



Training Students For The Development of Pupils' Mathematical Abilities on The Historical Material With The Use Of Ict

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ABSTRACT

The contents and methods of training the students for the development of mathematical abilities of the pupils at the lessons of the history of mathematics with the use of information and communication technologies are stated.

The most important methods included: an interactive method; search and heuristic method for the solution of educational tasks; decision of the historical tasks; independent work of students; scientific-research work of students.

KEYWORDS : information and communication technology (ICT); mathematical ability; an interactive method.

Introduction

Changes in socio-economic development of Kazakhstan, calling for reform in all spheres, caused radical changes in the education system.

Informatization of education is one of the directions of the State program for developing education in Kazakhstan till 2020 [1]. The tasks set by the President and the Government of the Republic of Kazakhstan in front of the universities require improvement of the quality of training specialists in the conditions of Informatization of mathematical education and the modernization of preparation of future teachers to the profile training of mathematics and to achieve a level of the world standards. In this case, there is a necessity of competitive teachers preparation with information competence.

Because of its importance the problem of development of mathematical abilities of the future expert considered on a range of areas. Relevant to the issue of development of mathematical abilities of the individual in the educational process of professional schools are investigating the spatial imagination (Yakimanskaya, I.S.)

[2], the mental activity of students in the process of solving mathematical problems (Menchinskaya, HA) [3], sources of development and the structure of mathematical abilities (VA Krutetskiy) [4]. Of particular interest in the study of problems of development of mathematical abilities of the future specialists are the works of foreign scientists Alexander V. Borovik, Tony Gardiner [5], Joakim Samuelsson, Karin Forslund Frykedal [6].

Alexander Borovik and Tony Gardiner studied mathematical ability and mathematical skills in children. Joakim Samuelsson and Karin Forslund Frykedal investigated the development of mathematical abilities of students.

Features of teaching of information and communication technologies and the use of ICT in mathematical education are investigated in the works Hammond, Michael [7], Geer, R. & Sweeney, T.[8], Knop, N. & Lanmaster, K.9], Anderson, S., Groulx, J. & Maninger, R [10], Lapchik, M.P.[11].

However, despite the undoubted theoretical and applied significance of the research, it should be noted that the problem of development of mathematical abilities of students in the context of their future professional activity remains open for theoretical research and experimental study.

In the structure of the secondary and higher pedagogical education history of mathematics is considered to be one of the most important subjects. There is a huge gap in the preparation of mathematics teachers: lack of qualified experts-lecturers on the history of mathematics in universities, a lack of literature on the history of science. This

circumstance complicates the use of historical information by teacher in his further practical work. Therefore, implies the need to prepare students to use historical materials at the history of mathematics lessons in bachelor degree and in special courses in magistracy. Thus, preparation of students for the development of pupils' mathematical abilities on the historical material with the use of ICT is relevant.

Nowadays typical usage of information and communication technologies (ICT) in pedagogical activities open for school teachers and university professors of mathematics unique opportunities for intensification of processes of knowledge, individual and collective cognitive activity of learners.

Computer technologies in training of mathematics can be used not only as means of automation of training and knowledge control but also as a tool for the implementation of new didactic approaches to the actualization of the research of mathematical activity, broadening the outlook and developing useful practical skills of a pupil and student based on the inclusion of means and methods of ICT into the subject mathematical activities.

Methodology

The best results are achieved by conducting mathematics lessons with the use of interactive methods of training in the computer class, equipped with multimedia means which have already become traditional (projector, interactive blackboard and etc.) that allows full use of instrumental technology. Interactive training is a training, built on the interaction of learner with the educational environment, the learning environment, which is the area of mined experience [12].

The use of computer technology extends the intellectual abilities of the student, and also carries out training of future teachers to use ICT effectively.

Usage of interactive training for preparation of future teachers solves simultaneously three tasks:

- cognitive, which is connected with the educational situation and mastering of educational information;
- communicative development, which is connected with development of basic communication skills inside and outside the group and with the development of learners' mathematical abilities;
- socio-orientation, which is connected with the education of civil qualities needed for the proper socialization of the individual in the community.

The basis for the implementation of interactive approaches to the content of education is the development and use of interactive exercises and tasks that will be performed by the learners. The main difference of interactive exercises and tasks from the ordinary ones is that

To solve the equation means any way to describe a set of M points of the circle, the coordinates of which are rational numbers.

