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Higher Dimensional cosmological Models with Accelerated Expansion in Lyra's Geometry

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ABSTRACT

In this paper we have constructed a higher dimensional (m+1 dimensional) spherically symmetric cosmological model in Lyra's geometry. FRW models of the universe have been studied in the cosmological theory based on Lyra's manifold. Exact solutions of the field equation are obtained with constant acceleration parameters. Some physical properties of the models are also discussed.

KEYWORDS : Cosmology;FRWuniverse, Lyra's geometry

Introduction

The Kaluza-Klein theory has a long and venerable history. However the original kaluza version of this theory suffered from the assumption that the 5-dimensional metric does not depend on the extra coordinate. Hence the proliferation in recent year of versions of Kaluza-Klein theory, upergravity and superstring. In the last years number of author (Wesson 1992), Chatterjee (1994a), Chatterjee (1994b), Chakraborty and Roy (1999) have considered multi dimensional cosmological model. Kaluza-Klein achievements are shown that five dimensional general relativity contains both Einstein's four-dimensional theory of gravity and Maxell's theory of electromagnetism.

"Einstein Universe" is one of Friedman's solutions of Einstein's fiel equations for the value of cosmological constant (). This is only stationary solution of all Friedman's solutions, and because it is stationary, it is thought to be non- physical by majority of astronomers. Those astronomers think that universe is interpreted by those astronomers as a Doppler's shift caused by galaxies moving away from our own Galaxy. Therefore, it is thought that the real solution of Einstein's field equation cannot be stationary. As discussed earlier by many researchers [1] that the constant () cannot explain the huge difference between the cosmological constant inferred from observation and energy density resulting from quantum field theories. In the year 1930 and onwards eminent cosmologists such as A.S. Eddington and Abbe Lemaitre [2,3] felt that the -term introduced certain attractive features into cosmology and that model based on it should also be discussed.

The geometrization of gravitation by Einstein in his general theory of relativity inspired several authors to geometrize other physical fields. Wey l[4] proposed a unified theory to geometrize gravitation and electromagnetism. But due to the non-inerrability of length transfer this theory was never considered seriously. However, this theory inspired Gehard Lyra's to develop what is called Lyra's geometry. Lyra's [5] proposed a new modification of Riemannian geometry by introducing a gauge function of remove the non inerrability of the length of the vector under parallel transport. Subsequently Sen, [6] Sen and Dunn [7] suggested a new scalar –tensor theory of gravitation and constructed an analog of the Einstein field equation based on Lyra's geometry , which in normal gauge be written as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \emptyset_i \emptyset_j - \frac{3}{4} g_{ij} \emptyset_k \emptyset^k = -\chi T_{ij} \qquad (1)$$

Where is the displacement vector c=1, and other symbols their usual meaning in the Riemannian geometry ρ

Halford [8] observation is the constant displacement vector field in Lyra's geometry plays the role of a cosmological constant in the normal general relativistic treatment. Halford [9] showed the scalar –tensor treatment based on Lyra's geometry predicts the same effect, within observation limits as in Einstein's theory. Several authors [10] have studies cosmological models based on Lyra's geometry with a constant displacement field vector. However this restriction of the displacement field to the constant is a coincidence and there is no a priori reason for it. Singh et al. [11] have studies Bianchi I, III, Kantowski –Sachs and a new class of the models with a time dependent displacement field. Recently Pradhan et al. [11] and Rahaman et al. [12] have studied cosmological models based on Lyra's geometry with constant and time time dependent displacement field in different context.

Motivated by the above investigations, we have considered the present study of constructed a higher-dimensional spherically symmetric cosmological model in Lyra's geometry. Exact solution of the field equation

2. Field equation

Here we consider the (m+1) –dimensional spherically symmetric

$$ds^2 = dt^2 - R^2 t \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right] \qquad (2)$$

Where is the line element foe unit (m-1) sphere and k=+1, -1, 0

The (m+1) dimensional time –like displacement vector in (1) is defined as

Ø_i=(β(t),0,0,0……..,0) (3)

The energy –momentum tensor is taken as

$$T_{ij} = (\rho + p) u_i u_j - p g_{ij} \qquad (4)$$

Together with the coordinates satisfying

Where, are pressure energy density and (m+1) dimensional velocity vector of the fluid distribution respectively that is has the components (1,0,0,0,0.................,0)

The field equation (1) has the metric with equation (3) and (4) becomes

$$\left[\frac{1}{2} m(m-1)\right] H^2 + \left[\frac{1}{2} m(m-1)\right] \frac{k}{R^2} - \frac{3}{4} \alpha^2 = \chi\rho \qquad (5)$$

$$\left[(m-1)\dot{H}\right] + \left[\frac{1}{2} m(m-1)\right] H^2 + \left[\frac{1}{2}(m-1)(m-2)\right] \frac{k}{R^2} - \frac{3}{4} \alpha^2 = -\chi p \qquad (6)$$

Where Hubble's parameter and dot is denotes differentiation with respect to t.

Now differentiating equation (5) with respect to t we get

$$\chi\rho = m(m-1) H \dot{H} - m(m-1) k \frac{\dot{R}}{R^3} - \frac{3}{2} \alpha\dot{\alpha} \qquad (7)$$

Adding equation (5) and equation (6) we get

$$\chi(\rho + p) = -(m-1) H + \frac{1}{2}(m-1) \frac{k}{R^2} - \frac{3}{2} \alpha^2 \qquad (8)$$

$$\chi(\rho + p) + \frac{3}{2} \alpha^2 = (m-1) \frac{k}{R^2} - (m-1) \dot{H}$$

From equation (7) and (8) we get

$$\chi\dot{\rho} = mH\left[(m-1)\dot{H} - (m-1)\frac{k}{R^2}\right] - \frac{3}{2}\alpha\dot{\alpha}$$

$$\chi\rho^{\cdot} + \frac{3}{2}\,\alpha\dot{\alpha} = -mH\left[\chi(\rho+p) + \frac{3}{2}\alpha^2\right]$$

$$\chi\rho^{\cdot} + \frac{3}{2}\,\alpha\dot{\alpha} + mH\left[\chi(\rho+p) + \frac{3}{2}\alpha^2\right] = 0 \qquad (9)$$

tEquation is the continuity.

