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Embedding of Non Skolem Differenece Mean Graphs

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ABSTRACT A graph G =(V,E) with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices x ϵ V with distinct elements f (x) from {1,2,…,p+q} in such a way that the edge e =uv is labeled with |f(u)-f(v)|/2 if |f(u)-f(v) | is even and (|f(u)-f(v) |+1)/2 if |f(u)-f(v) | is odd and the resulting labels of the edges are distinct and are {1,2,…,q}. A graph that admits skolem difference mean labeling is called a skolem difference mean graph. The necessary condition for a graph to be skolem difference mean is that p ≥q. In this paper, the graphs for which p <q are considered. They are embedded in skolem difference mean graphs and their skolem difference mean labeling is studied.

KEYWORDS : Path, Star, Cycle, embedding, Skolem difference mean labeling

1.INTRODUCTION

Throughout this paper, only finite, undirected, simple graphs are considered. Let G = (V, E) be a graph with p vertices and q edges. For graph theoretic terminologies and notations Harary [4] is followed.

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Rosa [9] introduced β -valuation of a graph and Golomb [5] called it as graceful labeling. There are several types of graph labeling and a detailed survey can be found in Gallian [6].A systematic study of various applications of graph labeling is carried out in Bloom and Golomb[3].

The concept of skolem difference mean was introduced in [8].The necessary condition for a graph to be skolem difference mean is that p ≥ q. Acharya et.al have proved in [2] that every connected graph can be embedded as an induced sub graph of a connected graceful graph. This inspired of the authors to study the embedding of some non skolem difference mean graphs in skolem difference mean graphs [7].The following definitions are necessary for the present study.

1.1 Definition: A path P of length n in a graph G is a sequence of distinct vertices $\{v_0, v_1, \dots, v_n\}$ where $e_i = v_i v_{i+1}$ for $i = 0,1,\dots,n-1$.

1.2 Definition: A star is a complete bigraph$K_{1,n}$.

1.3 Definition: A Cycle in a graph G is a sequence of distinct vertices $\{v_0, v_1 \dots v_{n-1}, v_0\}$ where v_i and v_{i+1} are adjacent for all $i= 0,1\dots n-2$ and v_{n-1} and v_0 are adjacent in G. A cycle with n ≥ 3 vertices is denoted by C_n.

1.4 Definition[1]: A graph G is said to be embedded in a graph G', written as $G \preccurlyeq G'$, if there exists an induced sub graph of G' which is isomorphic to G.

In this paper, the author studied the embedding of some non skolem difference mean graphs in skolem difference mean graphs.

2. MAIN RESULTS

In this section, some new graphs in which p < q are defined and they are embedded in skolem difference mean graphs by inserting atleast q-p isolated vertices and their skolem difference mean labeling is studied.

2.1 Definition: The graph $K_{1,m} \dagger K_{1,n}$ is obtained from $K_{1,m}$ and $K_{1,n}$ by identifying the vertices of $K_{1,m}$ with the vertices of $K_{1,n}$ other than the central vertex where m, n ≥ 1.

2.2 Theorem: The graph $(K_{1,m} \dagger K_{1,n}) \cup (n-2)K_1$ is skolem difference mean for all $m,n \geq 3$ and where m $\geq n$.

Proof: Let G be the graph$(K_{1,m} \dagger K_{1,n}) \cup (n-2)K_1$ .Let $V(G) = \{u, u_i, v, v_j, w_k; 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq n-2\}$ and $E\ (G) = \{uv_j, vv_j, uu_i; 1 \leq j \leq n, n+1 \leq i \leq m\}$. Then $|V(G)| = |E(G)| = m+n$. Let $f\colon V(G) \to \{1,2,\dots,2m+2n\}$ be defined as follows.

