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Fourier Transformation on Model Fitting for Sri Lankan Share Market Returns

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ABSTRACT *Sri Lankan share market returns have wave like patterns. A wave can be viewed either in time domain or in the frequency domain. The frequency domain analysis is known as Spectral Analysis or Fourier analysis. This study was focused on applying Fourier transformation on model fitting for stock returns of Sri Lankan share market. Also it was intended to compare the forecasting ability of above models and Auto Regressive Integrated Moving Average (ARIMA). First Fourier transformation was used to transform monthly returns to series of trigonometric series and multiple regression analysis was used for estimating amplitudes. Monthly returns also were analyzed by ARIMA. Root Mean Square Errors of models fitted with spectral analysis were less than those of ARIMA models. It was concluded that Fourier transformation along with multiple regressions are suitable for forecasting individual company returns of Sri Lankan share market.*

KEYWORDS : Spectral analysis, ARIMA

INTRODUCTION

Wave like patterns are a common feature in natural sciences. Analysis of electrical signals, image processing, sound spectrograms or the analysis of seismic waves is just a few examples. There are primarily two ways of viewing any type of a wave; in the time domain, or in the frequency domain. The traditional way of observing such waves is to view them in the time domain. The time domain is a record of what happens to a parameter of the system versus time or space. The time domain analysis is known as "Time Series Analysis". Box-Jenkins (1976) methodology or Auto Regressive Integrated Moving Average (ARIMA) methodology is a forecasting technique comes under time domain analysis. ARIMA methodology would be successful in forecasting stationary type of series. A stationary series has constant mean and constant conditional variance; as such a stationary series has a wave like pattern.

Waves generally have a period, which can be viewed as the time/ distance of when the wave repeats itself. A period is the distance between two peaks/ troughs of a wave or time between two peaks/ troughs of a wave. A closely related property of the wave period is the frequency. Frequency is the speed at which the wave cycles its periods, and is always expressed in proper speed units, as in `cycles per second'. Frequency domain analyzes a signal with respect to the frequency. This frequency domain representation of signals is called the spectrum of the signal and frequency

domain analysis is known as "Spectral Analysis".

The most important concept in frequency domain analysis is the transformation. Transformation is used to convert a time domain function to a frequency domain and vice versa. This technique was a finding of great mathematician, Baron Jean Baptiste Joseph Fourier of France in years 1768~1830. Fourier has shown that any waveform that exists in the real world can be generated by adding up sine/ cosine waves. Fourier transformation was the first milestone of spectral analysis. As such Spectral analysis is also known as Fourier analysis.

Spectral analysis was initially established in natural sciences such as physics, engineering, geophysics, oceanography, atmospheric science, astronomy etc. and not much used in the field of economics. According to Ehrich (1966), Economists made little use of Spectral analysis in analyzing economic time series at his time. Even today the time domain view is more popular than frequency domain view in analyzing economic variables. It may be due to the lack of understanding in advanced mathematical techniques; trigonometry, calculus and complex numbers.

Granger & Hakanta (1964) emphasized the power of spectral analysis in studying economic variables that exhibit a cyclical behavior or seasonal behavior. According to Granger & Hakanta (1964), Spectral analysis would be successful in analyzing stationary series.

Problem Statement

Share trading is an important part of the economy of a country from both the industry's point of view as well as the investor's point of view. For example, whenever a company wants to raise funds for further expansion or settling up a new business venture, instead of taking loans it can issue shares of the company. On the other hand an investor can get the part ownership of the company through buying shares. Investors have the ability to quickly and easily sell securities. This is an attractive feature of investing in stocks, compared to other less liquid investments such as real estate. In a stock trading system, most important activity that helps to judge the market risk and grab scarce opportunities is forecasting. As such predictability of asset returns in share market has given an immense interest over the past decades.

Box-Jenkins (1976) methodology or Auto Regressive Integrated Moving Average (ARIMA) methodology has been widely applied in forecasting stock returns. But Konarsinghe & Abeynayake (2014-a) has shown that ARIMA models were not sufficient to forecast returns of Sri Lankan share market. Also Konarsinghe & Abeynayake (2014-b) has concluded that returns of individual companies of Colombo Stock Exchange (CSE), returns of sectors as well as returns of the total market were stationary type. It was unable to find any application of spectral analysis in forecasting Sri Lankan share market returns.

This study was mainly focused on model fitting for returns of individual companies of CSE with the help of Fourier transformation. Also it was intended to compare the forecasting ability of models based on

Spectral analysis and ARIMA in Sri Lankan context.

METHODOLOGY

Fourier transformation (spectral analysis) along with the multiple regression analysis and ARIMA models were tested on returns.

Spectral Analysis

The basic idea of spectral analysis is to re express the original time-series Y(t) as a new sequence Y(f), which determines the importance of each frequency component f in the dynamics of the original series. In other words, spectral analysis transform the time domain type series Y(t) into frequency domain type series Y(f). This is achieved using the discrete version of Fourier transformation;

$$Y(f) = \sum_{-\infty}^{\infty} Y(t) e^{-i2\pi ft} \qquad (1)$$

Where f denotes the frequency at which Y (f) is evaluated (Philippe, 2008).

De Moivre's theorem allows writing $e^{-i2\pi ft}$ as;

$$e^{-i2\pi ft} = \cos 2\pi ft + i \sin 2\pi ft \quad (2)$$

Where, *i* is a complex number.

