



PRINCIPLE AND DEVELOPMENT OF EVOLUTIONARY EQUATIONS OF SOLITON PROPAGATING IN NONLINEAR MEDIA

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ABSTRACT

Soliton are narrow and high intensity pulses which can retain their shape by compensating the effects of Self phase modulation and group velocity dispersion mechanisms and non-linear Schrodinger equation (NLSE) is used to describe the propagation of solitons in nonlinear medium like an optical fiber.

KEYWORDS : soliton, Kerr effect, modulation and dispersion.

INTRODUCTION:

The soliton wave concept was suggested by John Scott-Russell [7] which can travel rapidly and unattenuated over very long distance even thousands of kilometers maintaining its shape and size. They interact with other soliton as normal waves but unlike normal waves after interaction emerge out by retaining their shape and amplitude with phase change (or) splits into two solitary waves with the same shape and velocity as before collision as shown in figure 1.

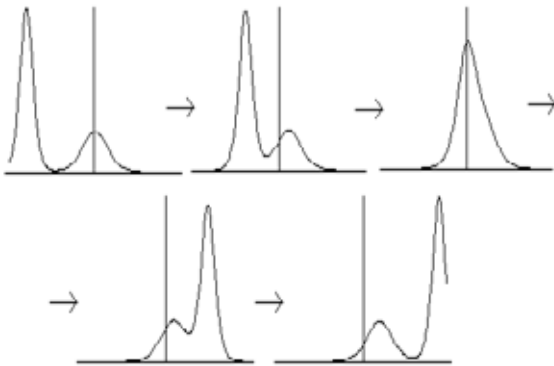


Fig: 1: Solitons before and after collision.
Sources: Wikipedia

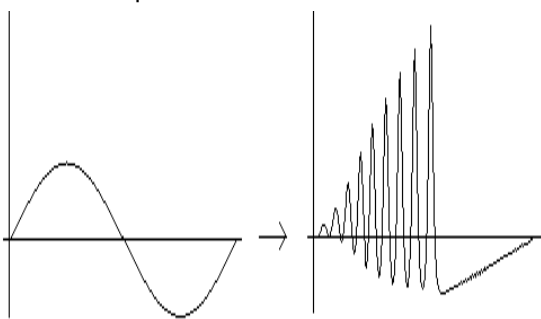


Fig: 2: Solitons break into a train of solitons.
Sources: Report on wave

1.1.2 Properties associated with soliton:

In general a wave cannot travel very long distances and gets affected after collision with another wave with regard to its amplitude, velocity, etc.

But a new kind of waves called as soliton waves could travel rapidly and unattenuated over very long distance even thousands of kilometer [1] and also a soliton of large height and greater velocity when merge with soliton with slower waves of less height, the large height soliton emerges out undistorted with their shape and identity unchanged but there would be change in phase only.

Following are the properties associated with solitons [4,6]

They interact with other soliton as normal waves.

After interaction emerge out by retaining their shape and amplitude but there is phase change

They can travel long distances.

They are permanent and localized waves.

In soliton kerr effect cancel the dispersion effect and thus pulse shape is maintained.

John Scott-Russell set up experiments where he would drop weights at one end of a long but shallow water channel.

From the experiments, he made two key discoveries:

The existence of a solitary wave, i.e, a long, shallow water wave of permanent form.

For these waves, Russell showed that the speed of the wave, v, was given by

$$v = \sqrt{g(h + \zeta)}$$

η is the amplitude of the wave measured from the plane of the water, h is the depth of the channel and g is the measure of gravity ($\eta/h < 1$ for Russell's experiments). Russell noted that there was not an appropriate mathematical theory which described his results.

The existence of solitary waves was verified mathematically, When Korteweg and de Vries derived an equation (the so-called KdV equation) which describes shallow water waves [5].

In the development of soliton concept, paper of Korteweg and deVries set a very important milestone.

Results and Discussion:

1.2.1 Wave equation (NLSE) of soliton:

A wave is represented by a wave equation mathematically. For example, matter wave is represented by Schrodinger wave equation which may be time dependent (or) time independent. Similarly, non linear Schrodinger equation (NLSE) is used to describe the propagation of solitons in optical fiber [9]

It is given by

$$i \left(\frac{\partial u}{\partial z} \right) - \frac{s}{2} \left(\frac{\partial^2 u}{\partial t^2} \right) + N^2 |u|^2 u + i \left(\frac{\dot{a}}{2} \right) u = 0$$

The solution can be written as

$$u(z, t) = \text{sech } h(t) \exp(i \xi / 2)$$

(1.10)

Here $\text{sech}(t)$ represents the hyperbolic secant function. This is a bell-shaped pulse used for soliton pulses. The first term of NLSE equation gives the GVD effects and second term gives non-linear factor [5] through self phase modulation (SPM).

The third term gives the alternation or amplification in other words the loss or gain of energy respectively.

1.2.2: Effect of Self Phase Modulation mechanism in soliton:

Self phase modulation (SPM) [8] solitons are narrow and high intensity pulses that maintains its shape by balancing the pulse compression occurring by SPM and pulse broadening occurring by GVD (group velocity dispersion).

The root cause of SPM(self phase modulation)[2,8] is the change in frequency which is in turn changes by the phase difference introduced (or) developed by the refractive index of fiber which in turn depends on the intensity.