$f(u) = 2m+2n; f(v) = 2n$
$f(v_j) = 2j-1; 1 \leq j \leq n$
$f(u_i) = 2i-1; n+1 \leq i \leq m$
$f(w_k) = 2k; 1 \leq k \leq n-2$

Let f^* be the induced edge labeling of f. Then $f^*(uv_j) = m+n+1-j; 1 \leq j \leq n$
$f^*(uu_i) = m+n+1-i; n+1 \leq i \leq m$
$f^*(vv_j) = n+1-j; 1 \leq j \leq n$

The induced edge labels are distinct and are $1,2,\dots, m+n$. ■

2.3 Definition: Let n ≥ 1.Then the graph $K_{1,n} \boxplus K_{1,n}$ is obtained from two copies of $K_{1,n}$ by joining the pendant vertices of the first copy of $K_{1,n}$ with the central vertex of the second copy of $K_{1,n}$.

2.4 Theorem: $(K_{1,n} \boxplus K_{1,n}) \cup (n-2)K_1$ is skolem difference mean for all $n \geq 3$.

Proof: Let G be the graph $(K_{1,n} \boxplus K_{1,n}) \cup (n-2)K_1$ .Let $V(G) = \{u, u_i, v, v_i, w_j; 1 \leq i \leq n, 1 \leq j \leq n-2\}$ and $E\ (G) = \{uu_i, uv_i, vv_i; 1 \leq i \leq n\}$ Then $|V(G)| =$

$|E(G)| = 3n$ .Let $f: V(G) \to \{1,2,\dots,6n\}$ be defined as follows.
$f(u) = 6n; f(u_i) = 2i - 1; 1 \le i \le n$
$f(v_i) = 2n - 1 + 2i; 1 \le i \le n$
$f(v) = 4n; f(w_j) = 2j; 1 \le j \le n - 2$
Let f^* be the induced edge labeling of *f*. Then $f^*(uu_i) = 3n + 1 - i; 1 \le i \le n$
$f^*(uv_i) = 2n + 1 - i; 1 \le i \le n$
$f^*(vv_i) = n + 1 - i; 1 \le i \le n$
The induced edge labels are distinct and are 1,2,…,3n. ■

2.5 Definition [11]: The fan graph f_n is obtained by taking $n-2$ concurrent chords in cycle C_{n+1}. The vertex at which all vertices are concurrent is called the apex vertex. It is also given by $f_n = P_n + K_1$.

2.6 Theorem: The graph $f_n \cup (n-2)K_1$ is skolem difference mean for all $n \ge 3$.

Proof: Let G be the graph $f_n \cup (n-2)K_1$. Let $V(G) = \{u, u_i, w_j; 1 \le i \le n, 1 \le j \le n-2\}$ and $E(G) = \{uu_i, u_j u_{j+1}; 1 \le i \le n, 1 \le j \le n-1\}$.Then $|V(G)| = |E(G)| = 2n - 1$. Let $f: V(G) \to \{1,2,\dots,4n-2\}$ be defined as follows.

Case (i) : when *n* is odd
$f(u) = 1$
$f(u_{2i+1}) = 4n - 2 - 4i; 0 \le i < \frac{n+1}{2}$
$f(u_{2i}) = 4i - 1; 1 \le i < \frac{n+1}{2}$
$f(w_j) = 2j; 1 \le j \le n - 2$
Let f^* be the induced edge labeling of *f*.
$f^*(uu_{2i+1}) = 2n - 1 - 2i; 0 \le i < \frac{n+1}{2}$
$f^*(uu_{2i}) = 2i - 1; 1 \le i < \frac{n+1}{2}$
$f^*(u_i u_{i+1}) = 2n - 2i; 1 \le i \le n - 1$

Case(ii): when *n* is even
$f(u) = 1$
$f(u_{2i+1}) = 4n - 2 - 4i; 0 \le i < \frac{n}{2}$
$f(u_{2i}) = 4i - 1; 1 \le i \le \frac{n}{2}$
$f(w_j) = 2j; 1 \le j \le n - 2$
Let f^* be the induced edge labeling of *f*.
$f^*(uu_{2i+1}) = 2n - 1 - 2i; 0 \le i < \frac{n}{2}$
$f^*(uu_{2i}) = 2i - 1; 1 \le i \le \frac{n}{2}$
$f^*(u_i u_{i+1}) = 2n - 2i; 1 \le i \le n - 1$
In both the cases, the induced edge labels are 1,2,…,2n-1 which are distinct. ■

2.7 Definition [**10**]:A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing all edges incident to v and adding edges joining v to every vertex which are not adjacent to v in G.

