Volume-3, Issue-11, Nov-2014 • ISSN No 2277 - 8160

Research Paper | Statistics

Analysis and General Solution of Normal Equations for Five Associate Class PBIB Designs

Davinder Kumar Garg — Associate Professor and Head, Department of Statistics, Punjabi University, Patiala

Gurinder Pal Singh — Lecturer Department of Applied Sciences B.G.I.E.T, Sangrur

ABSTRACT *With the availability of the general solution of the normal equations in the intra – block analysis of PBIB designs, it becomes very easy to get the estimates of various treatment effects and also to get the estimates of different elementary treatment contrasts. In this paper, we have attempted to find the general solution of the normal equations in the intra – block analysis of PBIB designs having five associate classes which is not available in the available literature. Here, we have provided the general solution of normal equations of PBIB designs with five associate classes by solving five normal equations simultaneously and get the efficiencies factors for five kinds of comparisons as well as average efficiency factor (A.E.F).*

KEYWORDS : Partially balanced incomplete block designs, association class, normal equations, intra – block analysis

Introduction

With the construction of new PBIB designs having higher associate classes, the need arises to analysis these designs. The general analysis of PBIB designs with m (m ≥ 2) associate classes and the general solution of the normal equations in the intra – block analysis and general solution of normal equations for (m=2) two associate class PBIB designs given by Dey (1986) is available in literature. Also Rao (1947) provides the general solution of normal equations of three associate classes PBIB designs. Thereafter, the general solution of normal equations for four associate class PBIB designs was given by Garg and Mishra (2013). After studying the available literature, we have successfully extended the literature and obtained the general solution of the normal equations in the intra – block analysis of PBIB designs for five associate class PBIB designs. We have verified it for m = 2,3,4 and also obtained general solution for five associate class PBIB designs. For m = 2, solution obtained is equivalent to Dey (1986) for m= 3 and m= 4, solution obtained are equivalent to Rao (1947b) and Garg and Mishra (2013) respectively.

2. General Analysis of Higher Associate Class PBIB Design

Let us first reproduce an m associate class PBIB designs having v treatments in b blocks with plot size k and replicates r. Then, reduced normal equations of intra block model for treatment effects are

Let us first reproduce an m associate class PBIB designs having v treatments in b blocks with plot size k and replicates r. Then, reduced normal equations of intra block model for treatment effects are

r(k-1)τi – λ1S1(τi) - λ 2S2(τi) - … - λ mSm(τi) = k Qi --- (2.1)

where i = 1,2,…,m.

Sj(τi) represent the sum of treatment effects those nj treatments which are j – th associates of treatment 'i' for j = 1,2,…,m.

Suppose, nj be the jth associates of ith treatment effects αi1, αi2 ,…, αinj .

Now, Rewriting the normal equations in respect of αi1, αi2 ,…, αinj , we get

r(k-1) αi1 – λ1S1(αi1) - λ 2S2(αi1) - λ 3S3(αi1) - … - λ mSm(αi1) = k Q(αi1)

r(k-1) αi2 – λ1S1(αi2) - λ 2S2(αi2) - λ 3S3(αi2)- … - λ mSm(αi2) = k Q(αi2)

r(k-1) αi3 – λ1S1(αi3) - λ 2S2(αi3) - λ 3S3(αi3) - … - λ mSm(αi3) = k Q(αi3)

. . . . . .

r(k-1) αinj – λ1S1(αinj) - λ 2S2(αinj) - λ 3S3(αinj) - … - λ mSm(αinj) = k Q(αinj)

--- (2.2)

Summing over n_j equations as given above , we get an equation given below as

$r(k-1)\, S_j(\tau_i) - \lambda_1 P_1 - \lambda_2 P_2 - \lambda_3 P_3 - \dots - \lambda_m P_m = k\, S_j\,(Q_i)$ --- (2.3)

where $P_1, P_2, \dots, P_m$ are linear functions of treatment effects and $S_j\,(Q_i)$. Now, from the definition of a PBIB design, we write

Pu =∑ptjuSt(τi) for u ≠ j , u =1, 2,…, m.

Pu = njτi +∑ptjuSt(τi)

Now, substituting for P_u's in above equation (7.3) and rearrange, we get the equation as given below

r(k-1) Sj(τi) – S1(τi)∑ λ t p1jt–S2(τi) ∑ λ t p2jt – S3(τi) ∑ λ t p3jt - …– Sm(τi) ∑ λ t pmjt - nj λ j τi = k Sj (Qi) --- (2.4)

and solved these equations to get a solution for τ_i by taking the restriction for conveniently is

$\tau_i + S_1(\tau_i) + S_2(\tau_i) + S_3(\tau_i) + \dots + S_m(\tau_i) = 0$

---(2.5)

3. Reduced Normal Equations for Five Associate Class PBIB Design

Let us consider a five associate class PBIB design (m = 5) having 'v' treatments, 'b' blocks, 'r' replications with 'k' block size. Applying the least square method of estimation, the reduced normal equation of linear model are given by

$r(k-1)\, \tau_i - [\, \lambda_1 S_1(\tau_i) + \lambda_2 S_2(\tau_i) + \lambda_3 S_3(\tau_i) + \lambda_4 S_4(\tau_i) + \lambda_5 S_5(\tau_i)] = k\, Q_i$ ---(3.1)

where i = 1,2,3,….v and $S_j(\tau_i)$ denoted the sum of those n_j treatment effects which are the j^{th} associates of i, for j =1,2,3,4,5. Now, let the n_j , j^{th} associates of i^{th} treatment effects $\alpha^i_1, \alpha^i_2, \dots, \alpha^i_{nj}$. Rewriting the normal equations in respect of $\alpha^i_1, \alpha^i_2, \dots, \alpha^i_{nj}$, we get

$r(k\text{-}1)\,\alpha^i_1 - \lambda_1 S_1(\alpha^i_1) - \lambda_2 S_2(\alpha^i_1) - \lambda_3 S_3(\alpha^i_1) - \lambda_4 S_4(\alpha^i_1) - \lambda_5 S_5(\alpha^i_1) = k\,Q(\alpha^i_1)$

$r(k\text{-}1)\,\alpha^i_2 - \lambda_1 S_1(\alpha^i_2) - \lambda_2 S_2(\alpha^i_2) - \lambda_3 S_3(\alpha^i_2) - \lambda_4 S_4(\alpha^i_2) - \lambda_5 S_5(\alpha^i_2) = k\,Q(\alpha^i_2)$

$r(k\text{-}1)\,\alpha^i_3 - \lambda_1 S_1(\alpha^i_3) - \lambda_2 S_2(\alpha^i_3) - \lambda_3 S_3(\alpha^i_3) - \lambda_4 S_4(\alpha^i_3) - \lambda_5 S_5(\alpha^i_3) = k\,Q(\alpha^i_3)$

. . . . . .

