Research Paper **Mathematics**

Graph Theory: Importance and Scope

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ABSTRACT *Graphs are discrete structures consisting of vertices and nodes that connect these vertices. In real world, there are many problems that can be represented with the help of graph. In this paper the main aim is to explore the importance of graphs in various fields. In this paper, an overview is presented to demonstrate its importance in multiple fields which includes Computer Science, Pure Mathematics, Operation Research, Bio Chemistry, Sociology and other Sciences.*

INTRODUCTION

A graph is symbolic representation of a network and connectivity. It is concerned that how networks can be encoded. A graph is an ordered pair of set of vertices V and set of edges E. Vertices can be presented with the points in a plane and these data types are called nodes. A line connecting these nodes is called an edge. If a graph is directed then edges are ordered pairs and if a graph is undirected then its edges are unordered pairs. The order of the graph is equal to the number of vertices in it. LEONHARD EULAR became the father of graph theory when he settled a famous unsolved problem called the Konigsberg Bridge Problem. Graphs has many features like data flow diagram, decision making ability, displays relationships among objects, easy alterations and modifications in existing system etc.

AREAS OF GRAPH THEORY

A concept of graph theory is widely growing and moving into the mainstream of mathematics as it is playing a vital role in following areas:

- In Pure Mathematics
- In Computer Science
- In Operation Research
- In Sociology
- In Science

IMPORTANCE IN PURE MATHEMATICS
Cantor Schroder-Bernstein Theorem

Graph theory is used in this theorem. This theorem states that For two sets A and B, if there is an injective mapping from A to B and an injective mapping from B to A then there is a one-one and onto mapping from A to B i.e. A and B have the same number of elements. Now, A and B are two disjoint sets. Each component is either a one-way infinite path or a two way infinite path. There is a set of edges in each component such that each vertex is incident with one of these edges. In this, bipartite graph is used. Therefore, in each component, the vertices from A have same number of elements as the subset of vertices from B.

Fermat's Theorem

Fermat's theorem can be proved in different ways; the first proof is given by Euler. Fermat's theorem states that let *'a'* be a natural number and *'p'* be a prime number such that *'a'* is not divisible by *'p'* then a^p-a is divisible by *'p'*. In this theorem, graphs are used. We take a sequence of natural numbers between 1 to *'a'* as a vertex set. The vertex set has a^p-a elements. Let x= $(x_1, x_2 \ldots x_p)$, y = $(x_p, x_1, x_2 \ldots x_{p-1})$ Therefore each vertex is of degree 2 and each component is a cycle of length p. Therefore number of component is $(a^p$-a)/p. Hence a^p-a is divisible by *'p'.*

IMPORTANCE IN COMPUTER SCIENCE
Network System and its Security

Graph theory is widely used in representation of Network System. Graph theory in networking can be observed in two categories: Graphical representation (Topology) and Network theory. Topol-

ogy is the way to represent a network structure in various formats that can help in making the problem easier and deriving more accurate results. The term network and graph are similar as both refer to topology (structure) in which vertices and edges are arranged. Some basic topologies are Star topology, Ring topology, Bus topology, Mesh topology, Tree topology and Mesh topology. The term Network theory represents the different methodologies to analyze a graph and applying network theory using a topology.

The team of computer scientists at the Virology and Cryptology Lab, ESAT and the French Navy, ESCANSIC2-4, has recently used the vertex cover algorithm to design optimal strategies for protecting the network against virus attacks in real time. The main idea is to find a minimum vertex cover in the graph whose vertices are the routing servers and whose edges are the connections between routing servers. This is an optimal solution for designing the network defense strategy. The network activity is used to solve large number of combinatorial problems.

Data Mining

Graph theory plays a important role in data mining as graph mining. Graph mining describes the relational aspect of the data. The different approaches of graph mining are Sub graph categories, Sub graph isomorphism, Mining measures, Solution methods and invariants.

Data Structures

The structuring or organizing of data into information so that operations like traversing, searching, sorting, merging, insertion, deletion, etc. becomes easy, Such logical and mathematical model is called as 'Data Structures'5. The choice of data structuring model depends upon two factors:

It must depict actual relationship among data.

The structure should be simple and easy to process the data into information if required.

The non linear representation of data into memory is possible using graph theory. Arbitrary relationship among data is represented by a graph and its adjacency matrix. Many graph algorithms requires traversing the nodes and edges of a graph systematically. There are two standard ways to traverse a graph:

Breadth – First search
Depth – First Search

The Breadth – First search technique uses queue data structure and Depth – First search technique uses stack data structure.