2.8 Theorem: The graph $G_v \cup (n-5)K_1$ where G_v is the graph obtained by switching a vertex in a cycle C_n is skolem difference mean for all $n \ge 6$.

Proof: Let G be the given graph. Let $V(G) = \{v, v_i, w_j; 2 \le i \le n, 1 \le j \le n-5\}$ and $E(G) = \{v_i v_{i+1}, vv_j; 2 \le i \le n-1, 3 \le j \le n-1\}$ Then $|V(G)| = |E(G)| = 2n - 5$. Let f: V(G) → {1,2,…,4n-10} be defined as follows.

Case(i) when *n* is odd
$f(v) = 4n - 10$
$f(v_{2i}) = 2n - 3 - 2i; 2 \le i \le \frac{n-1}{2}$
$f(v_{2i+1}) = 2i - 1; 1 \le i \le \frac{n-3}{2}$
$f(v_2) = 2n - 3; f(v_n) = 3n - 8$
$f(w_j) = 2j; 1 \le j \le n - 5$

Case(ii) when *n* is even
$f(v) = 4n - 10$
$f(v_{2i+1}) = 2i - 1; 1 \le i < \frac{n}{2}$
$f(v_{2i}) = 2n - 5 - 2i; 1 \le i < \frac{n-2}{2}$
$f(v_2) = 2n - 3$
$f(v_n) = 3n - 10$ when $n = 6$
$= 3n - 9$ when $n \ne 6$
$f(w_j) = 2j; 1 \le j \le n - 5$
In both the cases, let f^* be the induced edge labeling of f. Then $f^*(vv_i) = 2n - 4 - \frac{i-1}{2}$; $3 \le i \le n-1$; *i* odd $= n - 3 + \frac{i}{2}; 3 \le i \le n - 1$; *i* even;
$f^*(v_i v_{i+1}) = n - 1 - i; 3 \le i \le n - 2;$
$f^*(v_1 v_2) = n - 2;$
$f^*(v_{n-1} v_n) = n - 3.$
The induced edge labels are distinct and are $1,2,\dots,2n - 5$. ■

2.9 Definition: The graph $P_m \oslash K_{1,n}$ is obtained from the path P_m, $m \geq 2$ and the star $K_{1,n}$, $n \geq 1$ by identifying the i^{th} vertex of P_m and $K_{1,n}$.

2.10 Theorem: The graph $(P_m \oslash K_{1,n}) \cup (S-2)K_1$, where S = $\min\{m,n\}$ is skolem difference mean for all $m, n \geq 3$.

Proof: Let G be the given graph

Case (i) When $n = m$

Let $V(G) = \{v, v_i, u_i, w_k ; 1 \leq i \leq n, 1 \leq k \leq n-2\}$ and $E(G) = \{vu_i, u_j u_{j+1}; 1 \leq i \leq n, 1 \leq j \leq n-1\}$. Then $|V(G)| = |E(G)| = 2n-1$. Let $f: V(G) \to \{1, 2, \ldots, 4n-2\}$ be defined as follows.

Sub case $i(a)$ when n is odd

$f(v) = 4n-2$

$f(u_{2i+1}) = 2i+1; 0 \leq i < \frac{n+1}{2}$

$f(u_{2i}) = 2n+1-2i; 1 \leq i < \frac{n+1}{2}$

$f(w_k) = 2k; 1 \leq k \leq n-2$

Let f^* be the induced edge labeling of f.