$r(k\text{-}1)\,\alpha^i_{nj} - \lambda_1 S_1(\alpha^i_{nj}) - \lambda_2 S_2(\alpha^i_{nj}) - \lambda_3 S_3(\alpha^i_{nj}) - \lambda_4 S_4(\alpha^i_{nj}) - \lambda_5 S_5(\alpha^i_{nj}) = k\,Q(\alpha^i_{nj})$ --- (3.2)

Summing over nj equations in set (7.2.2) for j = 1,2,3,4,5 we get an equation as

$r(k\text{-}1)\,S_j(\tau_i) - \lambda_1 P_1 - \lambda_2 P_2 - \lambda_3 P_3 - \lambda_4 P_4 - \lambda_5 P_5 = k\,S_j(Q_i)$ ---(3.3)

where P_1, P_2, P_3, P_4 and P_5 are linear functions of treatment effects and $S_j(Q_i)$. From the definition of a PBIB design, we write

Pu =∑ptjuSt(τi) for u ≠ j , u =1, 2,3,4,5

Pj = nji + by Pu = njτi +∑ptjuSt(τi)

Now, substituting for P_u's in equation (3.3) and rearranging, we get equations as given below

r(k-1) Sj(τi) – S1(τi)∑ λ t p1jt–S2(τi) ∑ λ t p2jt – S3(τi) ∑ λ t p3jt -S4(τi) ∑ λ t p4jt -S5(τi) ∑ λ tp5jt - nj λ j τi = k Sj (Qi) --- (3.4)

Now, we expand equation (3.4) for j = 1,2,3,4,5 as given below respectively

$r(k\text{-}1)\,S_1(\tau_i) - S_1(\tau_i)\,[\lambda_1 p^1_{11} + \lambda_2 p^1_{12} + \lambda_3 p^1_{13} + \lambda_4 p^1_{14} + \lambda_5 p^1_{15}] - S_2(\tau_i)\,[\lambda_1 p^2_{11} + \lambda_2 p^2_{12} + \lambda_3 p^2_{13} + \lambda_4 p^2_{14} + \lambda_5 p^2_{15}] - S_3(\tau_i)\,[\lambda_1 p^3_{11} + \lambda_2 p^3_{12} + \lambda_3 p^3_{13} + \lambda_4 p^3_{14} + \lambda_5 p^3_{15}] - S_4(\tau_i)\,[\lambda_1 p^4_{11} + \lambda_2 p^4_{12} + \lambda_3 p^4_{13} + \lambda_4 p^4_{14} + \lambda_5 p^4_{15}] - S_5(\tau_i)\,[\lambda_1 p^5_{11} + \lambda_2 p^5_{12} + \lambda_3 p^5_{13} + \lambda_4 p^5_{14} + \lambda_5 p^5_{15}] - n_1 \lambda_1 \tau_i = k\,S_1(Q_i)$ ---(3.5)

$r(k\text{-}1)\,S_2(\tau_i) - S_1(\tau_i)\,[\lambda_1 p^1_{21} + \lambda_2 p^1_{22} + \lambda_3 p^1_{23} + \lambda_4 p^1_{24} + \lambda_5 p^1_{25}] - S_2(\tau_i)\,[\lambda_1 p^2_{21} + \lambda_2 p^2_{22} + \lambda_3 p^2_{23} + \lambda_4 p^2_{24} + \lambda_5 p^2_{25}] - S_3(\tau_i)\,[\lambda_1 p^3_{21} + \lambda_2 p^3_{22} + \lambda_3 p^3_{23} + \lambda_4 p^3_{24} + \lambda_5 p^3_{25}] - S_4(\tau_i)\,[\lambda_1 p^4_{21} + \lambda_2 p^4_{22} + \lambda_3 p^4_{23} + \lambda_4 p^4_{24} + \lambda_5 p^4_{25}] - S_5(\tau_i)\,[\lambda_1 p^5_{21} + \lambda_2 p^5_{22} + \lambda_3 p^5_{23} + \lambda_4 p^5_{24} + \lambda_5 p^5_{25}] - n_2 \lambda_2 \tau_i = k\,S_2(Q_i)$ --- (3.6)

$r(k\text{-}1)\,S_3(\tau_i) - S_1(\tau_i)\,[\lambda_1 p^1_{31} + \lambda_2 p^1_{32} + \lambda_3 p^1_{33} + \lambda_4 p^1_{34} + \lambda_5 p^1_{35}] - S_2(\tau_i)\,[\lambda_1 p^2_{31} + \lambda_2 p^2_{32} + \lambda_3 p^2_{33} + \lambda_4 p^2_{34} + \lambda_5 p^2_{35}] - S_3(\tau_i)\,[\lambda_1 p^3_{31} + \lambda_2 p^3_{32} + \lambda_3 p^3_{33} + \lambda_4 p^3_{34} + \lambda_5 p^3_{35}] - S_4(\tau_i)\,[\lambda_1 p^4_{31} + \lambda_2 p^4_{32} + \lambda_3 p^4_{33} + \lambda_4 p^4_{34} + \lambda_5 p^4_{35}] - S_5(\tau_i)\,[\lambda_1 p^5_{31} + \lambda_2 p^5_{32} + \lambda_3 p^5_{33} + \lambda_4 p^5_{34} + \lambda_5 p^5_{35}] - n_3 \lambda_3 \tau_i = k\,S_3(Q_i)$ --- (3.7)

