



Some Regular Properties of the Strong Product of Two Fuzzy Graphs

Dr. K. RADHA

P.G & Research Department of Mathematics, Periyar E.V.R. College, Tiruchirapalli-620023, Tamilnadu, India.

Mr. S. ARUMUGAM

B.T. ASSISTANT (MATHEMATICS), Government High School, Thinnanur, Tiruchirapalli-621006., Tamilnadu, India.

ABSTRACT

In this paper, regular properties of the strong product of two fuzzy graphs are studied. It is illustrated that when two fuzzy graphs are regular then their strong product need not be a regular fuzzy graph.

With some restrictions it is proved that the strong product of two regular fuzzy graphs is regular and it is proved that the strong product of two full regular fuzzy graphs is full regular.

**KEYWORDS :** Fuzzy Graph, Strong Product, Regular Fuzzy Graph and Full Regular Fuzzy Graph.

I. INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld[9] in 1975. Later on, Bhattacharya[1] gave some remarks on fuzzy graphs. Some operations on fuzzy graphs were introduced by Mordeson.J.N. and Peng.C.S.[4]. We defined the direct sum of two fuzzy graphs and studied the properties of that operation[6]. Also we defined the strong product of two fuzzy graphs and studied some of its properties[8].

In this paper we study the regular properties of the strong product of two fuzzy graphs. We illustrate that the strong product of two regular fuzzy graphs need not be regular. Then we provide the conditions under which the strong product of two regular fuzzy graphs is regular as well as full regular.

II. PRELIMINARIES

First let us recall some preliminary definitions and results that can be found in [1]-[9].

A fuzzy graph G is a pair of functions  $(\sigma, \mu)$  where  $\sigma$  is a fuzzy subset of a nonempty set V and  $\mu$  is a symmetric fuzzy relation on  $\sigma$ . The underlying crisp graph of  $G:(\sigma, \mu)$  is denoted by  $G^*(V, E)$  where  $E \subseteq V \times V$ .

The degree of a vertex u is defined as

$$d_G(u) = \sum_{u \neq v} \mu(uv) = \sum_{uv \in E} \mu(uv).$$

If  $d_G(v)=k$  for all  $v \in V$ , G is said to be a regular fuzzy graph of degree 'k' or a k-regular fuzzy graph.

The regular fuzzy graph  $G:(\sigma, \mu)$  is called a full regular fuzzy graph if its underlying crisp graph  $G^*$  is a regular graph and a complete regular fuzzy graph if its underlying crisp graph  $G^*$  is a complete graph.

If  $G_1:(\sigma_1, \mu_1)$  and  $G_2:(\sigma_2, \mu_2)$  are two fuzzy graphs such that  $\sigma_1 \leq \mu_2$  then  $\sigma_2 \geq \mu_1$ .

Let  $G_1:(\sigma_1, \mu_1)$  and  $G_2:(\sigma_2, \mu_2)$  denote two fuzzy graphs with underlying crisp graphs  $G_1^*:(V_1, E_1)$  and  $G_2^*:(V_2, E_2)$  respectively. The normal product of  $G_1^*$  and  $G_2^*$  is  $G^*=G_1^* \circ G_2^*:(V, E)$  where  $V=V_1 \times V_2$  and  $E=\{(u_1, v_1)(u_2, v_2)/u_1=u_2, v_1 v_2 \in E_2 \text{ or } v_1=v_2, u_1 u_2 \in E_1 \text{ or } u_1 u_2 \in E_1 \text{ and } v_1 v_2 \in E_2\}$ .

