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Solving Fully Fuzzy Transportation Problems with Mixed Fuzzy Constraints Using Linear Programming Problems

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ABSTRACT

A new method namely, split and bounded method is proposed to find an optimal more-for-less fuzzy solution to fully fuzzy transportation problems (FFTP) with mixed fuzzy constraints by using crisp linear programming (LP) technique. The split and bounded method do not require any initial basic feasible solution and an optimal solution (MODI method) to solve FFTP with mixed fuzzy constraints. In this method, the optimal fuzzy solutions do not contain any negative part of the values of the decision variables. Also, fuzzy ranking functions were not used. The proposed method is an appropriate method to find an optimal fuzzy solution to FFTP with mixed fuzzy constraints occurring in real life situations.

KEYWORDS : Triangular fuzzy number, linear programming problems, fuzzy linear programming, fuzzy transportation problems with mixed constraints, split and bounded method.

INTRODUCTION

A fuzzy transportation problem (FTP) is a transportation problem (TP) in which the transportation costs, supply and demand quantities are fuzzy quantities. The aim of the FTP is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and fuzzy demand limits. Bellman and Zadeh [3] and Zadeh [9] introduced the notion of fuzziness.. Many researchershave developed various algorithms to solve FTP with equality constraints. In real life, most of the TPs have mixed constraints accommodating many applications that go beyond transportation related problems to include job scheduling, production inventory, production distribution, allocation problems and investment analysis. The TPs with mixed constraints are not addressed in the literature because of the rigor required to solve these problems optimally.

The more-for-less (MFL) paradox in a TP occurs when it is possible to ship more total goods for less (or equal) total cost while shipping the same amount or more from each origin and to each destination keeping all shipping costs nonnegative. There are various methods for finding MFL solution for TPs in the existing literature [1,2, 4, 5, 8]. The major goal of the MFL method is to minimize the total cost and not merely maximize the shipment load transported.

Based on the literature study, it was found that there is a little work carried out to find an optimal MFL fuzzy solution to FFTP with mixed fuzzy constraints. Recently, Pandian and Natarajan [7] have suggested a new algorithm based on fuzzy zero point method for finding an optimal MFL fuzzy solution for FFTP with mixed fuzzy constraints.

PRELIMINARIES

We need the following mathematical orientated definitions of fuzzy set, fuzzy number and membership function, which can be found in Zadeh [9].

DEFINITION 1 Let A be a classical set and $\mu_A(x)$ be a membership function from A to [0,1]. A fuzzy set $\widetilde{A}$ with the membership function $\mu_A(x)$ is defined by

$$\widetilde{A} = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1]\}.$$

DEFINITION 2A fuzzy number $\widetilde{a}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) where a_1, a_2 and a_3 are real numbers and its membership function $\mu_{\widetilde{a}}(x)$ is given below:

$$\mu_{\widetilde{a}}(x) = \begin{cases} (x-a_1)/(a_2-a_1) & \text{for } a_1 \le x \le a_2 \\ (a_3-x)/(a_3-a_2) & \text{for } a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

DEFINITION 3 Let (a_1, a_2, a_3) and (b_1, b_2, b_3) be two triangular fuzzy numbers. Then

(i) $(a_1,a_2,a_3) \oplus (b_1,b_2,b_3) = (a_1+b_1, a_2+b_2, a_3+b_3)$.

(ii) $(a_1,a_2,a_3) \ominus (b_1,b_2,b_3) = (a_1-b_3, a_2-b_2, a_3-b_1)$.

(iii) $k(a_1,a_2,a_3) = (ka_1, ka_2, ka_3)$, for $k \ge 0$.

(iv) $k(a_1,a_2,a_3) = (ka_3, ka_2, ka_1)$, for $k < 0$.

(v) $$(a_1,a_2,a_3) \otimes (b_1,b_2,b_3) = \begin{cases} (a_1b_1, a_2b_2, a_3b_3), & a_1 \ge 0, \\ (a_1b_3, a_2b_2, a_3b_3), & a_1 < 0, a_3 \ge 0, \\ (a_1b_3, a_2b_2, a_3b_1), & a_3 < 0. \end{cases}$$

Let $F(R)$ be the set of all real triangular fuzzy numbers.