$r(k\text{-}1)\,S_4(\tau_i) - S_1(\tau_i)\,[\lambda_1 p^1_{41} + \lambda_2 p^1_{42} + \lambda_3 p^1_{43} + \lambda_4 p^1_{44} + \lambda_5 p^1_{45}] - S_2(\tau_i)\,[\lambda_1 p^2_{41} + \lambda_2 p^2_{42} + \lambda_3 p^2_{43} + \lambda_4 p^2_{44} + \lambda_5 p^2_{45}] - S_3(\tau_i)\,[\lambda_1 p^3_{41} + \lambda_2 p^3_{42} + \lambda_3 p^3_{43} + \lambda_4 p^3_{44} + \lambda_5 p^3_{45}] - S_4(\tau_i)\,[\lambda_1 p^4_{41} + \lambda_2 p^4_{42} + \lambda_3 p^4_{43} + \lambda_4 p^4_{44} + \lambda_5 p^4_{45}] - S_5(\tau_i)\,[\lambda_1 p^5_{41} + \lambda_2 p^5_{42} + \lambda_3 p^5_{43} + \lambda_4 p^5_{44} + \lambda_5 p^5_{45}] - n_4 \lambda_4 \tau_i = k\,S_4(Q_i)$ ---(3.8)

$r(k\text{-}1)\,S_5(\tau_i) - S_1(\tau_i)\,[\lambda_1 p^1_{51} + \lambda_2 p^1_{52} + \lambda_3 p^1_{53} + \lambda_4 p^1_{54} + \lambda_5 p^1_{55}] - S_2(\tau_i)\,[\lambda_1 p^2_{51} + \lambda_2 p^2_{52} + \lambda_3 p^2_{53} + \lambda_4 p^2_{54} + \lambda_5 p^2_{55}] - S_3(\tau_i)\,[\lambda_1 p^3_{51} + \lambda_2 p^3_{52} + \lambda_3 p^3_{53} + \lambda_4 p^3_{54} + \lambda_5 p^3_{55}] - S_4(\tau_i)\,[\lambda_1 p^4_{51} + \lambda_2 p^4_{52} + \lambda_3 p^4_{53} + \lambda_4 p^4_{54} + \lambda_5 p^4_{55}] - S_5(\tau_i)\,[\lambda_1 p^5_{51} + \lambda_2 p^5_{52} + \lambda_3 p^5_{53} + \lambda_4 p^5_{54} + \lambda_5 p^5_{55}] - n_5 \lambda_5 \tau_i = k\,S_5(Q_i)$ ---(3.9)

For simplicity, we can solved these equation by taking a condition or restriction

$\tau_i + S_1(\tau_i) + S_2(\tau_i) + S_3(\tau_i) + S_4(\tau_i) + S_5(\tau_i) = 0$

---(3.10)

4. Solution of Normal Equations for Five Associate Class PBIB Design

To get the solution of τ_i for m=5, equation (2.1) can be written as

$r(k\text{-}1)\,\tau_i - \lambda_1 S_1(\tau_i) - \lambda_2 S_2(\tau_i) - \lambda_3 S_3(\tau_i) - \lambda_4 S_4(\tau_i) - \lambda_5 S_5(\tau_i) = k\,Q_i$

Substitute '$S_5(\tau_i)$' from equation (3.10) in above equation and we get

$r(k\text{-}1)\,\tau_i - \lambda_1 S_1(\tau_i) - \lambda_2 S_2(\tau_i) - \lambda_3 S_3(\tau_i) - \lambda_4 S_4(\tau_i) + \lambda_5 [\tau_i + S_1(\tau_i) + S_2(\tau_i) + S_3(\tau_i) + S_4(\tau_i)] = k\,Q_i$

or

$[r(k\text{-}1) + \lambda_5]\,\tau_i + (\lambda_5 - \lambda_1)\,S_1(\tau_i) + (\lambda_5 - \lambda_2)\,S_2(\tau_i) + (\lambda_5 - \lambda_3)\,S_3(\tau_i) + (\lambda_5 - \lambda_4)\,S_4(\tau_i) = k\,Q_i$ ---(4.1)

Substitute '$S_5(\tau_i)$' from equation (3.10) in equation (3.5), we get

$r(k\text{-}1)\,S_1(\tau_i) - S_1(\tau_i)\,[\lambda_1 p^1_{11} + \lambda_2 p^1_{12} + \lambda_3 p^1_{13} + \lambda_4 p^1_{14} + \lambda_5 p^1_{15}] - S_2(\tau_i)\,[\lambda_1 p^2_{11} + \lambda_2 p^2_{12} + \lambda_3 p^2_{13} + \lambda_4 p^2_{14} + \lambda_5 p^2_{15}] - S_3(\tau_i)\,[\lambda_1 p^3_{11} + \lambda_2 p^3_{12} + \lambda_3 p^3_{13} + \lambda_4 p^3_{14} + \lambda_5 p^3_{15}] - S_4(\tau_i)\,[\lambda_1 p^4_{11} + \lambda_2 p^4_{12} + \lambda_3 p^4_{13} + \lambda_4 p^4_{14} + \lambda_5 p^4_{15}] + [\tau_i + S_1(\tau_i) + S_2(\tau_i) + S_3(\tau_i) + S_4(\tau_i)]\,[\lambda_1 p^5_{11} + \lambda_2 p^5_{12} + \lambda_3 p^5_{13} + \lambda_4 p^5_{14} + \lambda_5 p^5_{15}] - n_1 \lambda_1 \tau_i = k\,S_1(Q_i)$

or

$[\lambda_1 p^5_{11} + \lambda_2 p^5_{12} + \lambda_3 p^5_{13} + \lambda_4 p^5_{14} + \lambda_5 p^5_{15} - n_1 \lambda_1]\,\tau_i + S_1(\tau_i)\,[\lambda_1 p^5_{11} + \lambda_2 p^5_{12} + \lambda_3 p^5_{13} + \lambda_4 p^5_{14} + \lambda_5 p^5_{15} - (\lambda_1 p^1_{11} + \lambda_2 p^1_{12} + \lambda_3 p^1_{13} + \lambda_4 p^1_{14} + \lambda_5 p^1_{15}) + r(k\text{-}1)] + S_2(\tau_i)\,[\lambda_1 p^5_{11} + \lambda_2 p^5_{12} + \lambda_3 p^5_{13} + \lambda_4 p^5_{14} + \lambda_5 p^5_{15} - (\lambda_1 p^2_{11} + \lambda_2 p^2_{12} + \lambda_3 p^2_{13} + \lambda_4 p^2_{14} + \lambda_5 p^2_{15})] + S_3(\tau_i)\,[\lambda_1 p^5_{11} + \lambda_2 p^5_{12} + \lambda_3 p^5_{13} + \lambda_4 p^5_{14} + \lambda_5 p^5_{15} - (\lambda_1 p^3_{11} + \lambda_2 p^3_{12} + \lambda_3 p^3_{13} + \lambda_4 p^3_{14} + \lambda_5 p^3_{15})] + S_4(\tau_i)\,[\lambda_1 p^5_{11} + \lambda_2 p^5_{12} + \lambda_3 p^5_{13} + \lambda_4 p^5_{14} + \lambda_5 p^5_{15} - (\lambda_1 p^4_{11} + \lambda_2 p^4_{12} + \lambda_3 p^4_{13} + \lambda_4 p^4_{14} + \lambda_5 p^4_{15})] = k\,S_1(Q_i)$

