



Solving Intuitionistic Fuzzy Transportation Problems With Mixed Intuitionistic Fuzzy Constraints

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ABSTRACT

A new method namely, breaking method is proposed to find an optimal more-for-less fuzzy solution for intuitionistic fuzzy transportation problems (IFTP) with mixed intuitionistic fuzzy constraints (IFC) by using crisp linear programming (LP) technique. The breaking method does not require any initial basic feasible solution and an optimal solution (MODI method) to solve IFTP with mixed intuitionistic fuzzy constraints. In this method, the optimal fuzzy solutions do not contain any negative part of the values of the decision variables. Also, fuzzy ranking functions were not used. The proposed method is an appropriate method to find an optimal more-for-less fuzzy solution to IFTP with mixed intuitionistic fuzzy constraints occurring in real life situations.

KEYWORDS : Triangular fuzzy number, fuzzy linear programming, fuzzy transportation problems with mixed constraints, split and bounded method.

INTRODUCTION

A fuzzy transportation problem (FTP) is a transportation problem (TP) in which the transportation costs, supply and demand quantities are fuzzy quantities. The objective of the FTP is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and fuzzy demand limits. To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh [4] and Zadeh [14] introduced the notion of fuzziness. In the literature, many researchers have developed various algorithms to solve FTP with equality constraints. In real life, most of the TPs have mixed constraints accommodating many applications that go beyond transportation related problems to include job scheduling, production inventory, production distribution, allocation problems, and investment analysis. The TPs with mixed constraints are not addressed in the literature because of the rigor required to solve these problems optimally.

The more-for-less (MFL) paradox in a TP occurs when it is possible to ship more total goods for less (or equal) total cost while shipping the same amount or more from each origin and to each destination keeping all shipping costs nonnegative. The occurrence of MFL in distribution problems is observed in nature. The existing literature [1,2,3,10,11,12,13] has demonstrated the identifying cases where MFL paradoxical situation exists and also, has provided various methods for finding MFL solution for TPs. The primary goal of the MFL method is to minimize the total cost and not merely maximize the shipment load transported.

Based on the literature study, it was found that there is a little work carried out to find an optimal MFL fuzzy solution to FFTP with mixed fuzzy constraints. Recently, Pandian and Natarajan [9] have proposed a new algorithm based on fuzzy zero point method for finding an optimal MFL fuzzy solution for FFTP with mixed fuzzy constraints. Jahir Hussain and Senthil Kumar [6] introduced a new ranking procedure to obtain an optimal solution for an intuitionistic fuzzy transportation problem. Jahir Hussain and Senthil Kumar [7] have proposed an algorithm namely intuitionistic fuzzy zero point method to find an optimal – more-for-less solution for triangular intuitionistic fuzzy numbers.

PRELIMINARIES

We need the following mathematical orientated definitions of IF set, triangular IF number and membership function and non-membership function of an IF set/number which can be found [5,6,7].

DEFINITION 1 Let X denote a universe of discourse and $A \subseteq X$. Then, an IF set of A in X , \tilde{A}^I is defined as follows: $\tilde{A}^I = \{ (x, \mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x)); x \in X \}$

Where $(\mu_{\tilde{A}^I}(x), \mathcal{G}_{\tilde{A}^I}(x)): X \rightarrow [0, 1]$ are functions such that $0 \leq \mu_{\tilde{A}^I}(x) + \mathcal{G}_{\tilde{A}^I}(x) \leq 1$, for all $x \in X$. For each x in X , $\mu_{\tilde{A}^I}(x)$ and $\mathcal{G}_{\tilde{A}^I}(x)$ represent the degree of membership and non-membership values of x in the set $A \subseteq X$.

DEFINITION 2A A fuzzy number \tilde{a}^I is a triangular IF number denoted by $(a_2, a_3, a_4)(a_1, a_3, a_5)$ where a_1, a_2, a_3, a_4 and a_5 are real numbers such that $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$ and its membership function $\mu_{\tilde{A}^I}(x)$ and non-membership function $\mathcal{G}_{\tilde{A}^I}(x)$ are given below:

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x-a_2}{a_3-a_2} & ; \quad a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & ; \quad a_3 \leq x \leq a_4 \\ 0 & ; \quad \text{otherwise} \end{cases} \text{ and } \mathcal{G}_{\tilde{A}^I}(x) = \begin{cases} \frac{a_3-x}{a_3-a_1} & ; \quad a_1 \leq x \leq a_3 \\ \frac{x-a_5}{a_5-a_3} & ; \quad a_3 \leq x \leq a_5 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Let $IF(R)$ be a set of all triangular IF numbers over R , a set of real numbers. Based on ordering relation in interval theory/fuzzy set theory, we define the following:

DEFINITION 3 Let $\tilde{a}^I = (a_2, a_3, a_4)(a_1, a_3, a_5)$ and $\tilde{b}^I = (b_2, b_3, b_4)(b_1, b_3, b_5)$ be in $IF(R)$. Then,

- (a) \tilde{a}^I and \tilde{b}^I are said to be equal if $a_i = b_i, i = 1, 2, 3, 4, 5$;
- (b) \tilde{a}^I is said to be less than or equal \tilde{b}^I if $a_i \leq b_i, i = 1, 2, 3, 4, 5$;
- (c) \tilde{a}^I is said to be greater than or equal \tilde{b}^I if $a_i \geq b_i, i = 1, 2, 3, 4, 5$;
- (d) \tilde{a}^I is said to be equal \tilde{b}^I if $a_i = b_i, i = 1, 2, 3, 4, 5$;
- (e) \tilde{a}^I is said to be positive $(\tilde{a}^I \succeq \tilde{0}^I)$ if $a_i \geq 0, i = 1, 2, 3, 4, 5$;
- (f) \tilde{a}^I is said to be integer if $a_i \geq 0, i = 1, 2, 3, 4, 5$ are integers.