Then $f^*(vu_{2i+1}) = 2n-1-i; 0 \leq i < \frac{n+1}{2}$

$f^*(vu_{2i}) = n-1+i; 1 \leq i < \frac{n+1}{2}$

$f^*(u_i u_{i+1}) = n-i; 1 \leq i \leq n-1$

Sub case $i(b)$ when n is even

$f(v) = 4n-2; f(u_{2i+1}) = 2i+1; 0 \leq i < \frac{n}{2}$

$f(u_{2i}) = 2n+1-2i; 1 \leq i \leq \frac{n}{2}$

$f(w_k) = 2k; 1 \leq k \leq n-2$

Let f^* be the induced edge labeling of f.

Then $f^*(vu_{2i+1}) = 2n-1-i; 0 \leq i < \frac{n}{2}$

$f^*(vu_{2i}) = n-1+i; 1 \leq i \leq \frac{n}{2}$

$f^*(u_i u_{i+1}) = n-i; 1 \leq i \leq n-1$

Case (ii) when $m > n$

Let $V(G) = \{v, v_i, u_j, w_k; 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq k \leq n-2\}$ and E(G) = {vu$_i$ $u_j u_{j+1}$; $1 \leq i \leq n, 1 \leq j \leq m-1\}$. Then $|V(G)| = |E(G)| = m+n-1$. Let $f: V(G) \to \{1, 2, \ldots, 2m+2n-2\}$ be defined as follows.

Sub case $ii(a)$ when m and n are odd

$f(v) = 2m+2n-2$

$f(u_{2i+1}) = 2i+1; 0 \leq i < \frac{n+1}{2}$

$= n+1+2\left(i - \frac{n-1}{2}\right); \frac{n+1}{2} \leq i < \frac{m+1}{2}$

$f(u_{2i}) = 2n+1-2i; 1 \leq i < \frac{n+1}{2}$

$= 2m+n-2\left(i - \frac{n-1}{2}\right); \frac{n+1}{2} \leq i < \frac{m+1}{2}$

$f(w_k) = 2k; 1 \leq k \leq n-2$

Let f^* be the induced edge labeling of f.

$f^*(vu_{2i+1}) = m+n-1-i; 0 \leq i < \frac{n+1}{2}$

$f^*(vu_{2i}) = m-1+i; 1 \leq i < \frac{n+1}{2}$

$f^*(u_i u_{i+1}) = n-i; 1 \leq i \leq n-1$

$= m+n-1-i; n \leq i \leq m-1$

Sub case $ii(b)$ when m is odd and n is even

$f(v) = 2m+2n-2$

$f(u_{2i+1}) = 2i+1; 0 \leq i < \frac{n}{2}$

$= 2m+n+1-2\left(i - \frac{n-2}{2}\right); \frac{n}{2} \leq i < \frac{m+1}{2}$

$f(u_{2i}) = 2n+1-2i; 1 \leq i \leq \frac{n}{2}$

$= 2n-2+2\left(i - \frac{n+2}{2}\right); \frac{n+2}{2} \leq i < \frac{m+1}{2}$

$f(w_k) = 2k; 1 \leq k \leq n-2$

Let f^* be the induced edge labeling of f.

$f^*(vu_{2i+1}) = m+n-1-i; 0 \leq i < \frac{n}{2}$

$f^*(vu_{2i}) = m-1+i; 1 \leq i \leq \frac{n}{2}$

$f^*(u_i u_{i+1}) = n-i; 1 \leq i \leq n-1$

$= m+n-1-i; n \leq i \leq m-1$

Sub case $ii(c)$ when m is even and n is odd

$f(v) = 2m+2n-2$

$f(u_{2i+1}) = 2i+1; 0 \leq i < \frac{n+1}{2}$

$= n+1+2\left(i - \frac{n-1}{2}\right); \frac{n+1}{2} \leq i < \frac{m}{2}$

$f(u_{2i}) = 2n+1-2i; 1 \leq i < \frac{n+1}{2}$

$= 2m+n-2\left(i - \frac{n-1}{2}\right); \frac{n+1}{2} \leq i \leq \frac{m}{2}$

$f(w_k) = 2k; 1 \leq k \leq n-2$

Let f^* be the induced edge labeling of f.

