



Homomorphism and Anti homomorphism in Interval Valued Q-L Fuzzy Subhemirings of a Hemiring

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ABSTRACT

In this paper, we study some of the properties of interval valued Q-L fuzzy subhemiring of a hemiring under homomorphism, antihomomorphism and prove some results on these.

2000 AMS SUBJECT CLASSIFICATION: 03F55, 08A72, 20N25.

KEYWORDS : Interval valued fuzzy subset, interval valued Q-L fuzzy subhemiring, and interval valued Q-L fuzzy normal subhemiring.

INTRODUCTION: There are many concepts of universal algebras generalizing an associative ring $(R, +, \cdot)$. Some of them in particular, near rings and several kinds of semirings have been proven very useful. Semirings (called also half rings) are algebras $(R, +, \cdot)$ share the same properties as a ring except that $(R, +)$ is assumed to be a semi group rather than a commutative group. Semi rings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra $(R, +, \cdot)$ is said to be a semi ring $(R, +)$ and (R, \cdot) are semi groups satisfying $a \cdot (b+c) = a \cdot b + a \cdot c$ and $(b+c) \cdot a = b \cdot a + c \cdot a$ for all a, b and c in R . A semi ring R is said to be additively commutative if $a+b = b+a$ for all a, b and c in R . A semi ring R may have an identity 1 , defined by $1 \cdot a = a = a \cdot 1$ and a zero 0 , defined by $0+a = a = a+0$ and $a \cdot 0 = 0 = 0 \cdot a$ for all a in R . A semi ring R is said to be a hemi ring if it is an additively commutative with zero. Interval valued fuzzy sets were introduced independently by Zadeh [13], Grattan-Guinness [4], Jahn [6], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval-valued membership function Jun. Y.B and Kin.K [7] defined an interval valued fuzzy R-subgroups of nearrings. Solairaju. A and Nagarajan.R [10] defined the characterization of interval valued Anti fuzzy Left h-ideals over hemirings. Azriel Rosenfeld [2] defined fuzzy groups. Osman Kazanci, Sultan yamark and serife yilmaz in [8] have introduced the Notion of intuitionistic Q-fuzzification of N-subgroups (subnear rings) in a near-ring and investigated some related properties. Solairaju. A and Nagarajan.R [11], have given a new structure in the construction of Q-fuzzy groups and subgroups [12]. We introduced the concept of interval valued Q-L fuzzy subhemiring of a hemiring under homomorphism, anti homomorphism and established some results.

PRELIMINARIES:

1.1 Definition: Let X be a non-empty set and $L = (L, \geq)$ be a lattice with least element 0 and greatest element 1.

1.2 Definition: Let X be any nonempty set. A mapping $[M]: X \rightarrow D[0,1]$ is called an interval valued L -fuzzy subset (briefly, IVLFS) of X , where $D[0,1]$ denoted the family of all closed subintervals of $[0,1]$ and $[M](x) = [M^-(x), M^+(x)]$, for all x in X , where M^- and M^+ are fuzzy subsets of X such that $M^-(x) \leq M^+(x)$, for all x in X . Thus $[M](x)$ is an interval (a closed subset of $[0,1]$) and not number from the interval $[0,1]$ as in the case of fuzzy subset. Note that $[0] = [0,0]$ and $[1] = [1,1]$.

1.3 Definition: Let X be a non-empty set and Q be a non-empty set. A Q , L -fuzzy subset A of X is function $A: X \times Q \rightarrow [0,1]$.

1.2 Remark: Let D^X be the set of all interval valued L -fuzzy subsets of X , where D means $D[0,1]$.

1.4 Definition: Let $[M] = \{ \langle (x, q), [M^-(x, q), M^+(x, q)] \rangle / x \in X, q \in Q \}$, $[N] = \{ \langle (x, q), [N^-(x, q), N^+(x, q)] \rangle / x \in X, q \in Q \}$ be any two interval valued L -fuzzy subsets of x . We define the following relations and operations:

- i) $[M] \subseteq [N]$ if and only if $M^-(x, q) \leq N^-(x, q)$ and $M^+(x, q) \leq N^+(x, q)$, for all x in X .
- ii) $[M] = [N]$ if and only if $M^-(x, q) = N^-(x, q)$ and $M^+(x, q) = N^+(x, q)$, for all x in X .
- iii) $[M] \cap [N] = \{ \langle (x, q), [M^-(x, q) \wedge N^-(x, q), M^+(x, q) \wedge N^+(x, q)] \rangle / x \in X, q \in Q \}$.
- iv) $[M] \cup [N] = \{ \langle (x, q), [M^-(x, q) \vee N^-(x, q), M^+(x, q) \vee N^+(x, q)] \rangle / x \in X, q \in Q \}$.
- v) $[M]^c = [1] - [M] = \{ \langle (x, q), [1 - M^+(x, q), 1 - M^-(x, q)] \rangle / x \in X, q \in Q \}$

1.5 Definition: Let $(R, +, \cdot)$ be a hemiring. A interval valued Q - L fuzzy subset $[M]$ of R is said to be an interval valued $Q - L$ fuzzy subhemiring (IVFSHR) of R if the following conditions are satisfied:

- (i) $[M](x + y, q) \geq ([M](x, q) \wedge [M](y, q))$
- (ii) $[M](xy, q) \geq ([M](x, q) \wedge [M](y, q))$, for all x and y in R , and q in Q .

1.6 Definition: Let $(R, +, \cdot)$ be a hemiring. A interval valued Q -fuzzy subhemiring $[A]$ of R is said to be an interval valued Q -fuzzy normal subhemiring (IVFNSHR) of R if $[A](xy, q) = [A](yx, q)$, for all x and y in R and q in Q .

1.7 Definition: Let $X \times Q$ and $X' \times Q$ be any two sets. Let $f: X \times Q \rightarrow X' \times Q$ be any function and $[A]$ be an interval valued Q -fuzzy subset in X , $[V]$ be an interval valued Q -fuzzy subset in $f(X) = X'$, defined by $[V](y, q) = \sup_{x \in f^{-1}(y)} [A](x, q)$ for all x in X and y in X' and q in Q . Then $[A]$ is called a pre-image of $[V]$ under f and is denoted by $f^{-1}([V])$.

1. PROPERTIES OF INTERVAL VALUED Q-FUZZY SUBHEMIRINGS

2.1 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The homomorphic image of an interval valued Q-L fuzzy subhemiring of R is an interval valued Q-L fuzzy subhemiring of R' .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a homomorphism. Then, $f(x + y, q) = f(x, q) + f(y, q)$ and $f(xy, q) =$

$f(x, q)f(y, q)$, for all x and y in R . Let $[V] = f([A])$, where $[A]$ is an interval valued Q-L fuzzy subhemiring of R . We have to prove that $[V]$ is an interval valued Q-L fuzzy subhemiring of R' . Now, for $f(x), f(y)$ in R' & q in Q .

$$[V]((f(x), q) + (f(y), q)) = [V](f(x + y, q)) \geq [A](x + y, q) \geq \{[A](x, q) \wedge [A](y, q)\}$$

Which implies that $[V]((f(x), q) + (f(y), q)) \geq ([V](f(x), q) \wedge [V](f(y), q))$. Again,

$$[V]((f(x), q)(f(y), q)) = [V](f(xy, q)) \geq [A](xy, q) \geq \{[A](x, q) \wedge [A](y, q)\} \quad \text{which}$$

implies that $[V]((f(x), q)(f(y), q)) \geq ([V](f(x), q) \wedge [V](f(y), q))$. Hence $[V]$ is an interval valued Q-L fuzzy subhemiring of R' .

2.2 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The homomorphic preimage of an interval valued Q-L fuzzy subhemiring of R' is interval valued Q-L fuzzy subhemiring of R .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a homomorphism.

Then, $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R . Let $[V] = f([A])$, where $[V]$ is an interval valued Q-L fuzzy subhemiring of R' . We have to prove that $[A]$ is an interval valued Q-L fuzzy subhemiring of R . Let x and y in R and q in Q .

$$\begin{aligned} \text{Then } [A](x + y, q) &= [V](f(x + y, q)) \geq [V](f(x, q) + f(y, q)) \geq \\ &\{[V](f(x, q)) \wedge [V](f(y, q))\} = \{[A](x, q) \wedge [A](y, q)\}, \text{ which implies that } [A](x + y, q) = \\ &\{[A](x, q) \wedge [A](y, q)\} \end{aligned}$$

$$\begin{aligned} \text{Again, } [A](xy, q) &= [V](f(xy, q)) \geq [V](f(x, q)f(y, q)) \geq \{[V](f(x, q)) \wedge [V](f(y, q))\} \\ &= \{[A](x, q) \wedge [A](y, q)\} \text{ which implies that } [A](xy, q) = \{[A](x, q) \wedge [A](y, q)\}. \end{aligned}$$

Hence $[A]$ is an interval valued Q-L fuzzy subhemiring of R .