using

$n_1 = p^5_{11} + p^5_{12} + p^5_{13} + p^5_{14} + p^5_{15}$

$p^1_{15} = n_1 - 1 - p^1_{11} - p^1_{12} - p^1_{13} - p^1_{14}$

$p^2_{15} = n_1 - p^2_{11} - p^2_{12} - p^2_{13} - p^2_{14}$

$p^3_{15} = n_1 - p^3_{11} - p^3_{12} - p^3_{13} - p^3_{14}$

$p^4_{15} = n_1 - p^4_{11} - p^4_{12} - p^4_{13} - p^4_{14}$

$p^5_{15} = n_1 - p^5_{11} - p^5_{12} - p^5_{13} - p^5_{14}$

in above equation (3.5), it becomes and after solving it , we get

$[(\lambda_2 - \lambda_1)p^5_{12} + (\lambda_3 - \lambda_1)p^5_{13} + (\lambda_4 - \lambda_1)p^5_{14} + (\lambda_5 - \lambda_1)p^5_{15}]\,\tau_i + [r(k\text{-}1) + \lambda_5 + (\lambda_5 - \lambda_1)(p^1_{11} - p^5_{11}) + (\lambda_5 - \lambda_2)(p^1_{12} - p^5_{12}) + (\lambda_5 - \lambda_3)(p^1_{13} - p^5_{13}) + (\lambda_5 - \lambda_4)(p^1_{14} - p^5_{14})]\,S_1(\tau_i) + [(\lambda_5 - \lambda_1)(p^2_{11} - p^5_{11}) + (\lambda_5 - \lambda_2)(p^2_{12} - p^5_{12}) + (\lambda_5 - \lambda_3)(p^2_{13} - p^5_{13}) + (\lambda_5 - \lambda_4)(p^2_{14} - p^5_{14})]\,S_2(\tau_i) + [(\lambda_5 - \lambda_1)(p^3_{11} - p^5_{11}) + (\lambda_5 - \lambda_2)(p^3_{12} - p^5_{12}) + (\lambda_5 - \lambda_3)(p^3_{13} - p^5_{13}) + (\lambda_5 - \lambda_4)(p^3_{14} - p^5_{14})]\,S_3(\tau_i) + [(\lambda_5 - \lambda_1)(p^4_{11} - p^5_{11}) + (\lambda_5 - \lambda_2)(p^4_{12} - p^5_{12}) + (\lambda_5 - \lambda_3)(p^4_{13} - p^5_{13}) + (\lambda_5 - \lambda_4)(p^4_{14} - p^5_{14})]\,S_4(\tau_i) = k\,S_1(Q_i)$ --- (4.2)

Substitute '$S_5(\tau_i)$' from equation (3.10) in equation (3.6), we get

$r(k\text{-}1)\,S_2(\tau_i) - S_1(\tau_i)\,[\lambda_1 p^1_{21} + \lambda_2 p^1_{22} + \lambda_3 p^1_{23} + \lambda_4 p^1_{24} + \lambda_5 p^1_{25}] - S_2(\tau_i)\,[\lambda_1 p^2_{21} + \lambda_2 p^2_{22} + \lambda_3 p^2_{23} + \lambda_4 p^2_{24} + \lambda_5 p^2_{25}] - S_3(\tau_i)\,[\lambda_1 p^3_{21} + \lambda_2 p^3_{22} + \lambda_3 p^3_{23} + \lambda_4 p^3_{24} + \lambda_5 p^3_{25}] - S_4(\tau_i)\,[\lambda_1 p^4_{21} + \lambda_2 p^4_{22} + \lambda_3 p^4_{23} + \lambda_4 p^4_{24} + \lambda_5 p^4_{25}] + [\tau_i + S_1(\tau_i) + S_2(\tau_i) + S_3(\tau_i) + S_4(\tau_i)]\,[\lambda_1 p^5_{21} + \lambda_2 p^5_{22} + \lambda_3 p^5_{23} + \lambda_4 p^5_{24} + \lambda_5 p^5_{25}] - n_2 \lambda_2 \tau_i = k\,S_2(Q_i)$

or

$[\lambda_1 p^5_{21} + \lambda_2 p^5_{22} + \lambda_3 p^5_{23} + \lambda_4 p^5_{24} + \lambda_5 p^5_{25} - n_2 \lambda_2]\,\tau_i + [\lambda_1 p^5_{21} + \lambda_2 p^5_{22} + \lambda_3 p^5_{23} + \lambda_4 p^5_{24} + \lambda_5 p^5_{25} - (\lambda_1 p^1_{21} + \lambda_2 p^1_{22} + \lambda_3 p^1_{23} + \lambda_4 p^1_{24} + \lambda_5 p^1_{25})]\,S_1(\tau_i) + [r(k\text{-}1) - \lambda_1 p^2_{21} - \lambda_2 p^2_{22} - \lambda_3 p^2_{23} - \lambda_4 p^2_{24} - \lambda_5 p^2_{25} + \lambda_1 p^5_{21} + \lambda_2 p^5_{22} + \lambda_3 p^5_{23} + \lambda_4 p^5_{24} + \lambda_5 p^5_{25}]\,S_2(\tau_i) + [\lambda_1 p^5_{21} + \lambda_2 p^5_{22} + \lambda_3 p^5_{23} + \lambda_4 p^5_{24} + \lambda_5 p^5_{25} - \lambda_1 p^3_{21} - \lambda_2 p^3_{22} - \lambda_3 p^3_{23} - \lambda_4 p^3_{24} - \lambda_5 p^3_{25}]\,S_3(\tau_i) + [\lambda_1 p^5_{21} + \lambda_2 p^5_{22} + \lambda_3 p^5_{23} + \lambda_4 p^5_{24} + \lambda_5 p^5_{25} - \lambda_1 p^4_{21} - \lambda_2 p^4_{22} - \lambda_3 p^4_{23} - \lambda_4 p^4_{24} - \lambda_5 p^4_{25}]\,S_4(\tau_i)] = k\,S_2(Q_i)$