Different parts of travelling pulse have different intensity suffer different phase shift and this results in frequency change (or) also called as frequency chirping. The phase introduced by the fiber after travelling a fiber length L is given by equation

$$\dot{\phi} = \frac{2\delta}{\epsilon} (n_e + n_{ne} I) L_{eff} \quad (1.1)$$

The first term of above expression gives the linear portion of phase and second term gives the non linear phase constant [3, 12].

As phase is varying with time it leads to frequency variation also called as chirping frequency due to SPM.

As phase is varying it leads to change in frequency spectrum or broadening of a pulse due to self phase modulation [10, 11] as shown in figure below.

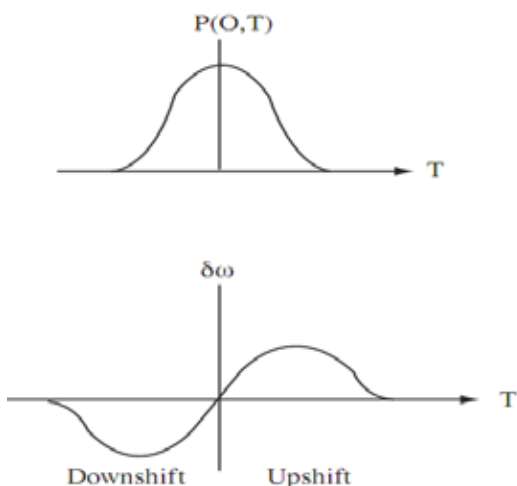


Figure 3: changes in frequency spectrum or broadening of a pulse due to self phase modulation.

1.2.3: Effect of GVD mechanism in soliton:

The effect of GVD is the overlapping of neighboring pulses which is caused by the group velocity of the signal. Due to overlapping error occurs at the receiver.

$$u_g = c \left(\frac{d\beta}{dk} \right)^{-1}$$

The group velocity is given by

And the delay difference over a length L is given by

$$\delta t = \frac{d\tau_g}{d\omega} \delta\omega = \frac{d}{d\omega} \left(\frac{L}{u_g} \right) \delta\omega = L \left(\frac{d^2\beta}{d\omega^2} \right) \delta\omega$$

Where $\hat{a}_2 \equiv \frac{d^2\hat{a}}{d\hat{u}^2}$ is called the GVD parameter.

The first term of NLSE equation gives the GVD effects in which dispersion tends to broaden pulse. The second term gives non-linear factor which shows the relationship between refractive index of the fiber and intensity of light. This leads to broadening of frequency spectrum of pulse through self phase modulation (SPM).

Thus, the dispersive and nonlinear terms are complementary phase shifts[10] and upon integration leads to phase shift but maintain its shape and size.

The third term gives the alternation or amplification in other words the loss or gain of energy.

CONCLUSIONS:

Soliton are narrow and high intensity pulses which can retain their shape by compensating the effects of SPM and GVD Mechanisms and are represented by as nonlinear Schrödinger equation.

If a wave is in motion, the compression of pulses occurs by SPM (Self phase modulation) and broadening of pulse occurs by GVD (Group velocity dispersion).

If these two mechanisms compensate each other the pulse do not change shape (called fundamental solitons).If Pulses undergo periodic change in shape then they are called higher order soliton.

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REFERENCES

1) Agrawal G.P. (1997) "Fiber Optic Communication Systems", 2nd edition, Wiley, New York. | 2) Ablowitz M.J, Kaup, D.J., Newell, A.C. and H. Segur (1974) "The inverse scattering transform – Fourier analysis for nonlinear problems", Stud. Appl. Math, 53, 249. | 3) Biswas and A and S. Konar (2005) "Soliton-solitons interaction with kerr law non-linearity," Journal of Electromagnetic Waves and Applications, Vol. 19, No. 11, 1443–1453. | 4) Biswas.A et al (2006) "Soliton-soliton interaction with parabolic law nonlinearity," Journal of Electromagnetic Waves and Applications, Vol. 20, No. 7, 927–939. | 5) Gardner C.S et al (1974) "The Korteweg-deVries equation and generalizations. VI. Methods for exact solution", Comm. Pure Appl. Math, 27, 97. | 6) Gardner.C.S, Greene, J.M., Kruskal, M.D. and Miura, R.M., (1967) "Method For Solving The Korteweg deVries Equation", Phys. Rev. Lett., 19, 1095. | 7) Hasegawa.A and Y. Kodma (1995) "Soliton in Optical Communication", Clarendon Press, Oxford. | 8) Haus.H and W. S. Wong (1996) "Soliton in optical communications," Rev. Mod. Phys., Vol. 68, 432–444. | 9) Haus .H. A. (1993) "Optical fiber solitons: Their properties & uses", Proc. IEEE, Vol. 81, 970–983. | 10) Scott Russell. J (1844) "Report on waves", fourteenth meeting of the British Association for the Advancement of Science. | 11) Stolen .R. H. and C. Lin (1978) "Self-phase modulation in silica optical fibers," Physical Review, Vol. 17, No. 4, 1448–1453. | 12) Singh .S.P. & N. Singh (2007) "Nonlinear effects in optical fibers: Origin, management and applications," Progress In Electromagnetics Research, PIER 73, 249–275. |