2.3 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The anti-homomorphic image of an interval valued Q-L fuzzy subhemiring of R is an interval valued Q-L fuzzy subhemiring of R' .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be an anti-homomorphism. Then, $f(x + y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R . Let $[V] = f([A])$, where $[A]$ is an interval valued Q-L fuzzy subhemiring of R . We have to prove that $[V]$ is an interval valued Q-L fuzzy subhemiring of R' . Now, for $f(x), f(y) \in R'$ & $q \in Q$. Now, $[V]((f(x), q) + (f(y), q)) = [V]((f(y), q) + (f(x), q)) \geq [A](y + x, q) \geq \{[A](y, q) \wedge [A](x, q)\}$ which implies that $[V]((f(x), q) + (f(y), q)) \geq ([V](f(x), q) \wedge [V](f(y), q))$. Again, $[V]((f(x), q)(f(y), q)) = [V](f(yx), q) \geq [A](yx, q) \geq \{[A](y, q) \wedge [A](x, q)\} = \{[A](x, q) \wedge [A](y, q)\}$, which implies that $[V]((f(x), q)(f(y), q)) \geq ([V](f(x), q) \wedge [V](f(y), q))$. Hence $[V]$ is an interval valued Q-fuzzy subhemiring of R' .

2.4 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The anti-homomorphic preimage of an interval valued Q-L fuzzy subhemiring of R' is an interval valued Q-L fuzzy subhemiring of R .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be an anti-homomorphism. Then, $f(x + y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R . Let $[V] = f([A])$ where V is an anti Q-L fuzzy subhemiring of R' . Let x and y in R & $q \in Q$. where $[V]$ is an interval valued Q-L fuzzy subhemiring of R' . We have to prove that $[A]$ is an interval valued Q-L fuzzy subhemiring of R . Let x and y in R and $q \in Q$. Then $[A](x + y, q) = [V](f(x + y), q) = [V](f(y + x), q) \geq \{[V](f(y), q) \wedge [V](f(x), q)\} = \{[A](x, q) \wedge [A](y, q)\}$, which implies that $[A](x + y, q) = \{[A](x, q) \wedge [A](y, q)\}$. Again, $[A](xy, q) = [V]((f(xy), q)) \geq [V](f(x), q)(f(y), q) \geq \{[V](f(y), q) \wedge [V](f(x), q)\} \geq \{[V](f(x), q) \wedge [V](f(y), q)\} = \{[A](x, q) \wedge [A](y, q)\}$. which implies that $[A](xy, q) = \{[A](x, q) \wedge [A](y, q)\}$. Hence $[A]$ is an interval valued Q-L fuzzy subhemiring of R .

2.5 Theorem: Let $[A]$ be an interval valued Q-L fuzzy subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H . Then $[A] \circ f$ is an interval valued Q-L fuzzy subhemiring of R .

Proof: Let x and y in R and $[A]$ be an interval valued Q-fuzzy subhemiring of a hemiring H . Then we have, $([A] \circ f)(x + y, q) = [A](f(x + y), q) = [A](f(x), q) + (f(y), q) \geq \{[A](f(x), q) \wedge [A](f(y), q)\} = \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. which implies that $([A] \circ f)(x + y, q) \geq \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. And, $([A] \circ f)(xy, q) = [A](f(xy), q) = [A]((f(x), q)(f(y), q)) \geq \{[A](f(x), q) \wedge [A](f(y), q)\} = \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. which implies that $([A] \circ f)(xy, q) \geq \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. Therefore $([A] \circ f)$ is an interval valued Q-fuzzy subhemiring of R .

2.6 Theorem: Let $[A]$ be an interval valued Q-L fuzzy subhemiring of hemiring H and f is an anti-isomorphism from a hemiring R onto H . Then $[A] \circ f$ is an interval valued Q-L fuzzy subhemiring of R .