Definition 2.4 Let $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ be in $F(R)$, then

(i) $\widetilde{A} \approx \widetilde{B}$ iff $a_i = b_i, i = 1,2,3$; (ii) $\widetilde{A} \preceq \widetilde{B}$ iff $a_i \le b_i, i = 1,2,3$

(iii) $\widetilde{A} \succeq \widetilde{B}$ iff $a_i \ge b_i, i = 1,2,3$ (iv) $\widetilde{A} \succeq \widetilde{0}$ iff $a_i \ge 0, i = 1,2,3$.

(v) $\widetilde{A}$ is said to be integer if $a_i \ge 0, \forall i = 1$ to 3 are integers.

Consider the following fully fuzzy transportation problem in which all parameters, that is, decision variables, transportation costs, supplies and demands, are triangular fuzzy numbers:

(P) $\quad$ Minimize $\widetilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c}_{ij} \otimes \widetilde{x}_{ij}$

Subject to

$$\sum_{i=1}^{m} \widetilde{x}_{ij} \succeq \widetilde{a}_i\,,\ j \in Q\ ;\ \sum_{i=1}^{m} \widetilde{x}_{ij} \preceq \widetilde{a}_i\,,\ j \in T\ ;\ \sum_{i=1}^{m} \widetilde{x}_{ij} \approx \widetilde{a}_i\,,\ j \in S \tag{1}$$

$$\sum_{j=1}^{n} \widetilde{x}_{ij} \succeq \widetilde{b}_j\,,\ i \in U\ ;\ \sum_{j=1}^{n} \widetilde{x}_{ij} \preceq \widetilde{b}_j\,,\ i \in V\ ;\ \sum_{j=1}^{n} \widetilde{x}_{ij} \approx \widetilde{b}_j\,,\ i \in W \tag{2}$$

$$\widetilde{x}_{ij} \succeq \widetilde{0}\,, \text{ for } i = 1, 2,...., m\,;\ j = 1, 2,...., n \text{ and are integers} \tag{3}$$

wherem = the number of supply points; n = the number of demand points; Q, T *and* S are pairwise disjoint subsets of $\{1,2,3,...,n\}$ such that $Q \cup T \cup S = \{1,2,3,...,n\}$; U, V *and* W are pairwise disjoint subsets of $\{1,2,3,...,m\}$ such that $U \cup V \cup W = \{1,2,3,...,m\}$; $\widetilde{x}_{ij} \approx (x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3})$ is the number of fuzzy units shipped from supply point i to demand point j; $\widetilde{c}_{ij} \approx (c_{ij}^{1}, c_{ij}^{2}, c_{ij}^{3})$ is the number of fuzzy cost of shipping one unit from supply point i to the demand point j; $\widetilde{a}_i \approx (a_i^{1}, a_i^{2}, a_i^{3})$ is the number of fuzzy supply at supply point i and $\widetilde{b}_j \approx (b_j^{1}, b_j^{2}, b_j^{3})$ is the number of fuzzy demand at demand point j.

REMARK 1 If $Q = R = U = V = \phi$, the problem (P) becomes the FFTP with equality fuzzy constraints.

DEFINITION 5A set of triangular fuzzy numbers $\widetilde{X}_{ij} = \{\, \widetilde{x}_{ij} = (x_{ij}, y_{ij}, t_{ij})\,,\ i = 1,2,...,m$ and $j = 1,2,...,n\}$ is said to be a feasible fuzzy solution to the problem (P) if it satisfies the conditions (1) to (3).

DEFINITION 6 A feasible fuzzy solution $\widetilde{X}_{ij} = \{\, \widetilde{x}_{ij} = (x_{ij}, y_{ij}, t_{ij})\,, i = 1,2,...,m$ and $j = 1,2,...,n\}$ of the problem (P) is said to be an optimal fuzzy solution to the problem (P) if $\widetilde{Z}(\widetilde{X}) \le \widetilde{Z}(\widetilde{U})$ for all feasible $\widetilde{U}$ of the problem (P).