In this study, Fourier transformation was used to transform a series of returns R_t into a series of trigonometric functions as;

$$R_t = \sum_{k=1}^{n} a_k \cos k\omega t + b_k \sin k\omega t \quad (3)$$

$$\omega = \frac{2\pi f}{N} \quad (4)$$

$$k = \begin{cases} n/2; & \text{if n is even} \\ (n-1)/2; & \text{if n is odd} \end{cases} \quad (5)$$

Where a_k and b_k are amplitudes, f= number of peaks/ troughs of series, N= number of observations in the series, n= number of observations per season (12 month seasonality in this analysis) and k is the harmonic of ω (Stephen, 1998). In this study multiple regression analysis was used to estimate the amplitudes.

Auto Regressive Integrated Moving Average (ARIMA)
ARIMA model is a time domain type model. It is a General Linear Process which is given by the formula;

$$\phi_p(B)\Delta^d Y_t = \theta_q(B)\varepsilon_t \quad (6)$$

Where, Y_t = present value, ε_t : present error, B = backshift operator, d= difference

ARIMA models were tested on series of returns R_t.

368 companies were listed in Colombo Stock Exchange representing 20 business sectors in year 2011. Random sample of two business sectors and three companies from each sector were selected. They were; Development Finance Corporation of Ceylon (DFCC), Commercial Bank (COMBANK), Sampath Bank (SAMPATH) from Bank Finance & Insurance (BFI) sector and Agalawatte Plantation Limited (AGAL), Bogawantalawa Tea Estate Limited (BOGAW), Watawala Plantations PLC (WATA) from Plantation (PLT) sector. Daily closing price data for the period 1991 to 2011 were obtained from CSE data library and monthly average prices (P_t) were calculated. Then monthly returns (R_t) for each company were calculated by formula;

$$R_t = \left(\frac{P_t - P_{t-1}}{P_t}\right)100 \quad (7)$$

Where; P_t is the monthly average price.

Monthly returns from year 1991-2008 were used for model fitting and rest of the data was used for model verification. Auto Correlation Functions (ACF) and Partial Autocorrelation Functions (PACF) were obtained to test the stationary of data series. Root Mean Square Errors (RMSE) were used for model selection.

RESULTS AND DISCUSSION
Time series plots were obtained for monthly returns of all six companies. Figures 1 is an example: It clearly shows a wave like pattern without any trend. Other time series also showed similar patterns.

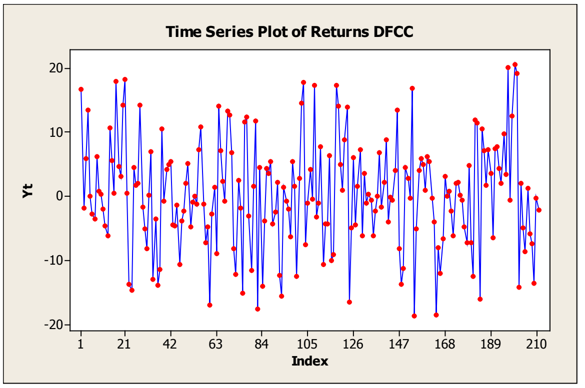

Figure 1: Time Series Plot of DFCC
Stationary of all six return series were confirmed by ACF's and PACF's. Then Fourier transformation method; was tested on each series. Table1 is the summary of outputs:

$$R_t = \sum_{k=1}^{6} a_k \cos k\omega t + b_k \sin k\omega t$$

Table 1: Summary of Regression Analysis with Fourier Transformation

Company	Model	RMSE-Model Fitting	RMSE- Model Verification
DFCC	$R_t = -0.252 - 1.64 Sin 5\omega t$	7.8	10.3
COMBANK	$R_t = 1.33 + 2.76 Cos 3\omega t - 2.59 Cos 4\omega t - 2.24 Sin 4\omega t$	6.6	6.9
SAMPATH	$R_t = -0.647 - 2.28 Cos 4\omega t$	6.4	9.4
AGAL	$R_t = -0.156 - 0.120 Cos 2\omega t - 0.250 Sin 3\omega t$	0.69	1.15
BOGA	Model parameters were not significant	-	-
WATA	Model parameters were not significant	-	-

P values of models DFCC, COMBANK, SAMPATH and AGAL were less than significance level (0.05). It confirmed a linear relationship between returns and trigonometric terms. RMSE of all the models were small in both model fitting and model verification. Residuals of the models were independent and normally distributed.

ARIMA models also were tested on all the series and Table 2 gives the summary of outputs:

Table 2: Summary of ARIMA

Company	Model	RMSE in Model Fitting	RMSE in Model Verification
DFCC	ARIMA (0,1,2)	8.5	10.4
COMBANK	ARIMA (0,0,1)	6.9	6.5
SAMPATH	ARIMA (0,0,1)	6.7	8.8
AGALA	ARIMA (0,1,1)	0.73	1.18
BOGAW	ARIMA (0,1,1)	0.82	1.2
WATA	ARIMA (0,1,1)	0.74	1.32

P values of all the models were less than significance level (0.05) and RMSE of all the models were small in both model fitting and model verification. Residual of the models were normally distributed and uncorrelated.

Results of Table 1 and Table 2 showed that RMSE of Fourier analysis was less than that of ARIMA in model fitting. But Fourier analysis was not successful in two of the six companies.

CONCLUSIONS

This study was mainly focused on testing Fourier transformation along with multiple regressions for forecasting returns of individual companies of CSE. Also it was intended to compare the forecasting ability of above mentioned models and ARIMA in Sri Lankan context. Random sample of six companies were selected for the study and monthly returns were analyzed in both approaches. RMSE's of all the fitted models were small and residuals were normally distributed and uncorrelated. It was concluded that both approaches are suitable in forecasting returns of individual companies of CSE. But RMSE's of models fitted by Fourier transformation along with multiple regressions were less than those of ARIMA models.

It was recommended to test Fourier analysis on more companies of CSE. Also it is suggested to combine the Fourier transformation and ARIMA in forecasting returns of CSE.

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