In task 9 of the book 11 Diophantus's "Arithmetic" it is required: "The given square divide into two squares". Diophantus explains how to solve a specific equation $x^2 + y^2 = 16$, and receives as decisions the numbers

$$x = \frac{16}{5}, y = \frac{12}{5} \quad . \text{ (he makes the substitution } y = kx, \text{ where } k = 2).$$

Only two kinds of uncertain equations were considered in mathematics till Diophantus. The equation

$x^2 + y^2 = z^2$ that had to be solved in integers, appeared in Ancient Babylon. The Pythagoreans found the formula for finding the infinite set of its solutions, but not all. Indeed, in their case.

$$x = \frac{m^2 - 1}{2}, y = m, z = \frac{m^2 + 1}{2}$$

Therefore $z - x = 1$

Let's consider the numbers $x = 56, y = 33, z = 65$. It is clear that they cannot be obtained from the formula of the Pythagoreans, and at the same time

$$(56)^2 + (33)^2 = (65)^2$$

which you can easily verify by direct calculations.

The most general formula for the solution of the equation

$$x^2 + y^2 = z^2 \text{ has led Euclid in the "Basis"}$$

$$x = 2pq, y = p^2 - q^2, z = p^2 + q^2.$$

They give with relatively simple p and q are all integer solutions of the equation which does not have a common divisor.

The equation

$$x^2 - ay^2 = 1$$

for the case when $a = 2$, solved Euclid in his "Principles" not in rational, in integer numbers. His solution for any a number which is not a square, maybe knew Archimedes. He set for Eratosthenes the problem in which $a = 4729494$.

In the XVII century methods of Diophantus found new life in the works of P. Fermat. In comment to the eighth task of book 11 the "Arithmetic", in which it is required to arrange a square number a^2 in the sum of two squares, Fermat wrote: "on the Contrary, it is impossible to decompose or cube on two cubes, nor биквадрат two биквадрата, neither in General, the degree of a large square, on two levels with the same result; I opened this a truly wonderful proof that due to lack of space can not fit these fields" [13, P. 79].

This so-called great theorem, which asserts that the equation

$$x^m + y^n = z^n \text{ при } n > 2, x \cdot y \cdot z \neq 0$$

is not solvable in integer (and hence, in rational) numbers.

Thus, the works of Diophantus had fundamental importance for the development of algebra and number theory.

2. Equations of the third and fourth degrees.

Remarkable results were achieved by scientists of the countries of Islam in the study of equations of third and partly of the fourth degree.

Omar Khayyam (1048-1131) - an outstanding mathematician, astronomer, philosopher and poet, widely known as the author of "the Rubaiyat". He was born in Nishapur city in the field of Khorasan, lived and worked in Samarkand, Bukhara and in the other cities of Central Asia and Iran.

Khayyam developed the geometrical theory of cubic equations. In its algebraic treatise he suggested that the equations of the third degree cannot be solved with the help of ruler and compasses. Khayyam stressed that this decision may be changed only with the help of conic sections".

Khayyam considered equations with arbitrary positive coefficients and found a positive root. He gave a classification of cubic equations, selecting 19 classes, 5 classes of which are reduced to a linear or square

For other 14 classes Khayyam pointed out the method of solution with the help of conic sections: parabolas, hyperbolas, equilateral hyperbolas, branches and circles [14, P. 97].

Like all of mathematicians of the East, Khayyam verbally described equations. So the phrase "the Cube and the roots are equal to the number" it means the equation

To solve it, Khayyam considered the circumference and the parabola (Fig.1). Abscissa of the point of intersection of these curves, which does not coincide with the beginning of the coordinates, is the root of this equation. Indeed, it is possible to record the system

$$\begin{cases} x^2 + y^2 = \frac{ax}{b} \\ y = \frac{x^2}{\sqrt{b}} \end{cases}$$

Hence when $x \neq 0$ we get

$$x^2 + \frac{x^4}{b} = \frac{ax}{b}, \quad bx + x^3 = a$$

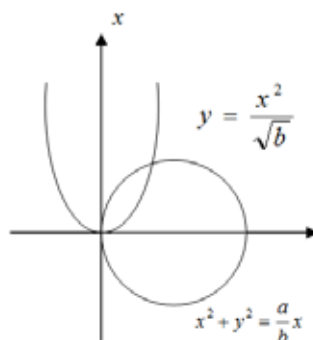


Fig.1. Graphical solution of cubic equations of Omar Khayyam

For each of the 14 classes Khayyam solved the question on the number of positive roots depending on the conditions imposed on the coefficients of the equations. He pointed out that the equation $x^3 + bx = a$ always has a unique positive root. He only considered class of equations

$$x^3 + bx = cx^2 + a.$$

Omar Khayyam forever entered in the history not only as a great mathematician, but also as an astronomer and philosopher:

I calculated - repeats people rumor -
The entire course of time.
I took forever of the calendar:
That what will not know tomorrow,

will not refund - yesterday [14, P.96].