THE SPLIT AND BOUNDED METHOD

Let the parameters $\widetilde{a}_i, \widetilde{c}_{ij}, \widetilde{x}_{ij}$ and $\widetilde{b}_j$ be the triangular fuzzy numbers $(a_i^{1}, a_i^{2}, a_i^{3}), (c_{ij}^{1}, c_{ij}^{2}, c_{ij}^{3})$, $(x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3})$ and $(b_j^{1}, b_j^{2}, b_j^{3})$ respectively. Then, the problem (P) can be written as follows:

Minimize $(Z_1, Z_2, Z_3) = \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij}^{1}, c_{ij}^{2}, c_{ij}^{3}) \otimes (x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3})$

Subject to

$$\sum_{j=1}^{n} (x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3})\ \{\preceq / \approx / \succeq\}\ (a_i^{1}, a_i^{2}, a_i^{3}), \text{ for } i = 1, 2,...., m$$

$$\sum_{i=1}^{m} (x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3})\ \{\preceq / \approx / \succeq\}\ (b_j^{1}, b_j^{2}, b_j^{3}), \text{ for } j = 1, 2,...., n$$

$$(x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3}) \succeq \widetilde{0}, \text{ for } i = 1, 2,...., m\,;\ j = 1, 2,...., n \text{ andare integers.}$$

Jayalakshmi and Pandian [6] have proposed a method namely, bound and decomposition method to a fully fuzzy linear programming (FFLP) problem to obtain an optimal fuzzy solution, by decomposing into three crisp LP problems namely, middle level problem (MLP), upper level problem (ULP) and lower level problem (LLP) as follows:

(MLP) Minimize $Z_2 = \sum_{j=1}^{n}$ middle value of $\left(\sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij}^{1}, c_{ij}^{2}, c_{ij}^{3}) \otimes (x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3}) \right)$

subject to

Constraints in the decomposition problem in which at least one decision variable of the (MLP) occurs and all decision variables are non-negative integers.

(ULP) Minimize $Z_3 = \sum_{j=1}^{n}$ upper value of $\left(\sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij}^{1}, c_{ij}^{2}, c_{ij}^{3}) \otimes (x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3}) \right)$

subject to

$$\sum_{j=1}^{n} \text{upper value of} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij}^{1}, c_{ij}^{2}, c_{ij}^{3}) \otimes (x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3}) \right) \geq Z_2^{\circ};$$

Constraints in the decomposition problem in which at least one decision variable of the (ULP) occurs and are not used in (MLP) ;

all variables in the constraints and objective function in (ULP) must satisfy the bounded constraints ;

replacing all values of the decision variables which are obtained in (MLP) and all decision variables are nonnegative integers.

and

(LLP) Minimize $Z_1 = \sum_{j=1}^{n}$ lower value of $\left(\sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij}^{1}, c_{ij}^{2}, c_{ij}^{3}) \otimes (x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3}) \right)$

subject to

$$\sum_{j=1}^{n} \text{lower value of} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij}^{1}, c_{ij}^{2}, c_{ij}^{3}) \otimes (x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3}) \right) \leq Z_2^{\circ}$$

Constraints in the decomposition problem in which at least one decision variable of the (LLP) occurs which are not used in (MLP) and (ULP) ;

all variables in the constraints and objective function in (LLP) must satisfy the bounded constraints ;

replacing all values of the decision variables which are obtained in (MLP) and (ULP) and all decision variables are nonnegative integers,where Z_2° is the optimal objective value of (MLP).

We, now introduce a new algorithm based on crisp linear programming problem for finding an optimal MFL fuzzy solution for FFTPs with mixed fuzzy constraints.

The proposedalgorithm proceeds as follows.

STEP 1: Convert the given FFTP with mixed fuzzy constraints into FFLP problems.

STEP 2: Construct (MLP), (ULP) and (LLP) problems from the given the FFLP problems.

STEP 3: Using existing linear programming technique, solve the (MLP) problem, then the (ULP) problem and then, the (LLP) problem in the order only and obtain the values of alldecision variables x_{ij}, y_{ij} *and* t_{ij}, for all $i = 1, 2,...., m$ and $j = 1, 2,...., n$ are non-negative integers and values of all objectives Z_1, Z_2 and Z_3. Let the decision variables values be $x_{ij}^{\circ}, y_{ij}^{\circ}$ *and* t_{ij}°, j = 1,2,…,m and objective values be Z_1°, Z_2° and Z_3°.