The fuzzy graph  $G:(\sigma, \mu)$ , where,  $\sigma(u_1, v_1)=\sigma_1(u_1) \wedge \sigma_2(v_1)$ , for all  $(u_1, v_1) \in V_1 \times V_2$  and  $\mu((u_1, v_1)(u_2, v_2))$

$$= \begin{cases} \sigma_1(u_1) \wedge \mu_2(v_1, v_2) & , \text{ if } u_1 = u_2, v_1 v_2 \in E_2 \\ \sigma_2(v_1) \wedge \mu_1(u_1, u_2) & , \text{ if } v_1 = v_2, u_1 u_2 \in E_1 \\ \mu_1(u_1 u_2) \wedge \mu_2(v_1 v_2) & , \text{ if } u_1 u_2 \in E_1, v_1 v_2 \in E_2 \end{cases}$$

is called the strong product of  $G_1$  and  $G_2$  and denoted by  $G_1 \circ G_2$ .

The following results give degree of a vertex in  $G_1 \circ G_2$  with some restrictions.

If  $\sigma_1 \geq \mu_2$  and  $\sigma_2 \geq \mu_1$  and  $\mu_1 \wedge \mu_2 = c$ , then

$$d_{G_1 \circ G_2}(u_i, v_j) = d_{G_2}(v_j) + d_{G_1}(u_i) + [d_{G_1}(u_i) d_{G_2}(v_j)]c \dots \dots \dots (2.1)$$

If  $\sigma_1 \geq \mu_2$  and  $\sigma_2 \geq \mu_1$  and  $\mu_1 \vee \mu_2 = C$ , then

$$d_{G_1 \circ G_2}(u_i, v_j) = [1 + d_{G_2}(v_j)]d_{G_1}(u_i) + [1 + d_{G_1}(u_i)]d_{G_2}(v_j) - [d_{G_1}(u_i) d_{G_2}(v_j)]C \dots \dots \dots (2.2)$$

If  $\sigma_1 \leq \mu_2$  and  $\mu_1 \wedge \mu_2 = c$ , then

$$d_{G_1 \circ G_2}(u_i, v_j) = d_{G_2}(v_j)\sigma_1(u_i) + d_{G_1}(u_i) + [d_{G_1}(u_i) d_{G_2}(v_j)]c \dots \dots \dots (2.3)$$

III. REGULAR PROPERTIES

If  $G_1:(\sigma_1, \mu_1)$  and  $G_2:(\sigma_2, \mu_2)$  are two regular fuzzy graphs then their strong product  $G_1 \circ G_2 : (\sigma, \mu)$  need not be a regular

fuzzy graph. It is illustrated through the following example.

**Example 3.1:**

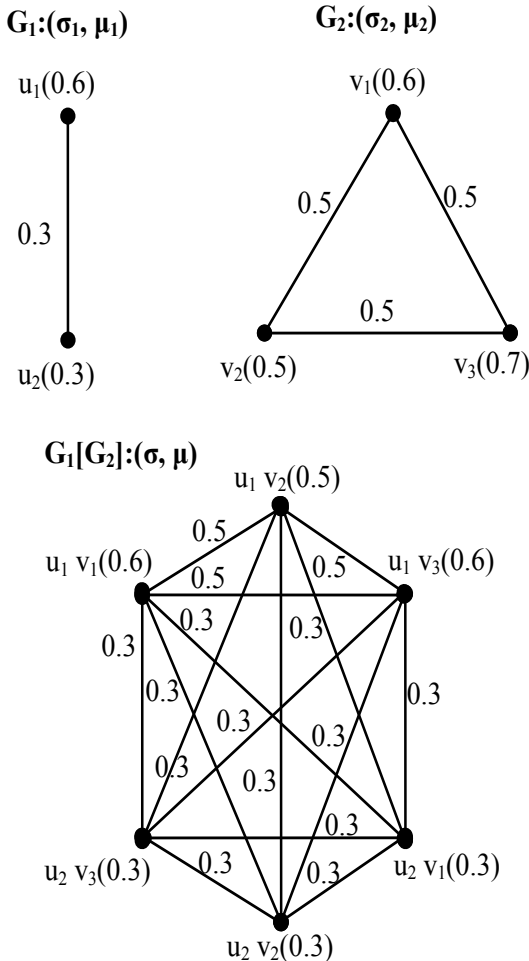


Figure-1

But with few restrictions it can be proved that the strong product of two regular fuzzy graphs is regular.