$f^*(vu_{2i+1}) = m+n-1-i; 0 \leq i < \frac{n+1}{2}$

$f^*(vu_{2i}) = m-1+i; 1 \leq i < \frac{n+1}{2}$

$f^*(u_i u_{i+1}) = n-i; 1 \leq i \leq n-1$

$= m+n-1-i; n \leq i \leq m-1$

Sub case $ii(d)$ when m and n are even

$f(v) = 2m + 2n - 2$

$f(u_{2i+1}) = 2i + 1; 0 \leq i < \frac{n}{2}$

$= 2m + n - 2\left(i - \frac{n-2}{2}\right); \frac{n}{2} \leq i < \frac{m}{2}$

$f(u_{2i}) = 2n + 1 - 2i;\ 1 \leq i \leq \frac{n}{2}$

$= 2n - 4 + 2\left(i - \frac{n+2}{2}\right); \frac{n+2}{2} \leq i \leq \frac{m}{2}$

$f(w_k) = 2k; 1 \leq k \leq n - 2$

Let f^* be the induced edge labeling of f.

$f^*(vu_{2i+1}) = m + n - 1 - i; 0 \leq i < \frac{n}{2}$

$f^*(vu_{2i}) = m - 1 + i; 1 \leq i \leq \frac{n}{2}$

$f^*(u_i u_{i+1}) = n - i; 1 \leq i \leq n - 1$

$= m + n - 1 - i; n \leq i \leq m - 1$

Case (iii) when $m < n$

Let $V(G) = \{v, v_i, u_j, w_k; 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq k \leq m-2$ and $E(G) = \{vv_i, u_j u_{j+1}; 1 \leq i \leq n, 1 \leq j \leq m - 1\}$.

Then $|V(G)| = |E(G)| = m + n - 1$ .Let f: $V(G) \rightarrow \{1,2,\dots,2m + 2n - 2\}$ be defined as follows.

Sub case $iii(a)$ when m is odd

$f(v) = 2m + 2n - 2$

$f(u_{2i+1}) = 2i + 1; 0 \leq i < \frac{m+1}{2}$

$f(u_{2i}) = 2m + 1 - 2i;\ 1 \leq i < \frac{m+1}{2}$

$f(u_{m+i}) = 2m - 1 + 2i; 1 \leq i \leq n - m$

$f(w_k) = 2k; 1 \leq k \leq m - 2$

Let f^* be the induced edge labeling of f.

$f^*(vu_{2i+1}) = m + n - 1 - i; 0 \leq i < \frac{m+1}{2}$

$f^*(vu_{2i}) = n - 1 + i; 1 \leq i < \frac{m+1}{2}$

$f^*(vu_{m+i}) = n - i; 1 \leq i \leq n - m$

$f^*(u_i u_{i+1}) = m - i; 1 \leq i \leq m - 1$

Sub case $iii(b)$ when m is even

$f(v) = 2m + 2n - 2$

$f(u_{2i+1}) = 2i + 1; 0 \leq i < \frac{m}{2}$

$f(u_{2i}) = 2m + 1 - 2i;\ 1 \leq i \leq \frac{m}{2}$

$f(u_{m+i}) = 2m - 1 + 2i; 1 \leq i \leq n - m$

$f(w_k) = 2k; 1 \leq k \leq n - 2$

Let f^* be the induced edge labeling of f.

$f^*(vu_{2i+1}) = m + n - 1 - i; 0 \leq i < \frac{m}{2}$

$f^*(vu_{2i}) = n - 1 + i; 1 \leq i \leq \frac{m}{2}$

$f^*(vu_{m+i}) = n - i; 1 \leq i \leq n - m$

$f^*(u_i u_{i+1}) = m - i; 1 \leq i \leq m - 1$

In all the cases the induced edge labels are 1,2,…,m+$n - 1$ which are distinct. ■

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