STEP 4: The fuzzy optimal solution to the given FFLP problems is $\widetilde{x}_{ij}^{\circ} = (x_{ij}^{\circ}, y_{ij}^{\circ}, t_{ij}^{\circ})$, for all $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$, the minimum fuzzy objective value is $(Z_1^{\circ}, Z_2^{\circ}, Z_3^{\circ})$ (by Bound and Decomposition Method [6]).

STEP 5: The optimal MFL fuzzy solution to FFTP with mixed fuzzy constraints is $\widetilde{x}_{ij}^{\circ} \approx (x_{ij}^{\circ}, y_{ij}^{\circ}, t_{ij}^{\circ})$, for all $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$, the total minimum fuzzy transportation cost is $\widetilde{Z} \approx (Z_1^{\circ}, Z_2^{\circ}, Z_3^{\circ})$.

REMARKS 2 In the case of FFTP with mixed fuzzy constraints involving trapezoidal fuzzy numbers and variables decompose it into four crisp LP problems and then, we solve the middle level problems (second and third problems) first. Then, solve the upper level and lower level problems and then, the optimal MFL fuzzy solution to the given FFTP with mixed fuzzy constraints is obtained.

Now, the split and bounded method for solving FFTP with mixed fuzzy constraints is illustrated using the following numerical examples.

EXAMPLE 1
Consider the following FFTP with mixed fuzzy constraints:

				Supply
	$(1, 2, 3)$	$(2, 5, 8)$	$(2, 4, 6)$	$\approx (2, 5, 8)$
	$(2, 6, 10)$	$(1, 3, 5)$	$(0, 1, 2)$	$\succeq (3, 6, 9)$
	$(4, 8, 12)$	$(3, 9, 15)$	$(1, 2, 3)$	$\preceq (3, 9, 15)$
Demand	$\approx (4, 8, 12)$	$\succeq (8, 10, 12)$	$\preceq (3, 5, 7)$	

Now, by solving the problems $(P2)$, $(P3)$ and $(P1)$ using the split and bounded method, we obtain the following optimal solutions of the crisp problems $(P2)$, $(P3)$ and $(P1)$ respectively:

$(P2)$: $y_{11} = 5,\ y_{21} = 3, y_{22} = 10$, $y_{12}, y_{13}, y_{23}, y_{31}, y_{32}, y_{33} = 0$ and Minimize Z_2=58.

$(P3)$: $z_{11} = 8, z_{21} = 4, z_{22} = 12$, $z_{12}, z_{13}, z_{23}, z_{31}, z_{32}, z_{33} = 0$ and Minimize Z_3=124.

$(P1)$: $x_{11} = 2, x_{21} = 2, x_{22} = 8$, $x_{12}, x_{13}, x_{23}, x_{31}, x_{32}, x_{33} = 0$ and Minimize Z_1=14.

Therefore, the optimal MFL fuzzy solution for the given FFTP with mixed fuzzy constraints is $\widetilde{x}_1 \approx (2, 5, 8)$, $\widetilde{x}_4 \approx (2, 3, 4)$, $\widetilde{x}_5 \approx (8, 10, 12)$, $\widetilde{x}_2 \approx \widetilde{x}_3 \approx \widetilde{x}_6 \approx \widetilde{x}_7 \approx \widetilde{x}_8 \approx \widetilde{x}_9 \approx (0, 0, 0)$ and the total minimum fully fuzzy transportation cost is $\widetilde{Z} \approx (14, 58, 124)$.

REMARK 3 The proposed algorithm can also be used for finding an optimal MFLsolution of FFTPs with equality constraints.

CONCLUSION
We attempted to develop a new algorithm based on the crisp LP problems to find an optimal MFL fuzzy solution to FFTP with mixed fuzzy constraints.Since, the proposed method is based only on crisp LP problem it is very easy to solve FFTP problems with mixed fuzzy constraints having more number of fuzzy variables with help of existing computer software. Fuzzy ranking functions were not used and there is no restriction on the elements of the cost matrix in the proposed method. Also, the optimal MFL fuzzy solution obtained by the proposed method does not contain any negative part; the solution is meaningful and can be applied in real life situations. The proposed method can serve managers by providing one of the best MFL solutions to a variety of distribution problems.

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