**Theorem 3.2:**

If  $G_1:(\sigma_1, \mu_1)$  and  $G_2:(\sigma_2, \mu_2)$  are two fuzzy graphs such that  $\sigma_1 \geq \mu_2$ ,  $\sigma_2 \geq \mu_1$  and  $\mu_1 \wedge \mu_2 = c$  with  $G_1^*:(V_1, E_1)$  and  $G_2^*:(V_2, E_2)$  are  $r$ -regular graphs then  $G_1 \circ G_2 : (\sigma, \mu)$  is regular if and only if  $G_1:(\sigma_1, \mu_1)$  and  $G_2:(\sigma_2, \mu_2)$  are regular.

**Proof:**

Let  $\sigma_1 \geq \mu_2$  and  $\sigma_2 \geq \mu_1$  and  $\mu_1 \wedge \mu_2 = c$  (a constant) and  $G_1^*:(V_1, E_1)$  and  $G_2^*:(V_2, E_2)$

be  $r$ -regular. Then for any vertex  $(u_i, v_j)$  in  $G_1 \circ G_2 : (\sigma, \mu)$ ,

$$\begin{aligned} d_{G_1 \circ G_2}(u_i, v_j) &= d_{G_2}(v_j) + d_{G_1}(u_i) + [d_{G_1^*}(u_i) d_{G_2^*}(v_j)]c \\ &= d_{G_2}(v_j) + d_{G_1}(u_i) + r^2 c \end{aligned}$$

Now assume that  $G_1$  and  $G_2$  are regular fuzzy graphs of degrees  $k_1$  and  $k_2$  respectively. Then,

$$d_{G_1 \circ G_2}(u_i, v_j) = k_1 + k_2 + r^2 c.$$

This is a constant since  $k_1, k_2, r$  and  $c$  are all constants. Hence the strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is regular.

Conversely assume that the strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is regular.

Then for any two vertices  $(u_1, v_1)$  and  $(u_2, v_2)$  in  $V_1 \times V_2$ ,

$$\begin{aligned} d_{G_1 \circ G_2}(u_1, v_1) &= d_{G_1 \circ G_2}(u_2, v_2) \\ \Rightarrow d_{G_2}(v_1) + d_{G_1}(u_1) + r^2 c &= d_{G_2}(v_2) + d_{G_1}(u_2) + r^2 c \\ \Rightarrow d_{G_2}(v_1) + d_{G_1}(u_1) &= d_{G_2}(v_2) + d_{G_1}(u_2) \dots (2.1) \end{aligned}$$

Fix  $v \in V_2$  and consider  $(u_1, v)$  and  $(u_2, v)$  in  $V_1 \times V_2$  where  $u_1, u_2 \in V_1$  are arbitrary.

$$\begin{aligned} \text{From (2.1), } d_{G_2}(v) + d_{G_1}(u_1) &= d_{G_2}(v) + d_{G_1}(u_2) \\ \Rightarrow d_{G_1}(u_1) &= d_{G_1}(u_2). \end{aligned}$$

This is true for all  $u_1, u_2 \in V_1$ . Thus  $G_1$  is regular.

Now fix  $u \in V_1$  and consider  $(u, v_1)$  and  $(u, v_2)$  in  $V_1 \times V_2$  where  $v_1, v_2 \in V_2$  are arbitrary.

$$\begin{aligned} \text{From (2.2), } d_{G_2}(v_1) + d_{G_1}(u) &= d_{G_2}(v_2) + d_{G_1}(u) \\ \Rightarrow d_{G_2}(v_1) &= d_{G_2}(v_2). \end{aligned}$$

This is true for all  $v_1, v_2 \in V_2$ . Thus  $G_2$  is also regular.