Software Engineering

Graphs are widely used in engineering the software at every level of the engineering model. The various engineering models are waterfall model, Spiral model, Prototype model, Iterative model etc. Following is the role of graph theory at levels of software engineering models:

Phases	Role of Graph Theory
Requirement Analysis and Specification	Data Flow Diagrams (DFD)
Design Phase	Graphical design is used for depicting relationship among modules
Testing	Control flow of a program associated with McCabe's complexity which describes directed graphs for representing the sequence of instructions executed.

Operating System

An operating system6 is a program that acts as an interface between user and computer hardware. The purpose of operating system is to provide an environment in which a user can execute programs in an efficient and convenient manner. Graph theory plays an important role in operating system in solving job scheduling and resource allocation problems. The concept of graph coloring is applied in job scheduling problems of CPU. Jobs are assumed as vertices of a graph and there will be an edge between two jobs that cannot be executed simultaneously. Graphs are also used in disk scheduling algorithms.

Database Designing

The graph structure has an important role in designing database7 as it gives fast implementation process. It uses a graph database that uses graph representation with nodes, edges and properties to represent and store data. It provides storage system with index free adjacency list and it has robust tool like query. Moreover it is easy to depict relationship among data in graph database representation.

Website Designing

The graph theory is used to model the website designing process, where web pages are represented by vertices and the hyper links between them are represented by edges in the graph. This concept is known as web graph. In graph theory such a graph is called as complete bipartite graph. Graph representation helps in finding all connected components and using directed graph we can evaluate website utility and link structure.

IMPORTANCE IN OPERATION RESEARCH
Time Table Management Problem

Graphs are widely used in solving Timetable management problem. Let us assume that in a school we have N teachers and M subjects, suppose we need a particular teacher to teach a particular subject in the respective period and authority needs to prepare a time table using the minimum possible number of periods. This is Timetable management problem. It can be solved by using Bipartite multiple graphs. To solve this problem we partitioned vertex set into two disjoint subsets. One is for teachers and other is for subjects. The relationship among both subsets is represented by edges.

Road Maps

Graphs can be used to demonstrate road maps. In road maps, Intersecting points are represented by vertices and roads are represented by edges. One way roads are represented by directed graphs and two way roads are represented by undirected graphs. Multiple undirected edges represent multiple two way roads connecting the same two intersections. The multiple one way roads that start at one intersection and end at other intersection are represented by multiple directed edges. The road having same starting and terminal point are represented by loops. Therefore to represent road maps, mixed graphs are required.

Travelling Sales Man Problem

Graphs can be used in travelling sales man problem to visit a route. For these we use Hamiltonian paths and circuits. Assuming a salesman has to visit 'n' cities. He wants to start from a particular city and visit each city once and returning to his home city without visiting single city twice. His main purpose is to select the sequence in which the cities are visited in such a manner that his total travelling time or distance is minimized.

Job Assignment

Graphs can be used in assigning optimal jobs for best throughput. Suppose there are 'n' employees in an organization and 'm' different jobs that need to be done where number of employees are less than number of jobs. Here each employee is trained to do one or more of these jobs. We can use a graph to model employee capabilities. To solve this problem we partitioned vertex set into two disjoint subsets. We represent employee and jobs as two disjoint set of vertex respectively. For each employee we include an edge from the vertex representing that employee to the vertices representing all jobs that the employee has been trained to do. To complete the assignment, we must assign jobs to the employees so that every job has an employee assigned to it and no employee is assigned more than one job.

Shortest Path Algorithm

A graph plays a vital role in selecting shortest and optimal path from available multiple paths. There are various algorithms for calculating shortest path like Dijkstra's algorithm, Minimum spanning tree, Kruskal Algorithm and Prim's Algorithm. In Dijkstra's algorithm, we obtain a set of vertices whose shortest path from source is already known. In this, all weights assigned to edges must always be positive. Firstly take a source vertex, and then we find all paths from source vertex to all other vertices without going through any other vertex. Now we take the vertex in vertex set which is nearest to source vertex and find the shortest path to all the vertices through this vertex and update the values. We repeat this procedure until all the vertices are not included in the set. After traversing all the vertices we obtain the shortest path to all the vertices from the source vertex.