using

$n_2 = p^5_{21} + p^5_{22} + p^5_{23} + p^5_{24} + p^5_{25}$

$p^1_{25} = n_2 - p^1_{21} - p^1_{22} - p^1_{23} - p^1_{24}$

$p^2_{25} = n_2 - 1 - p^2_{21} - p^2_{22} - p^2_{23} - p^2_{24}$

$p^3_{25} = n_2 - p^3_{21} - p^3_{22} - p^3_{23} - p^3_{24}$

$p^4_{25} = n_2 - p^4_{21} - p^4_{22} - p^4_{23} - p^4_{24}$

$p^5_{25} = n_2 - p^5_{21} - p^5_{22} - p^5_{23} - p^5_{24}$

in above equation (3.6), it becomes and after solving it

, we get

$$[(\lambda_1 - \lambda_2) p^5_{21} + (\lambda_3 - \lambda_2) p^5_{23} + (\lambda_4 - \lambda_2) p^5_{24} + (\lambda_5 - \lambda_2) p^5_{25}] \tau_i + [(\lambda_5 - \lambda_1)(p^1_{21} - p^5_{21}) + (\lambda_5 - \lambda_2)(p^1_{22} - p^5_{22}) + (\lambda_5 - \lambda_3)(p^1_{23} - p^5_{23}) + (\lambda_5 - \lambda_4)(p^1_{24} - p^5_{24})] S_1(\tau_i) +[r(k-1) + \lambda_5 + (\lambda_5 - \lambda_1)(p^2_{21} - p^5_{21}) + (\lambda_5 - \lambda_2)(p^2_{22} - p^5_{22}) + (\lambda_5 - \lambda_3)(p^2_{23} - p^5_{23}) + (\lambda_5 - \lambda_4)(p^2_{24} - p^5_{24})] S_2(\tau_i) + [(\lambda_5 - \lambda_1)(p^3_{21} - p^5_{21}) + (\lambda_5 - \lambda_2)(p^3_{22} - p^5_{22}) + (\lambda_5 - \lambda_3)(p^3_{23} - p^5_{23}) + (\lambda_5 - \lambda_4)(p^3_{24} - p^5_{24})] S_3(\tau_i) + [(\lambda_5 - \lambda_1)(p^4_{21} - p^5_{21}) + (\lambda_5 - \lambda_2)(p^4_{22} - p^5_{22}) + (\lambda_5 - \lambda_3)(p^4_{23} - p^5_{23}) + (\lambda_5 - \lambda_4)(p^4_{24} - p^5_{24})] S_4(\tau_i) = k S_2 (Q_i) \quad \text{---(4.3)}$$

Equation (3.7) becomes

$$r(k-1) S_3(\tau_i) - S_1(\tau_i) [\lambda_1 p^1_{31} + \lambda_2 p^1_{32} + \lambda_3 p^1_{33} + \lambda_4 p^1_{34} + \lambda_5 p^1_{35}] - S_2(\tau_i) [\lambda_1 p^2_{31} + \lambda_2 p^2_{32} + \lambda_3 p^2_{33} + \lambda_4 p^2_{34} + \lambda_5 p^2_{35}] - S_3(\tau_i) [\lambda_1 p^3_{31} + \lambda_2 p^3_{32} + \lambda_3 p^3_{33} + \lambda_4 p^3_{34} + \lambda_5 p^3_{35}] - S_4(\tau_i) [\lambda_1 p^4_{31} + \lambda_2 p^4_{32} + \lambda_3 p^4_{33} + \lambda_4 p^4_{34} + \lambda_5 p^4_{35}] + [\tau_i + S_1(\tau_i) + S_2(\tau_i) + S_3(\tau_i) + S_4(\tau_i)] [\lambda_1 p^5_{31} + \lambda_2 p^5_{32} + \lambda_3 p^5_{33} + \lambda_4 p^5_{34} + \lambda_5 p^5_{35}] - n_3 \lambda_3 \tau_i = k S_3 (Q_i)$$

or

$$[\lambda_1 p^5_{31} + \lambda_2 p^5_{32} + \lambda_3 p^5_{33} + \lambda_4 p^5_{34} + \lambda_5 p^5_{35} - n_3 \lambda_3] \tau_i + [\lambda_1 p^5_{31} + \lambda_2 p^5_{32} + \lambda_3 p^5_{33} + \lambda_4 p^5_{34} + \lambda_5 p^5_{35} - (\lambda_1 p^1_{31} + \lambda_2 p^1_{32} + \lambda_3 p^1_{33} + \lambda_4 p^1_{34} + \lambda_5 p^1_{35})] S_1(\tau_i) + [\lambda_1 p^5_{31} + \lambda_2 p^5_{32} + \lambda_3 p^5_{33} + \lambda_4 p^5_{34} + \lambda_5 p^5_{35} -(\lambda_1 p^2_{31} + \lambda_2 p^2_{32} + \lambda_3 p^2_{33} + \lambda_4 p^2_{34} + \lambda_5 p^2_{35})] S_2(\tau_i) + [r(k-1) - \lambda_1 p^3_{31} - \lambda_2 p^3_{32} - \lambda_3 p^3_{33} - \lambda_4 p^3_{34} - \lambda_5 p^3_{35} + \lambda_1 p^5_{31} + \lambda_2 p^5_{32} + \lambda_3 p^5_{33} + \lambda_4 p^5_{34} + \lambda_5 p^5_{35}] S_3(\tau_i) + [\lambda_1 p^5_{31} + \lambda_2 p^5_{32} + \lambda_3 p^5_{33} + \lambda_4 p^5_{34} + \lambda_5 p^5_{35} - \lambda_1 p^4_{31} - \lambda_2 p^4_{32} - \lambda_3 p^4_{33} - \lambda_4 p^4_{34} - \lambda_5 p^4_{35}] S_4(\tau_i) = k S_3 (Q_i)$$