Proof: Let x and y in R and $[A]$ be an interval valued Q-L fuzzy subhemiring of a hemiring H . Then we have, $([A] \circ f)(x + y, q) = [A](f(x + y), q) = [A](f(y, q) + f(x, q))$
 $\geq \min\{([A](f(x), q) \wedge [A](f(y), q))\} = \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. which implies that
 $([A] \circ f)(x + y, q) \geq \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. And, $([A] \circ f)(xy, q) =$
 $[A](f(yx), q) = [A](f(y), q)f(x, q) \geq \{([A](f(x), q) \wedge [A](f(y), q))\} = \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. which implies that $([A] \circ f)(xy, q) \geq \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. Therefore $[A] \circ f$ is an interval valued Q-L fuzzy subhemiring of the hemiring R .

2.7 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The homomorphic image of an interval valued Q-L fuzzy normal subhemiring of R is an interval valued Q-L fuzzy subhemiring of R' .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a homomorphism. Let $[A]$ is an interval valued Q-L fuzzy normal subhemiring of R . We have to prove that $[V]$ is an interval valued Q-L fuzzy normal subhemiring of $f(R) = R'$. Now, for $f(x), f(y)$ in R' & q in Q . Clearly, $[V]$ is an interval valued Q-L fuzzy subhemiring of $f(R) = R'$. since $[A]$ is an interval

valued Q-L fuzzy subhemiring of R . Now, $[V]((f(x), q)(f(y), q)) = [V](f(xy), q)$
 $\geq [A](xy, q) = [A](yx, q) \leq [V]\{f(yx), q\} = [V]((f(y), q)(f(x), q))$ which implies that
 Then we have, $([A] \circ f)(x + y, q) = [A](f(x + y), q) = [A](f(x), q) + (f(y), q) \geq$
 $\{([A](f(x), q) \wedge [A](f(y), q))\} = \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. which implies that
 $([A] \circ f)(x + y, q) \geq \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. And, $([A] \circ f)(xy, q) =$
 $[A](f(xy), q) = [A]((f(x), q)(f(y), q)) \geq \{([A](f(x), q) \wedge [A](f(y), q))\} = \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. which implies that $([A] \circ f)(xy, q) \geq \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. Therefore $[A] \circ f$ is an interval valued Q-fuzzy subhemiring of R .

2.6 Theorem: Let $[A]$ be an interval valued Q-L fuzzy subhemiring of hemiring H and f is an anti-isomorphism from a hemiring R onto H . Then $[A] \circ f$ is an interval valued Q-L fuzzy subhemiring of R .

Proof: Let x and y in R and $[A]$ be an interval valued Q-L fuzzy subhemiring of a hemiring H . Then we have, $([A] \circ f)(x + y, q) = [A](f(x + y), q) = [A](f(y, q) + f(x, q))$
 $\geq \min\{([A](f(x), q) \wedge [A](f(y), q))\} = \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. which implies that
 $([A] \circ f)(x + y, q) \geq \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. And, $([A] \circ f)(xy, q) =$
 $[A](f(yx), q) = [A](f(y), q)f(x, q) \geq \{([A](f(x), q) \wedge [A](f(y), q))\} = \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. which implies that $([A] \circ f)(xy, q) \geq \{([A] \circ f)(x, q) \wedge ([A] \circ f)(y, q)\}$. Therefore $([A] \circ f)$ is an interval valued Q-L fuzzy subhemiring of the hemiring R .

2.7 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The homomorphic image of an interval valued Q-L fuzzy normal subhemiring of R is an interval valued Q-L fuzzy subhemiring of R' .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a homomorphism. Let $[A]$ is an interval valued Q-L fuzzy normal subhemiring of R . We have to prove that $[V]$ is an interval valued Q-L fuzzy normal subhemiring of $f(R) = R'$. Now, for $f(x), f(y)$ in R' & q in Q . Clearly, $[V]$ is an interval valued Q – L fuzzy subhemiring of $f(R) = R'$.

since $[A]$ is an interval

valued Q – L fuzzy subhemiring of R . Now, $[V]((f(x), q)(f(y), q)) = [V](f(xy), q)$
 $\geq [A](xy, q) = [A](yx, q) \leq [V]\{f(yx), q\} = [V]((f(y), q)(f(x), q))$ which implies that
 $[V]((f(x), q)(f(y), q)) = [V]((f(y), q)(f(x), q))$. Hence $[V]$ is an interval valued Q-L fuzzy normal subhemiring of the hemiring R' .