**Theorem 3.3:**

If  $G_1:(\sigma_1, \mu_1)$  and  $G_2:(\sigma_2, \mu_2)$  are two fuzzy graphs such that  $\sigma_1 \geq \mu_2$  and  $\sigma_2 \geq \mu_1$  and

$\mu_1 \vee \mu_2 = c$  with  $G_1^*$  and  $G_2^*$  are r-regular, then the strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is regular if and only if  $G_1 : (\sigma_1, \mu_1)$  and  $G_2 : (\sigma_2, \mu_2)$  are regular.

**Proof:**

Let  $G_1 : (\sigma_1, \mu_1)$  and  $G_2 : (\sigma_2, \mu_2)$  be two fuzzy graphs such that  $\sigma_1 \geq \mu_2$  and  $\sigma_2 \geq \mu_1$  and  $\mu_1 \vee \mu_2 = c$  with  $G_1^*$  and  $G_2^*$  are r-regular.

Then for any vertex  $(u_i, v_j)$  in  $G_1 \circ G_2 : (\sigma, \mu)$ ,

$$\begin{aligned} & d_{G_1 \circ G_2}(u_i, v_j) \\ &= [1 + d_{G_2^*}(v_j)]d_{G_1}(u_i) + [1 + d_{G_1^*}(u_i)]d_{G_2}(v_j) \\ &\quad - [d_{G_1^*}(u_i)d_{G_2^*}(v_j)]c \\ &= [1 + r]d_{G_1}(u_i) + [1 + r]d_{G_2}(v_j) - r^2 c \\ &= [1 + r][d_{G_1}(u_i) + d_{G_2}(v_j)] - r^2 c \dots (2.2) \end{aligned}$$

Now assume that  $G_1$  and  $G_2$  are regular fuzzy graphs of degrees  $k_1$  and  $k_2$  respectively. Then,

$$d_{G_1 \circ G_2}(u_i, v_j) = [1 + r][k_1 + k_2] - r^2 c.$$

This is a constant since  $k_1, k_2, r$  and  $c$  are all constants.

Hence the strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is regular.

Conversely assume that the strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is regular.

Then for any two vertices  $(u_1, v_1)$  and  $(u_2, v_2)$  in  $V_1 \times V_2$ ,

$$d_{G_1 \circ G_2}(u_1, v_1) = d_{G_1 \circ G_2}(u_2, v_2)$$

From(2.2),

$$\begin{aligned} & [1 + r][d_{G_1}(u_1) + d_{G_2}(v_1)] - r^2 c \\ &= [1 + r][d_{G_1}(u_2) + d_{G_2}(v_2)] - r^2 c \\ \Rightarrow & [1 + r][d_{G_1}(u_1) + d_{G_2}(v_1)] \\ &= [1 + r][d_{G_1}(u_2) + d_{G_2}(v_2)] \\ \Rightarrow & d_{G_2}(v_1) + d_{G_1}(u_1) \\ &= d_{G_2}(v_2) + d_{G_1}(u_2) \dots (2.3) \end{aligned}$$

Fix  $v \in V_2$  and consider  $(u_1, v)$  and  $(u_2, v)$  in  $V_1 \times V_2$  where  $u_1, u_2 \in V_1$  are arbitrary.

$$\begin{aligned} & d_{G_2}(v) + d_{G_1}(u_1) = d_{G_2}(v) + d_{G_1}(u_2) \\ \Rightarrow & d_{G_1}(u_1) = d_{G_1}(u_2). \end{aligned}$$

This is true for all  $u_1, u_2 \in V_1$ . Thus  $G_1$  is regular.

Now fix  $u \in V_1$  and consider  $(u, v_1)$  and  $(u, v_2)$  in  $V_1 \times V_2$  where  $v_1, v_2 \in V_2$  are arbitrary. Then from(2.3)

$$\begin{aligned} & d_{G_2}(v_1) + d_{G_1}(u) = d_{G_2}(v_2) + d_{G_1}(u) \\ \Rightarrow & d_{G_2}(v_1) = d_{G_2}(v_2). \end{aligned}$$

This is true for all  $v_1, v_2 \in V_2$ . Thus  $G_2$  is also a regular fuzzy graph.