PERT AND CPM

Graph theory is also used widely in scheduling the project tasks. The most popular and successful applications of graphs in operation research is the planning and scheduling of large complicated projects. The best well known problems are PERT (Project Evaluation and Review Technique) and CPM (Critical Path Method)8. CPM and PERT are operation research techniques that were developed in the late 1950. Once the activity network representation has been worked out, resources are allocated to each activity. Resource allocation is typically done using a Gantt chart. After resource allocation is done, a Project Evaluation and Review Technique (PERT) chart representation is developed. A path in the activity network graph is any set of consecutive nodes and edges in this graph from the starting node to the last node. A critical path consists of set of dependent tasks that need to be performed in a sequence and which together take the longest time to complete.

IMPORTANCE IN SOCIOLOGY
Acquaintanceship Graph

Graphs are used to represent various relationships between people. For Example Simple graph is used to represent the acquaintance relationship among each other that whether they are known to each other or not. Each person is represented by a vertex and relationship is represented by edges. An undirected graph depicts a relationship that they are known to each other.

Influence Graph

It is observed that some people can influence the thinking of others; in this case directed graphs are used. Each person can be represented as a vertex and there is a directed edge when the person influences the other person. It does not contain loops and multiple edges. For Example, in a college Principal can influence teachers and teachers can influence students.

Measuring performance and Progress report

Graph theory is widely used in an organization to measure the performance of an employee for certain period. The graph can depict the progress or degrading of an employee or region. It will help an employee to motivate and work with more dedication.

Other Applications

Possible applications include targeted advertising, identifying leaders of terrorist networks, central hubs in transportation networks, or dominant species in an ecosystem.

IMPORTANCE IN SCIENCE
Niche overlap graph in Ecology

Graphs are used to represent the interaction of different species of animals. In an ecosystem, the competition between the species can be explained by niche overlap graph. Each species is represented as a vertex and an undirected edge represents that species compete. It does not contain loops and multiple edges.

Graphs in Chemistry

Graphs are used in the field of chemistry to model chemical compounds, study of molecules, construction of bonds and study of atoms. In computational biochemistry some sequences of cell samples have to be excluded to resolve the conflicts between two sequences. This is modeled in the form of graph where the vertices represent the sequences in the sample. An edge will be drawn between two vertices if and only if there is a conflict between the corresponding sequences. The aim is to remove possible vertices, (sequences) to eliminate all conflicts. In brief, graph theory has its unique impact in various fields and is growing large now a days. The subsequent section analyses the applications of graph theory especially in computer science.

Bioinformatics – DNA fragment assembly

The Swiss biochemist Frederich Miescher first observed DNA in the late 1869. DNA is Deoxyribo Nucleic Acid is a molecule that is found in every living organism. It contains the instruction an organism needs to develop, live and reproduce. These instructions are found in every cell, and are assed down from parents to their children. DNA9 is made up of molecules called nucleotides. Each nucleotide contains a phosphate group, a sugar group and a nitrogen base. These four types of nitrogen bases are Adenine (A), Thymine (T), Guanine (G) and Cytosine (C). DNA instructions are determined on the order of these bases. DNA sequencing and fragment assembly is the problem of reconstructing full strands of DNA based on the pieces of data recorded. The Eulerian circuit in Graph theory is implemented to solve the problem of DNA fragment assembly.

In Medical Sciences

The Graph theory is used in biology and conservation efforts where a vertex represents regions where certain species exist and the edges represent migration path or movement between the regions. This information is important when looking at breeding patterns or tracking the spread of disease, and to study the impact of migration that affect other species. The Graphs are also widely used in medical ultrasound projections. Medical ultrasound images is an important type of medical images and is widely used in medical diagnosis, Compared with other medical imaging methods, ultrasound imaging has the advantages of non-traumatic to human body, real-time display, low cost, ease to use. Now a day's 3D imaging techniques are widely used in medical sciences. The graph theory is used to construct a 3D graph is called 3D graph based segmentation algorithm. It can generate a set of minimum spanning trees each of which corresponds to a 3D sub region. Graph based segmentation model is much far better than 3D active contour model (3D snake) as it is less complex therefore results are computed in less time. It is more accurate as well.

CONCLUSION

The objective of this paper is to bring in the notice of readers about the importance and the relevance of the graph theory in every area like pure mathematics, Computer science, sociology, operation research and other science applications. There are a number of other interesting application areas where graph theory has played a vital role. An overview is presented especially to project the idea of graph theory.

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