the properties that

- a) The summation of all vertical deviations about it is zero, that is, $\Sigma(y - \hat{y}) = 0$
- b) The summation of all vertical deviations squared is a minimum, that is, $\Sigma(y - \hat{y})^2$ is the least and
- c) The line goes through the mean values of variables x and y. For linear equations, it is found by the simultaneous solution for a and b of the two normal equations:

$$\Sigma y = na + b\Sigma x \text{ and } \Sigma xy = a\Sigma x + b\Sigma x^2$$

Where the data can be coded so that $\Sigma x = 0$, two terms in three equations drop out and we have $\Sigma y = na$ and $\Sigma xy = b\Sigma x$

Coding is easily done with time-series data. For coding the data, we choose the centre of the time period as $x = 0$ and have an equal number of plus and minus periods on each side of the trend line which sum to zero.

Alternately, we can also find the values of constants a and b for any regression line as:

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n(\bar{x})^2} \text{ and } a = \bar{y} - b\bar{x}$$

Parabolic Trend Model

The curvilinear relationship for estimating the value of a dependent variable y from an independent variable x might take the form

$$\hat{y} = a + bx + cx^2$$

This trend line is called the parabola.

$$\hat{y} = a + bx - cx^2,$$

For a non-linear equation $\hat{y} = a + bx - cx^2$ the values of constants a, b, and c can be determined by solving three normal equations.

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$

$$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

When the data can be coded so that $\Sigma x = 0$ and $\Sigma x^3 = 0$, two term in the above expressions drop out and we have

$$\Sigma y = na + c\Sigma x^2$$

$$\Sigma xy = b\Sigma x^2$$

$$\Sigma x^2y = a\Sigma x^2 + c\Sigma x^4$$

To find the exact estimated value of the variable y, the values of constants a, b, and c needs to be calculated. The values of these constants can be calculated by using the following shortest method:

$$a = \frac{\Sigma y - c \Sigma x^2}{n}; b = \frac{\Sigma xy}{\Sigma x^2} \text{ and } c = \frac{n \Sigma x^2 y - \Sigma x^2 \Sigma y}{n \Sigma x^4 - (\Sigma x^2)^2}$$