Apparently, Omar here means his reform of the old Persian calendar, after which the calendar gave an error in one day for 5000 years, whereas our current Gregorian calendar, gives an error in one day for

3330 years. His reform was carried out in 1079 y., but later his calendar was replaced by the Muslim lunar calendar.

Mathematics of Islamic countries had a beneficial impact on the development of European science. Scientists of medieval Europe due to the translations from Arabic to Latin, met with the openings of the Egyptians, Babylonians, Indians, Greeks and scientists of Islamic countries.

The XV and XVI century entered to the history of Europe under the name "Renaissance". Flourishing of science occurs mainly in Italy, France and Germany, and later in the end of the XVI century, in the Netherlands, which at that time was experiencing the first in Europe the bourgeois revolution. Italian mathematics of the XVI century have made a major mathematical discovery. They found a formula for solutions of the equations of third and fourth degree.

Then interactive exercises are done collectively.

Here is an example of tasks on a theme "Solution of equations of the third degree".

Training exercises can be done by in two modes: independently or with the step by step hint. During training there is an access to theoretical material.

1. Task (10min).

There is given an arbitrary cubic equation

$$y^3 + ay^2 + by + c = 0 \quad \text{and show that using Solution:}$$

substitution $y = x - \frac{a}{3}$ it can be converted to the form $x^3 + px + q = 0$

Solution:

Let $y = x + \alpha$. We'll get:

$$\begin{aligned} (x + \alpha)^3 + a(x + \alpha)^2 + b(x + \alpha) + c &= 0, \\ x^3 + (3\alpha + a)x^2 + (3\alpha^2 + 2a\alpha + b)x + \dots &+ (\alpha^3 + a\alpha^2 + b\alpha + c) = 0 \end{aligned}$$

ut $3\alpha + a = 0$, i.e. $\alpha = -\frac{a}{3}$. Then this equation

takes the form $x^3 + px + q = 0$

Here a brief historical review is given (3min):

Such record first came to Descartes, who considered equation with both positive and negative coefficients. Mathematics of the XV century solved the equations only with positive coefficients. So they studied three types of cubic equations [13, P. 116]:

$$x^3 = ax + b, \quad x^3 + ax = b, \quad x^3 + b = ax$$

Luca Pacioli in the book "The Sum of arithmetic" (1491) said that for solving cubic equations "art algebra has not yet given way not given a way of squaring the circle". It was one of the first printed books on mathematics, written not in Latin but in Italian.

Words Pacioli became the starting point for studies of Italian scientists.

In 1505, a Professor of mathematics at the University of Bologna Scipio del ferro have found a formula for solving the equation $x^3 + ax = b$, but did not publish it, and said to his disciple Antonio Mario Fiore. In XVI century was widespread competition between scientists held in the form of a dispute. On the twelfth of February 1535, was scheduled tournament between Fiore and Tartaglia. Niccolo Tartaglia (1500-1557) was born in Brescia in a poor family. His mother could not pay for the teacher, and Niccolo learned in school for only the beginning of the alphabet to the letter "K". All other knowledge he possessed himself.

Tartaglia taught mathematics in Verona, Venice, Brescia. Before the tournament with Fiore he received from enemy 30 tasks, saw that they all boil down to the cubic equation

$$x^3 + ax = b, \quad a > 0, b > 0 \quad \text{and exerted every effort to address it.}$$

Finding the formula, Tartaglia solved all tasks, offered him Fiore, and won the tournament. A day after the fight he had found a formula for solving the equation

$$x^3 + ax = b, \quad a > 0, b > 0 \quad (1)$$

It was the greatest discovery. Tartaglia used substitution $x = \sqrt[3]{u} + \sqrt[3]{v}$

2 Task (7 min):

Using the substitution of Tartaglia solve the equation (1).