**Theorem 3.4:**

If  $G_1 : (\sigma_1, \mu_1)$  and  $G_2 : (\sigma_2, \mu_2)$  are two fuzzy graph such that  $\sigma_1 \leq \mu_2$ ,  $\sigma_1$  is a constant and  $\mu_1 \wedge \mu_2 = c$  with  $G_1^*$  and  $G_2^*$  are r-regular, then the strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is regular if and only if  $G_1 : (\sigma_1, \mu_1)$  is regular.

**Proof:**

Let  $\sigma_1 \leq \mu_2$ ,  $\sigma_1$  be a constant and  $\mu_1 \wedge \mu_2 = c$  with  $G_1^*$  and  $G_2^*$  are r-regular.

Then for any vertex  $(u_i, v_j)$  in  $G_1 \circ G_2 : (\sigma, \mu)$ ,

$$\begin{aligned} & d_{G_1 \circ G_2}(u_i, v_j) \\ &= d_{G_2^*}(v_j)\sigma_1(u_i) + d_{G_1}(u_i) + [d_{G_1^*}(u_i)d_{G_2^*}(v_j)]c \\ &= r\sigma_1 + d_{G_1}(u_i) + r^2 c \\ &= d_{G_1}(u_i) + r[\sigma_1 + r c] \dots (2.4) \end{aligned}$$

Now assume that  $G_1 : (\sigma_1, \mu_1)$  is a regular fuzzy graph of degree  $k_1$ .

Then the degree of any vertex  $(u_i, v_j)$  in the strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is given by,

$$d_{G_1 \circ G_2}(u_i, v_j) = k_1 + r[\sigma_1 + r c].$$

This is a constant since  $k_1, \sigma_1, r$  and  $c$  are all constants.

Hence the strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is regular.

Conversely assume that the strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is regular.

Then for any two vertices  $(u_1, v_1)$  and  $(u_2, v_2)$  in  $V_1 \times V_2$ ,  
 $d_{G_1 \circ G_2}(u_1, v_1) = d_{G_1 \circ G_2}(u_2, v_2)$   
 $\Rightarrow d_{G_1}(u_1) + r[\sigma_1 + r c] = d_{G_1}(u_2) + r[\sigma_1 + r c]$   
 $\Rightarrow d_{G_1}(u_1) = d_{G_1}(u_2)$

This is true for all  $u_1, u_2 \in V_1$ . Thus  $G_1$  is regular.

**IV. FULL REGULAR PROPERTIES**

In this chapter we assume that  $G_i^*$  is  $r_i$ -regular for  $i=1,2$ .

**Theorem 4.1:**

If  $G_1:(\sigma_1, \mu_1)$  and  $G_2:(\sigma_2, \mu_2)$  are full regular fuzzy graphs with underlying crisp graphs  $G_1^*:(V_1, E_1)$  and  $G_2^*:(V_2, E_2)$  respectively then the underlying crisp graph  $(G_1 \circ G_2)^*$  of their strong product is also regular.

**Proof:**

Since  $G_1:(\sigma_1, \mu_1)$  and  $G_2:(\sigma_2, \mu_2)$  are full regular,  $G_1^*:(V_1, E_1)$  and  $G_2^*:(V_2, E_2)$  are regular. Now, the degree of any vertex in the strong product is given by,

$$d_{G_1 \circ G_2}(u_i, v_j) = \sum_{u_i = u_k, v_j, v_\ell \in E_2} \sigma_1(u_i) \wedge \mu_2(v_j, v_\ell) + \sum_{u_i, u_k \in E_1, v_j = v_\ell} \mu_1(u_i, u_k) \wedge \sigma_2(v_j) + \sum_{u_i, u_k \in E_1, v_j, v_\ell \in E_2} \mu_1(u_i, u_k) \wedge \mu_2(v_j, v_\ell)$$