C-N

using

$n_3 = p^5_{31} + p^5_{32} + p^5_{33} + p^5_{34} + p^5_{35}$

$p^1_{35} = n_3 - p^1_{31} - p^1_{32} - p^1_{33} - p^1_{34}$

$p^2_{35} = n_3 - p^2_{31} - p^2_{32} - p^2_{33} - p^2_{34}$

$p^3_{35} = n_3 -1 - p^3_{31} - p^3_{32} - p^3_{33} - p^3_{34}$

$p^4_{35} = n_3 - p^4_{31} - p^4_{32} - p^4_{33} - p^4_{34}$

$p^5_{35} = n_3 - p^5_{31} - p^5_{32} - p^5_{33} - p^5_{34}$

in above equation (3.7), it becomes and after solving it , we get

$$[(\lambda_1 - \lambda_3) p^5_{31} + (\lambda_2 - \lambda_3) p^5_{32} + (\lambda_4 - \lambda_3) p^5_{34} + (\lambda_5 - \lambda_3) p^5_{35}] \tau_i + [(\lambda_5 - \lambda_1)(p^1_{31} - p^5_{31}) + (\lambda_5 - \lambda_2)(p^1_{32} - p^5_{32}) +(\lambda_5 - \lambda_3)(p^1_{33} - p^5_{33}) + (\lambda_5 - \lambda_4)(p^1_{34} - p^5_{34})] S_1(\tau_i) +[(\lambda_5 - \lambda_1)(p^2_{31} - p^5_{31}) + (\lambda_5 - \lambda_2)(p^2_{32} - p^5_{32}) +(\lambda_5 - \lambda_3)(p^2_{33} - p^5_{33}) + (\lambda_5 - \lambda_4)(p^2_{34} - p^5_{34})] S_2(\tau_i) + [r(k-1) + \lambda_5 + (\lambda_5 - \lambda_1)(p^3_{31} - p^5_{31}) + (\lambda_5 - \lambda_2)(p^3_{32} - p^5_{32}) +(\lambda_5 - \lambda_3)(p^3_{33} - p^5_{33}) + (\lambda_5 - \lambda_4)(p^3_{34} - p^5_{34})] S_3(\tau_i) + [(\lambda_5 - \lambda_1)(p^4_{31} - p^5_{31}) + (\lambda_5 - \lambda_2)(p^4_{32} - p^5_{32}) +(\lambda_5 - \lambda_3)(p^4_{33} - p^5_{33}) + (\lambda_5 - \lambda_4) (p^4_{34} - p^5_{34})] S_4(\tau_i) = k S_3 (Q_i) \quad \text{---(4.4)}$$

Equation (3.8) becomes

$$r(k-1) S_4(\tau_i) - S_1(\tau_i) [\lambda_1 p^1_{41} + \lambda_2 p^1_{42} + \lambda_3 p^1_{43} + \lambda_4 p^1_{44} + \lambda_5 p^1_{45}] - S_2(\tau_i) [\lambda_1 p^2_{41} + \lambda_2 p^2_{42} + \lambda_3 p^2_{43} + \lambda_4 p^2_{44} + \lambda_5 p^2_{45}] - S_3(\tau_i) [\lambda_1 p^3_{41} + \lambda_2 p^3_{42} + \lambda_3 p^3_{43} + \lambda_4 p^3_{44} + \lambda_5 p^3_{45}] - S_4(\tau_i) [\lambda_1 p^4_{41} + \lambda_2 p^4_{42} + \lambda_3 p^4_{43} + \lambda_4 p^4_{44} + \lambda_5 p^4_{45}] + [\tau_i + S_1(\tau_i) + S_2(\tau_i) + S_3(\tau_i) + S_4(\tau_i)] [\lambda_1 p^5_{41} + \lambda_2 p^5_{42} + \lambda_3 p^5_{43} + \lambda_4 p^5_{44} + \lambda_5 p^5_{45}] -n_4 \lambda_4 \tau_i = k S_4 (Q_i)$$

or

$$[\lambda_1 p^5_{41} + \lambda_2 p^5_{42} + \lambda_3 p^5_{43} + \lambda_4 p^5_{44} + \lambda_5 p^5_{45} -n_4 \lambda_4] \tau_i + [\lambda_1 p^5_{41} + \lambda_2 p^5_{42} + \lambda_3 p^5_{43} + \lambda_4 p^5_{44} + \lambda_5 p^5_{45} - (\lambda_1 p^1_{41} + \lambda_2 p^1_{42} + \lambda_3 p^1_{43} + \lambda_4 p^1_{44} + \lambda_5 p^1_{45})] S_1(\tau_i) + [\lambda_1 p^5_{41} + \lambda_2 p^5_{42} + \lambda_3 p^5_{43} + \lambda_4 p^5_{44} + \lambda_5 p^5_{45} - (\lambda_1 p^2_{41} + \lambda_2 p^2_{42} + \lambda_3 p^2_{43} + \lambda_4 p^2_{44} + \lambda_5 p^2_{45})] S_2(\tau_i) + [\lambda_1 p^5_{41} + \lambda_2 p^5_{42} + \lambda_3 p^5_{43} + \lambda_4 p^5_{44} + \lambda_5 p^5_{45} - (\lambda_1 p^3_{41} + \lambda_2 p^3_{42} + \lambda_3 p^3_{43} + \lambda_4 p^3_{44} + \lambda_5 p^3_{45})]S_3(\tau_i) + [r(k-1) - (\lambda_1 p^4_{41} + \lambda_2 p^4_{42} + \lambda_3 p^4_{43} + \lambda_4 p^4_{44} + \lambda_5 p^4_{45}) + \lambda_1 p^5_{41} + \lambda_2 p^5_{42} + \lambda_3 p^5_{43} + \lambda_4 p^5_{44} + \lambda_5 p^5_{45}] S_4(\tau_i) = k S_4 (Q_i)$$