Solution: (Students solve. The teacher via 10 minutes shows on the interactive board the solution for self-monitoring).

$$(\sqrt[3]{u} + \sqrt[3]{v})^3 = a(\sqrt[3]{u} + \sqrt[3]{v}) + b.$$

$$u + v + 3(\sqrt[3]{u} + \sqrt[3]{v})\sqrt[3]{u} \cdot \sqrt[3]{v} =$$

$$a(\sqrt[3]{u} + \sqrt[3]{v}) + b$$

$$\begin{cases} u + v = b \\ \sqrt[3]{u} \cdot \sqrt[3]{v} = \frac{a}{3} \end{cases} \Rightarrow \begin{cases} u + v = b \\ u \cdot v = \left(\frac{a}{3}\right)^3 \end{cases}$$

Therefore, they are the roots of the quadratic equation

$$y^2 - by + \left(\frac{a}{3}\right)^3 = 0; y_{1,2} = \frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - \left(\frac{a}{3}\right)^3}$$

$$x = \sqrt[3]{\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 - \left(\frac{a}{3}\right)^3}} + \sqrt[3]{\frac{b}{2} - \sqrt{\left(\frac{b}{2}\right)^2 - \left(\frac{a}{3}\right)^3}}$$

It is called now Cardano formula, since it was first published in 1545, in the book of Cardano "Great art, or about algebraic rules" [13, P. 117]. N.I. Lobachevsky (1792-1856) solved the equation of the third order (3) by substituting (4)

$$x^3 + px + q = 0(3)$$

$$x = z - \frac{p}{3z} (4)$$

3 Task: (5 min) Solve the equation (3) using substitution (4).

$$\begin{aligned} z^3 - 3z^2 \frac{p}{3z} + 3z \frac{p^2}{9z^2} - \frac{p^3}{27z^3} + \\ + pz - \frac{p^2}{3z} + q = z^3 - \frac{p^3}{27z^3} + q = 0 \\ 27z^6 + 27z^3q - p^3 = 0; \\ 27t^2 + 27qt - p^3 = 0 \end{aligned}$$

(Students solve. The teacher via 10 minutes shows on the interactive board the solution for self-monitoring. They put estimation for each other on the checklist).

Discussion of results in a group, search the task solution and the lesson is discussion of the results in the group, discussion of the questions asked each other and etc. are realized during the lesson. The element of creativity is manifested in the design of bookmarks, imagery or precision and conciseness of the comments to the decision.

As a result of such systematic work not only mathematical skills are developed, but also informational and communicative competence of students.

Only the marks "4" and "5" are considered in the control work, in case of lower result, it is offered to repeat training and re-do the control work.

Control works (10 min).

Solve the equation:

$$1) x^3 - 36x - 91 = 0; 2) x^3 = 15x + 4.$$

Here is the solution.

1) $x^3 - 36x - 91 = 0$ Use the replacement do Lobachevski (4)

$$x = z + \frac{36}{3z};$$

$$t^2 - 91t + 144 = 0$$

$$\text{Answer } (7, -\frac{1}{2}, -\frac{13}{2})$$

2) $x^3 = 15x + 4$. According to the Tartaglia formula (2) we have:

$$x = \sqrt[3]{2 + \sqrt{4 - 125}} + \sqrt[3]{2 - \sqrt{-121}}$$

$$= \sqrt[3]{2 + 11i} + \sqrt[3]{2 - 11i} = (2 + i) + (2 - i) = 4$$

$$x^3 - 15x - 4x = (x - 4)(x^2 + 4x + 1).$$

$$x^2 + 4x + 1 = 0$$

$$x_{2,3} = -2 \pm \sqrt{4 - 1} = -2 \pm \sqrt{3}; x_2 = -2 + \sqrt{3};$$

$$x_3 = -2 - \sqrt{3};$$

$$\text{Answer : } (4; -2 + \sqrt{3}; -2 - \sqrt{3}).$$

students are dealt checklists.

Summarizing (5min). The trainer summarizes the interactive sessions and puts marks by the control sheets.

Content of the lecture gives huge opportunities for awakening the interest of students to the subject and to develop their ideas about the emergence of new scientific directions.

Not only factual materials, but also stories about the development of mathematical concepts, their connections with practical needs of people, examples, which can be used at school for development of learners' mathematical abilities are included into the content of the course.

Conclusions.

Thus, we have justified that the history of mathematics is a rich source for the formation of the components of mathematical abilities and for education research skills of both students and pupils. The results of experimental work confirmed that the historical task, the historical facts and information, which investigate the origin, formation and development of the object or phenomenon, contribute to the development of mathematical abilities of students and instilling interest in mathematics. The use of ICT can provide show the educational material not only in traditional, but also more accessible for perception by students visual-verbal form. The use of these technologies, combined with traditional forms of educational work allows achieving high efficiency in preparation of future teachers of mathematics.

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