This implies,

$$\begin{aligned} d_{(G_1 \circ G_2)^*}(u_i, v_j) &= \sum_{u_i = u_k, v_j, v_\ell \in E_2} 1 + \sum_{u_i, u_k \in E_1, v_j = v_\ell} 1 + \sum_{u_i, u_k \in E_1, v_j, v_\ell \in E_2} 1 \\ &= d_{G_2^*}(v_j) + d_{G_1^*}(u_i) + [d_{G_1^*}(u_i) d_{G_2^*}(v_j)] \\ &= r_2 + r_1 + r_1 r_2. \end{aligned}$$

This is a constant and hence  $(G_1 \circ G_2)^*$  is a regular graph.

**Theorem 4.2:**

The strong product of two full regular fuzzy graphs  $G_1:(\sigma_1, \mu_1)$  and

$G_2:(\sigma_2, \mu_2)$  is full regular if  $\mu_1 = \mu_2 = k$ , where 'k' is a constant.

**Proof:**

Since  $G_1:(\sigma_1, \mu_1)$  and  $G_2:(\sigma_2, \mu_2)$  are full regular, their underlying crisp graphs  $G_1^*:(V_1, E_1)$  and  $G_2^*:(V_2, E_2)$  are regular.

Then by Theorem 4.1,  $(G_1 \circ G_2)^*$  is regular (n-regular (say)).

By the definition,  $(\mu_1 \circ \mu_2)((u_1, v_1)(u_2, v_2)) = k$  for all pairs of vertices in  $G_1 \circ G_2$ , since  $\mu_1 = \mu_2 = k$ .

Then,  $d_{G_1 \circ G_2}(u_i, v_j) = nk$ , where 'n' is the degree of each vertex in  $(G_1 \circ G_2)^*$ . Therefore  $G_1 \circ G_2$  is a full regular fuzzy graph.

**Example 4.3:**

Consider the full regular fuzzy graphs  $G_1:(\sigma_1, \mu_1)$  and  $G_2:(\sigma_2, \mu_2)$  with  $\mu_1 = \mu_2 = 0.4$ . Their strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is also full regular.

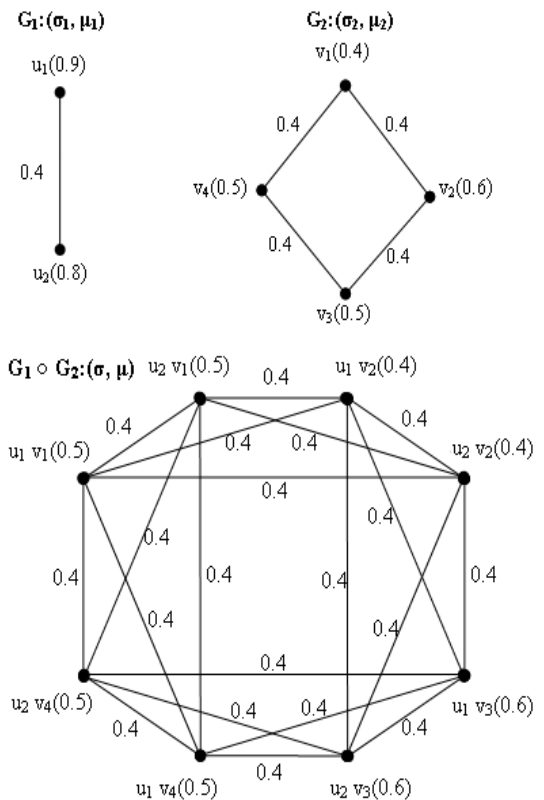


Figure-2

**Remark 4.4:**

The above theorem 3.2 holds for complete regular fuzzy graphs also.

**Theorem 4.5:**

If  $G_1:(\sigma_1,\mu_1)$  and  $G_2:(\sigma_2,\mu_2)$  are two full regular fuzzy graphs such that  $\sigma_1 \geq \mu_2$  and  $\sigma_2 \geq \mu_1$  and  $\mu_1 \wedge \mu_2 = c$  (a constant), then their strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is also a full regular fuzzy graph.