using

$n_4 = p^5_{41} + p^5_{42} + p^5_{43} + p^5_{44} + p^5_{45}$

$p^1_{45} = n_4 - p^1_{41} - p^1_{42} - p^1_{43} - p^1_{44}$

$p^2_{45} = n_4 - p^2_{41} - p^2_{42} - p^2_{43} - p^2_{44}$

$p^3_{45} = n_4 - p^3_{41} - p^3_{42} - p^3_{43} - p^3_{44}$

$p^4_{45} = n_4 -1 - p^4_{41} - p^4_{42} - p^4_{43} - p^4_{44}$

$p^5_{45} = n_4 - p^5_{41} - p^5_{42} - p^5_{43} - p^5_{44}$

in above equation (3.8), it becomes and after solving, we get

$$[(\lambda_1 - \lambda_4)p^5_{41} + (\lambda_2 - \lambda_4) p^5_{42} + (\lambda_3 - \lambda_4) p^5_{43} + (\lambda_5 - \lambda_4) p^5_{45}] \tau_i + [(\lambda_5 - \lambda_1) (p^1_{41} - p^5_{41}) + (\lambda_5 - \lambda_2) (p^1_{42} - p^5_{42}) + (\lambda_5 - \lambda_3) (p^1_{43} - p^5_{43}) + (\lambda_5 - \lambda_4) (p^1_{44} - p^5_{44})] S_1(\tau_i) + [(\lambda_5 - \lambda_1) (p^2_{41} - p^5_{41}) + (\lambda_5 - \lambda_2) (p^2_{42} - p^5_{42}) + (\lambda_5 - \lambda_3) (p^2_{43} - p^5_{43}) + (\lambda_5 - \lambda_4) (p^2_{44} - p^5_{44})] S_2(\tau_i) + [(\lambda_5 - \lambda_1) (p^3_{41} - p^5_{41}) + (\lambda_5 - \lambda_2) (p^3_{42} - p^5_{42}) + (\lambda_5 - \lambda_3) (p^3_{43} - p^5_{43}) + (\lambda_5 - \lambda_4) (p^3_{44} - p^5_{44})] S_3(\tau_i) + [r(k-1) + \lambda_5 + (\lambda_5 - \lambda_1) (p^4_{41} - p^5_{41}) + (\lambda_5 - \lambda_2) (p^4_{42} - p^5_{42}) + (\lambda_5 - \lambda_3) (p^4_{43} - p^5_{43}) + (\lambda_5 - \lambda_4) (p^4_{44} - p^5_{44})] S_4(\tau_i) = k S_4 (Q_i) \quad \text{--- (4.5)}$$

Therefore, we have

$A_1 \tau_i + A_2 S_1(\tau_i) + A_3 S_2(\tau_i) + A_4 S_3(\tau_i) + A_5 S_4(\tau_i) = k Q_i$

$B_1 \tau_i + B_2 S_1(\tau_i) + B_3 S_2(\tau_i) + B_4 S_3(\tau_i) + B_5 S_4(\tau_i) = k S_1(\tau_i)$

$C_1 \tau_i + C_2 S_1(\tau_i) + C_3 S_2(\tau_i) + C_4 S_3(\tau_i) + C_5 S_4(\tau_i) = k S_2(\tau_i)$

$D_1 \tau_i + D_2 S_1(\tau_i) + D_3 S_2(\tau_i) + D_4 S_3(\tau_i) + D_5 S_4(\tau_i) = k S_3(\tau_i)$

$E_1 \tau_i + E_2 S_1(\tau_i) + E_3 S_2(\tau_i) + E_4 S_3(\tau_i) + E_5 S_4(\tau_i) = k S_4(\tau_i)$

where

$A_1 = r(k-1) +$ $\quad A_2 = (\lambda_5 - \lambda_1)$

$A_3 = (\lambda_5 - \lambda_2)$ $\quad A_4 = (\lambda_5 - \lambda_3)$

$A_5 = (\lambda_5 - \lambda_4)$

$B_1 = (\lambda_1 - \lambda_1) p^5_{11} + (\lambda_2 - \lambda_1) p^5_{12} + (\lambda_3 - \lambda_1) p^5_{13} + (\lambda_4 - \lambda_1) p^5_{14} + + (\lambda_5 - \lambda_1) p^5_{15}$

$B_2 = r(k-1) + \lambda_5 + (\lambda_5 - \lambda_1)(p^1_{11} - p^5_{11}) + (\lambda_5 - \lambda_2) (p^1_{12} - p^5_{12}) + (\lambda_5 - \lambda_3) (p^1_{13} - p^5_{13}) + (\lambda_5 - \lambda_4) (p^1_{14} - p^5_{14})$

$B_3 = (\lambda_5 - \lambda_1)(p^2_{11} - p^5_{11}) + (\lambda_5 - \lambda_2)(p^2_{12} - p^5_{12}) + (\lambda_5 - \lambda_3)(p^2_{13} - p^5_{13}) + (\lambda_5 - \lambda_4)(p^2_{14} - p^5_{14})$

$B_4 = (\lambda_5 - \lambda_1)(p^3_{11} - p^5_{11}) + (\lambda_5 - \lambda_2)(p^3_{12} - p^5_{12}) + (\lambda_5 - \lambda_3)(p^3_{13} - p^5_{13}) + (\lambda_5 - \lambda_4)(p^3_{14} - p^5_{14})$

$B_5 = (\lambda_5 - \lambda_1)(p^4_{11} - p^5_{11}) + (\lambda_5 - \lambda_2)(p^4_{12} - p^5_{12}) + (\lambda_5 - \lambda_3)(p^4_{13} - p^5_{13}) + (\lambda_5 - \lambda_4)(p^4_{14} - p^5_{14})$

$C_1 = (\lambda_1 - \lambda_2)p^5_{21} + (\lambda_2 - \lambda_2) p^5_{22} + (\lambda_3 - \lambda_2) p^5_{23} + (\lambda_4 - \lambda_2) p^5_{24} + + (\lambda_5 - \lambda_2) p^5_{25}$

$C_2 = (\lambda_5 - \lambda_1)(p^1_{21} - p^5_{21}) + (\lambda_5 - \lambda_2) (p^1_{22} - p^5_{22}) + (\lambda_5 - \lambda_3)(p^1_{23} - p^5_{23}) + (\lambda_5 - \lambda_4)(p^1_{24} - p^5_{24})$

$C_3 = r(k-1) + \lambda_5 + (\lambda_5 - \lambda_1)(p^2_{21} - p^5_{21}) + (\lambda_5 - \lambda_2) (p^2_{22} - p^5_{22}) + (\lambda_5 - \lambda_3) (p^2_{23} - p^5_{23})$

$+ (\lambda_5 - \lambda_4) (p^2_{24} - p^5_{24})$

$C_4 = (\lambda_5 - \lambda_1)(p^3_{21} - p^5_{21}) + (\lambda_5 - \lambda_2)(p^3_{22} - p^5_{22}) + (\lambda_5 - \lambda_3)(p^3_{23} - p^5_{23}) + (\lambda_5 - \lambda_4) (p^3_{24} - p^5_{24})$

$C_5 = (\lambda_5 - \lambda_1)(p^4_{21} - p^5_{21}) + (\lambda_5 - \lambda_2)(p^4_{22} - p^5_{22}) + (\lambda_5 - \lambda_3)(p^4_{23} - p^5_{23}) + (\lambda_5 - \lambda_4)(p^4_{24} - p^5_{24})$