2.8 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The homomorphic preimage of an interval valued Q-L fuzzy normal subhemiring of R' is interval valued Q-L fuzzy normal subhemiring of R .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a homomorphism. Let $[V]$ is an interval valued Q-fuzzy normal subhemiring of $f(R) = R'$. We have to prove that $[A]$ is an interval valued Q – fuzzy normal subhemiring of R . Let x and y in R and q in Q . Then clearly, $[A]$ is an interval valued Q – fuzzy subhemiring of the hemiring R . Now, $[A](xy, q) = [V](f(xy), q) = [V]\{(f(x), q)(f(y), q)\} = [V]((f(y), q)(f(x), q)) =$
 $[V](f(yx), q) = [A](yx, q)$ which implies that $[A](xy, q) = [A](yx, q)$, for all x and y in R and q in Q . Hence $[A]$ is an interval valued Q-fuzzy normal subhemiring of the hemiring R .

2.9 Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The antihomomorphic image of an interval valued Q-L fuzzy normal subhemiring of R is an interval valued Q-L fuzzy subhemiring of R' .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be an anti homomorphism. Let $[A]$ is an interval valued Q-L fuzzy normal subhemiring of R . We have to prove that $[V]$ is an interval valued Q-L fuzzy normal subhemiring of $f(R) = R'$. Now, for $f(x), f(y)$ in R' & q in Q . Clearly, $[V]$ is an interval valued Q-L fuzzy subhemiring of R' . since $[A]$ is an interval valued Q-L fuzzy subhemiring of R . Now, $[V]((f(x), q)(f(y), q)) = [V](f(yx), q) \geq [A](yx, q) = [A](xy, q) \leq [V](f(xy), q) = [V]((f(y), q)(f(x), q))$ which implies that $[V]((f(x), q)(f(y), q)) = [V]((f(x), q)(f(y), q))$. Hence $[V]$ is an interval valued Q-L fuzzy normal subhemiring of the hemiring R' .

R . Then we have $([A] \circ f)(xy, q) = [A](f(xy), q) = [A]((f(y), q)(f(x), q)) = [A]((f(x), q)(f(y), q)) = [A](f(yx), q) = ([A] \circ f)(yx, q)$ which implies that $([A] \circ f)(xy, q) = ([A] \circ f)(yx, q)$, for all x and y in R and q in Q . Hence $([A] \circ f)$ is an interval valued Q-L fuzzy normal subhemiring of hemiring R .

REFERENCES

1. Akram.M and K.H.Dar On fuzzy d-algebras, Punjab university journal of mathematics,37,61-76(2005).
2. Azriel Rosenfeld, Fuzzy Groups, Journal of mathematical analysis and applications,35,512-517(1971).
3. Biswas.R, Fuzzy subgroups and anti-fuzzy subgroups, Fuzzy sets and systems, 35; 121-124, (1990).
4. Grattan-Guiness, Fuzzy membership mapped onto interval and many valued quantities, Z.math. Logik,Grundladen Math.22, 149-160(1975).
5. Indira.R, Arjunan.K and Palaniappan.N, Notes on IV-fuzzy rw-Closed,IV-fuzzy rw-Open sets in IV-fuzzy topological space, International Journal of Fuzzy Mathematics and systems, Vol.3,Num.1,pp 23-38(2013).
6. Jahn.K.U., Interval wertige mengen, Math Nach.68,115-132(1975).
7. Jun.Y.B and Kin.K.H, Interval valued fuzzy R-subgroups of nearrings, Indian Journal of Pure and Applied Mathematics,33(1),71-80(2002).
8. Osman kazanci, sultan yamark and serife yilmaz, 2007.On intuitionistic Q-fuzzy R-subgroups of near rings, International mathematical forum, 2(59):2899-2910.
9. Palaniappan.N &K.Arjunan Operation on fuzzy and anti fuzzy ideals, Antartical J.Math, 4(1); 59-64, 2007.
10. Solairaju .A and R.Nagarajan, Characterization of interval valued Anti fuzzy Left h-ideals over Hemirings, Advances in fuzzy Mathematics,Vol.4,No.2,129-136(2006).
11. Solairaju .A and R.Nagarajan, 2008.Q-fuzzy left R-subgroups of near rings w.r.t T-norms, Antarctica journal of mathematics, 5:1-2.
12. Solairaju .A and R.Nagarajan, 2009.A new structure and construction of Q-fuzzy groups, Advances in fuzzy mathematics, Volume4 (1):23-29.
13. Zadeh.L.A, The concept of a linguistic variable and its application to approximation reasoning-1, Inform.Sci.8,199-249(1975).