**Proof:**

Let  $G_1:(\sigma_1,\mu_1)$  and  $G_2:(\sigma_2,\mu_2)$  be two full regular fuzzy graphs such that  $\sigma_1 \geq \mu_2$  and  $\sigma_2 \geq \mu_1$  and  $\mu_1 \wedge \mu_2 = c$ . Then,

$$d_{G_1 \circ G_2}(u_i, v_j) = d_{G_2}(v_j) + d_{G_1}(u_i) + [d_{G_1^*}(u_i)d_{G_2^*}(v_j)]c.$$

Since  $G_1: (\sigma_1,\mu_1)$  is a full regular fuzzy graph,

$$d_{G_1}(u_i) = k_1 \text{ and } d_{G_1^*}(u_i) = r_1 \quad \forall u_i \in V_1.$$

Similarly since  $G_2:(\sigma_2,\mu_2)$  is a full regular fuzzy graph,

$$d_{G_2}(v_j) = k_2 \text{ and } d_{G_2^*}(v_j) = r_2 \quad \forall v_j \in V_2.$$

Then for any vertex  $(u_i, v_j)$  in  $G_1 \circ G_2 : (\sigma, \mu)$ ,

$$d_{G_1 \circ G_2}(u_i, v_j) = k_1 + k_2 + r_1 r_2 c.$$

This is a constant and hence the strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is a full regular fuzzy graph.

**Theorem 4.6:**

If  $G_1:(\sigma_1,\mu_1)$  and  $G_2:(\sigma_2,\mu_2)$  are two full regular fuzzy graphs such that  $\sigma_1 \geq \mu_2$  and  $\sigma_2 \geq \mu_1$  and  $\mu_1 \vee \mu_2 = C$  (a constant) with  $G_1^*$  and  $G_2^*$  are k-regular where ‘k’ is a constant, then their strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is also a full regular fuzzy graph.

**Proof:**

Let  $G_1:(\sigma_1,\mu_1)$  and  $G_2:(\sigma_2,\mu_2)$  be two full regular fuzzy graphs such that  $\sigma_1 \geq \mu_2$  and  $\sigma_2 \geq \mu_1$  and  $\mu_1 \vee \mu_2 = C$  (a constant) with  $G_1^*$  and  $G_2^*$  are k-regular. Then,

$$d_{G_1 \circ G_2}(u_i, v_j) = [1 + d_{G_2^*}(v_j)]d_{G_1}(u_i) + [1 + d_{G_1^*}(u_i)]d_{G_2}(v_j) - [d_{G_1^*}(u_i)d_{G_2^*}(v_j)]C.$$

Since  $G_1:(\sigma_1,\mu_1)$  is a full regular fuzzy graph with  $G_1^*$  is k-regular,

$$d_{G_1}(u_i) = k_1 \text{ and } d_{G_1^*}(u_i) = k \quad \forall u_i \in V_1.$$

Similarly since  $G_2:(\sigma_2,\mu_2)$  is a full regular fuzzy graph with  $G_2^*$  is k-regular,

$$d_{G_2}(v_j) = k_2 \text{ and } d_{G_2^*}(v_j) = k \quad \forall v_j \in V_2.$$

Then for any vertex  $(u_i, v_j)$  in  $G_1 \circ G_2 : (\sigma, \mu)$ ,

$$d_{G_1 \circ G_2}(u_i, v_j) = (1+k)k_1 + (1+k)k_2 - k^2 C = (1+k)(k_1 + k_2) - k^2 C.$$

This is a constant and hence the strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is a full regular fuzzy graph.

**Theorem 4.7:**

If  $G_1:(\sigma_1,\mu_1)$  is a full regular fuzzy graph and  $G_2:(\sigma_2,\mu_2)$  is a partially regular fuzzy graph such that  $\sigma_1 \leq \mu_2$ ,  $\sigma_1$  is a constant and  $\mu_1 \wedge \mu_2 = c$  (a constant), then their strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is a full regular fuzzy graph.