$D_1 = (\lambda_1 - \lambda_3) p^5_{31} + (\lambda_2 - \lambda_3) p^5_{32} + (\lambda_3 - \lambda_3) p^5_{32} + (\lambda_4 - \lambda_3) p^5_{34} + (\lambda_5 - \lambda_3) p^5_{35}$

$D_2 = (\lambda_5 - \lambda_1)(p^1_{31} - p^5_{31}) + (\lambda_5 - \lambda_2)(p^1_{32} - p^5_{32}) + (\lambda_5 - \lambda_3)(p^1_{33} - p^5_{33}) + (\lambda_5 - \lambda_4)(p^1_{34} - p^5_{34})$

$D_3 = (\lambda_5 - \lambda_1)(p^2_{31} - p^5_{31}) + (\lambda_5 - \lambda_2)(p^2_{32} - {}^5_{32}) + (\lambda_5 - \lambda_3)(p^2_{33} - p^5_{33}) + (\lambda_5 - \lambda_4)(p^2_{34} - p^5_{34})$

$D_4 = r(k-1) + \lambda_5 + (\lambda_5 - \lambda_1)(p^3_{31} - p^5_{31}) + (\lambda_5 - \lambda_2)(p^3_{32} - p^5_{32}) + (\lambda_5 - \lambda_3) (p^3_{33} - p^5_{33})$

$+ (\lambda_5 - \lambda_4) (p^3_{34} - p^5_{34})$

$D_5 = (\lambda_5 - \lambda_1)(p^4_{31} - p^5_{31}) + (\lambda_5 - \lambda_2)(p^4_{32} - p^5_{32}) + (\lambda_5 - \lambda_3)(p^4_{33} - p^5_{33}) + (\lambda_5 - \lambda_4)(p^4_{34} - p^5_{34})$

$E_1 = (\lambda_1 - \lambda_4) p^5_{41} + (\lambda_2 - \lambda_4) p^5_{42} + (\lambda_3 - \lambda_4) p^5_{42} + (\lambda_4 - \lambda_4) p^5_{44} + (\lambda_5 - \lambda_4) p^5_{45}$

$E_2 = (\lambda_5 - \lambda_1)(p^1_{41} - p^5_{41}) + (\lambda_5 - \lambda_2)(p^1_{42} - p^5_{42}) + (\lambda_5 - \lambda_3)(p^1_{43} - p^5_{43}) + (\lambda_5 - \lambda_4)(p^1_{44} - p^5_{44})$

$E_3 = (\lambda_5 - \lambda_1)(p^2_{41} - p^5_{41}) + (\lambda_5 - \lambda_2)(p^2_{42} - p^5_{42}) + (\lambda_5 - \lambda_3)(p^2_{43} - p^5_{43}) + (\lambda_5 - \lambda_4)(p^2_{44} - p^5_{44})$

$E_4 = (\lambda_5 - \lambda_1)(p^3_{41} - p^5_{41}) + (\lambda_5 - \lambda_2)(p^3_{42} - p^5_{42}) + (\lambda_5 - \lambda_3)(p^3_{43} - p^5_{43}) + (\lambda_5 - \lambda_4)(p^3_{44} - p^5_{44})$

$E_5 = r(k-1) + \lambda_5 + (\lambda_5 - \lambda_1)(p^4_{41} - p^5_{41}) + (\lambda_5 - \lambda_2)(p^4_{42} - p^5_{42}) + (\lambda_5 - \lambda_3)(p^4_{43} - p^5_{43}) + (\lambda_5 - \lambda_4)(p^4_{44} - p^5_{44})$

Using these constants, we get a solution

$= k[\, Q_i F - \,] / \Delta$

where

$\Delta = [A_1 F - A_2 G + A_3 H - A_4 I + A_5 J]$

$$F = \begin{vmatrix} B_2 & B_3 & B_4 & B_5 \\ C_2 & C_3 & C_4 & C_5 \\ D_2 & D_3 & D_4 & D_5 \\ E_2 & E_3 & E_4 & E_5 \end{vmatrix} \qquad G = \begin{vmatrix} A_2 & A_3 & A_4 & A_5 \\ C_2 & C_3 & C_4 & C_5 \\ D_2 & D_3 & D_4 & D_5 \\ E_2 & E_3 & E_4 & E_5 \end{vmatrix}$$

$$H = \begin{vmatrix} A_2 & A_3 & A_4 & A_5 \\ B_2 & B_3 & B_4 & B_5 \\ D_2 & D_3 & D_4 & D_5 \\ E_2 & E_3 & E_4 & E_5 \end{vmatrix} \qquad I = \begin{vmatrix} A_2 & A_3 & A_4 & A_5 \\ B_2 & B_3 & B_4 & B_5 \\ C_2 & C_3 & C_4 & C_5 \\ E_2 & E_3 & E_4 & E_5 \end{vmatrix}$$

$$J = \begin{vmatrix} A_2 & A_3 & A_4 & A_5 \\ B_2 & B_3 & B_4 & B_5 \\ C_2 & C_3 & C_4 & C_5 \\ D_2 & D_3 & D_4 & D_5 \end{vmatrix}$$

and the variance of an estimated elementary contrasts among treatment effects are

$V(\tau_i - \tau_j) = 2k\sigma^2 [F + G] / \Delta$

, if i and j are 1st associate

$V(\tau_i - \tau_j) = 2k\sigma^2 [F - H] / \Delta$

, if i and j are 2nd associate

$V(\tau_i - \tau_j) = 2k\sigma^2 [F + I] / \Delta$

,if i and j are 3rd associate

$V(\tau_i - \tau_j) = 2k\sigma^2 [F - J] / \Delta$

,if i and j are 4th associate

$V(\tau_i - \tau_j) = 2k\sigma^2 [F] / \Delta$

,if i and j are 5th associate

and the efficiency factors of five kinds are given by

$E_1 = \Delta / rk[F + G]$

$E_2 = \Delta / rk[F - H]$

$E_3 = \Delta / rk[F + I]$

$E_4 = \Delta / rk[F + J]$

$E_5 = \Delta / rk[F]$

The overall efficiency factor is

$E = (v-1)\Delta / rk[(v-1)F + n_1 G - n_2 H + n_3 I - n_4 J]$

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