3. Solution of field equation

There are two field equation with four unknown viz R, ρ, p,

Assuming the equation of state

$p=\gamma\rho, \quad 0\le\gamma\le1$ (10)

The number of unknowns is reduced to three

Making use of equation of state and eliminating (t) form equation (5) and (6) we get

$$(m-1)H^{\cdot} + \frac{1}{2}[m(m-1)(1+\gamma)]H^2 + \frac{1}{2}[(m-1)(m-2)+m\gamma]\frac{k}{R^2} + \frac{3}{4}\alpha^2(1-\gamma)=0 \qquad (11)$$

Here plays the role of cosmological term Λ (t) as one can see from field equations with cosmological term. There are three known and two independent equations (5) and (6). To obtain a unique solution, one more equation is needed. So we have considered the acceleration parameter to be constant.

We consider the acceleration parameter to be variable

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -\left(\frac{\dot{H}+H^2}{H^2}\right) = b \qquad (12)$$

The equation may be rewritten as

$$\frac{\ddot{R}}{R} + b\,\frac{\dot{R}^2}{R^2} = 0 \qquad (13)$$

The general solution of equation

$$\int e^{\int\frac{b}{R}dR}\,dR = t + c \qquad (14)$$

Where c is an integrating constant.

In order to solve the problem completely we have to chose in such a manner equation (14) be integral

Without any loss of generality, we consider

$$\int\frac{b}{R}\,dR = In\ L(R) \qquad (15)$$

This does not effect of nature of generality of solution.

Form equation (14) and (15) we obtain

$\int L(R)dR=t+c$ (16)

Where L(R) is quit arbitrary

$$\alpha^2 = \frac{2(m-1)}{3(\gamma-1)}\left[((m-2)+m(\gamma-2b))H^2 + ((m-2)+m\gamma)\frac{k}{R^2}\right] \qquad (17)$$

$$\chi\rho = \frac{m-1}{\gamma-1}\left[\left\{b-(m-1)H^2-(m-1)\frac{k}{R^2}\right\}\right] \qquad (18)$$

4. Flat models

With k=0 equation (17) and (18) reduced to

$$\alpha^2 = \frac{2(m-1)}{3(\gamma-1)}[((m-2)+m(\gamma-2b))H^2]$$

And (19)

$$\chi\rho = \frac{m-1}{\gamma-1}[(b-(m-1)H^2)] \qquad (20)$$

From equation (20) we see

5. Empty universe

Here p

Also we know that k=0 in equation (5) and (6) become

$$\left[\frac{1}{2}m(m-1)\right]H^2 - \frac{3}{4}\alpha^2 = 0 \qquad (21)$$

And

$$\left[(m-1)\dot{H} + \frac{1}{2}m(m-1)H^2 + \frac{3}{4}\alpha^2 = 0\right]$$

$$H + m\dot{H}^2 = 0 \qquad (22)$$

(22)On compare to equation (10) and (22)

We using equation (21) we get

$$\alpha^2 = \frac{2}{3}m(m-1)H^2 \qquad (23)$$

Thus equation (23), we observe that

6. Solution in the exponential form

We consider L(R) = where c1 is the arbitrary constant. We have integrating of equation (16) and its exact solution is given by

R (t) = $c_2 e^{\wedge c1t}$

Where is also arbitrary constant? We have using equation (11) and (5) we discussed of nature of and pressure and energy density p as

2 H˙+3(1+γ) H^2+(1+3γ) k/R^2 +3/4 α^2 (1-γ)=0

We have solving this equation we obtain the value of

$$\alpha^2 = -\frac{4(1+\gamma)}{(1-\gamma)}H^2 - \frac{4(1+3\gamma)}{3(1-\gamma)}\frac{k}{R^2}$$

Also we express the equation we have

$$\alpha^2 = -\frac{4(1+\gamma)c_1^{\,2}}{(1-\gamma)} - \frac{4(1+3\gamma)}{3(1-\gamma)}\frac{k}{c_2^{\,2}e^{2c1t}}$$

Putting the value of in equation number (5) we get

$$\chi\gamma\rho = \frac{2\gamma}{(1-\gamma)}\left[3c_1^{\,2} + \frac{2k}{c_2^{\,2}e^{2c1t}}\right]$$

Since Scale factor cannot be negative so R (t) is positive if the scale factor of the universe had been increased very slowly and universe had been accelerating. This is consistent with Big Bang scenario.

Conclusion

In this paper we have tried to present (m+1) dimensional spherically symmetric FRW cosmological models in Lyra's geometry. We have also discussed exact solution for constant accelerated parameter. Also we have tried to present q is time dependent and hence may be change it during cosmic evolution. Also we have obtained exact solution equation in Lyra's geometry for time dependent acceleration parameter in FRW space time. We have considered for different models (1) flat models (ii) empty models (iii) exponential form.

In recent past, there is an upsurge of interest in scalar fields in general relativity of inflationary cosmology therefore; the study of cosmological models in Lyra's geometry may be relevant for inflationary models. Further, the same dependence of the displacement field is important for inhomogeneous models for the early stages of evolution of the universe.

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