**Proof:**

Let  $G_1:(\sigma_1,\mu_1)$  be a full regular fuzzy graph and  $G_2:(\sigma_2,\mu_2)$  be a partially regular fuzzy graph such that  $\sigma_1 \leq \mu_2$ ,  $\sigma_1$  is a constant and  $\mu_1 \wedge \mu_2 = c$  (a constant).

Then for any vertex  $(u_i, v_j)$  in  $G_1 \circ G_2 : (\sigma, \mu)$ ,

$$d_{G_1 \circ G_2}(u_i, v_j) = d_{G_2^*}(v_j)\sigma_1(u_i) + d_{G_1}(u_i) + [d_{G_1^*}(u_i)d_{G_2^*}(v_j)]c.$$

Since  $G_1: (\sigma_1,\mu_1)$  is a full regular fuzzy graph,

$$d_{G_1}(u_i) = k_1 \text{ and } d_{G_1^*}(u_i) = r_1 \quad \forall u_i \in V_1.$$

Similarly since  $G_2:(\sigma_2,\mu_2)$  is a partially regular fuzzy graph,  $d_{G_2^*}(v_j) = r_2 \quad \forall v_j \in V_2$ .

Then the degree of any vertex  $(u_i, v_j)$  in the

strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is given by,

$$d_{G_1 \circ G_2}(u_i, v_j) = r_2 \sigma_1 + k_1 + r_1 r_2 c.$$

This is a constant and hence the strong product  $G_1 \circ G_2 : (\sigma, \mu)$  is a full regular fuzzy graph.

## V. CONCLUSION

In this paper, we have studied the regular properties of the strong product of two fuzzy graphs and illustrated that when two fuzzy graphs are regular then their strong product need not be regular. We have provided the conditions under which the strong product of two regular fuzzy graphs is regular and the strong product of two full regular fuzzy graphs is full regular. In addition to the existing properties these properties will be helpful to study the properties of large fuzzy graph as a combination of small fuzzy graphs.

## REFERENCES

- [1] Bhattacharya, P, Some Remarks on Fuzzy Graphs, Pattern Recognition Letter 6 (1987), 297-302. | [2] John N. Modeson and Premchand S.Nair, Fuzzy Graphs and Fuzzy Hypergraphs, Physica-verlag Heidelberg, 2000. | [3] Mordeson J.N. and Peng C.S., Operations on fuzzy graphs, Information Sciences 79 (1994), 159-170. | [4] Nagoorgani. A and Radha. K, Conjunction of Two Fuzzy Graphs, International Review of Fuzzy Mathematics, 2008, Vol. 3, 95-105. | [5] Nagoorgani. A and Radha. K, Regular Property of Fuzzy Graphs, Bulletin of Pure and Applied Sciences, Vol.27E ( No.2)2008, 411-419. | [6] Radha. K and Arumugam. S, On Direct Sum of Two Fuzzy Graphs, International Journal of Scientific and Research Publications, Volume 3, Issue 5, May 2013, ISSN 2250-3153. | [7] Radha. K and Arumugam. S, Path Matrices of Fuzzy Graphs, Proceedings of the International Conference on Mathematical Methods and Computation, Jamal Academic Research Journal, Special Issue, February 2014, ISSN 0973-0303. | [8] Radha. K and Arumugam. S, On Strong Product of Two Fuzzy Graphs, International Journal of Scientific and Research Publications, Volume 4, Issue 10, October 2014, ISSN 2250-3153. | [9] Rosenfeld. A, (1975) "Fuzzy graphs". In: Zadeh, L.A., Fu, K.S., Tanaka, K., Shimura, M. (eds.), Fuzzy Sets and their Applications to Cognitive and Decision Processes, Academic Press, New York, ISBN 9780127752600, pp.77-95. |