Consider the following fully IFTP in which all parameters, that is, decision variables, transportation costs, supplies and demands, are triangular IF numbers:

$$(P) \text{ Minimize } \tilde{Z}^I = \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij}^I \otimes \tilde{x}_{ij}^I$$

Subject to

$$\sum_{i=1}^m \tilde{x}_{ij}^I \succeq \tilde{a}_i^I, j \in Q; \sum_{i=1}^m \tilde{x}_{ij}^I \preceq \tilde{a}_i^I, j \in T; \sum_{i=1}^m \tilde{x}_{ij}^I \approx \tilde{a}_i^I, j \in S; \quad (1)$$

$$\sum_{j=1}^n \tilde{x}_{ij}^I \succeq \tilde{b}_j^I, i \in U; \sum_{j=1}^n \tilde{x}_{ij}^I \preceq \tilde{b}_j^I, i \in V; \sum_{j=1}^n \tilde{x}_{ij}^I \approx \tilde{b}_j^I, i \in W; \quad (2)$$

$$\tilde{x}_{ij}^I \succeq \tilde{0}^I, \text{ for } i = 1, 2, \dots, m; j = 1, 2, \dots, n \text{ and are integers} \quad (3)$$

Where m = the number of supply points; n = the number of demand points; Q, T and S are pairwise disjoint subsets of $\{1, 2, 3, \dots, n\}$ such that $Q \cup T \cup S = \{1, 2, 3, \dots, n\}$; U, V and W are pairwise disjoint subsets of $\{1, 2, 3, \dots, m\}$ such that $U \cup V \cup W = \{1, 2, 3, \dots, m\}$; $\tilde{x}_{ij}^I \approx (x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5)$ is the number of fuzzy units shipped from supply point i to demand point j ; $\tilde{c}_{ij}^I \approx (c_{ij}^2, c_{ij}^3, c_{ij}^4)(c_{ij}^1, c_{ij}^3, c_{ij}^5)$ is the number of fuzzy cost of shipping one unit from supply point i to the demand point j ; $\tilde{a}_i^I \approx (a_i^2, a_i^3, a_i^4)(a_i^1, a_i^3, a_i^5)$ is the number of fuzzy supply at supply point i and $\tilde{b}_j^I \approx (b_j^2, b_j^3, b_j^4)(b_j^1, b_j^3, b_j^5)$ is the number of fuzzy demand at demand point j .

REMARK 1 If $Q = R = U = V = \phi$, the problem (P) becomes the fully IFTP with equality IFC.

DEFINITION 4 A set of triangular IF numbers $\tilde{X}_{ij}^I = \{ \tilde{x}_{ij}^I = (x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5), i=1,2,...,m$ and $j=1,2,...,n \}$ is said to be a feasible fuzzy solution to the problem (P) if it satisfies the conditions (1) to (3).

DEFINITION 5 A feasible IF solution $\tilde{X}_{ij} = \{ \tilde{x}_{ij}^I = (x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5), i=1,2,...,m$ and $j=1,2,...,n \}$ of the problem (P) is said to be an optimal IF solution to the problem (P) if $\tilde{Z}(\tilde{X}^I) \leq \tilde{Z}(\tilde{U}^I)$ for all feasible \tilde{U}^I of the problem (P).

THE BREAKING METHOD

Here, the parameters $\tilde{a}_i^I, \tilde{c}_{ij}^I, \tilde{x}_{ij}^I$ and \tilde{b}_j^I be the IF triangular numbers $(a_i^2, a_i^3, a_i^4)(a_i^1, a_i^3, a_i^5), (c_{ij}^2, c_{ij}^3, c_{ij}^4)(c_{ij}^1, c_{ij}^3, c_{ij}^5), (x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5)$ and $(b_j^2, b_j^3, b_j^4)(b_j^1, b_j^3, b_j^5)$ respectively. Then, the problem (P) can be written as follows:

$$\text{Minimize } (Z_2, Z_3, Z_4)(Z_1, Z_3, Z_5) \approx \sum_{j=1}^n \sum_{i=1}^m (c_{ij}^2, c_{ij}^3, c_{ij}^4)(c_{ij}^1, c_{ij}^3, c_{ij}^5) \otimes (x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5)$$

subject to

$$\sum_{j=1}^n (x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5) \{ \leq / \approx / \geq \} (a_i^2, a_i^3, a_i^4)(a_i^1, a_i^3, a_i^5), \text{ for } i=1, 2, \dots, m$$

$$\sum_{i=1}^m (x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5) \{ \leq / \approx / \geq \} (b_j^2, b_j^3, b_j^4)(b_j^1, b_j^3, b_j^5), \text{ for } j=1, 2, \dots, n$$

$$(x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5) \geq \tilde{0}, \text{ for } i=1, 2, \dots, m; j=1, 2, \dots, n \text{ and are integers.}$$

Using separation and bound method proposed by Jayalakshmi and Pandian [5], the above problem can be decomposed into LP problems namely, (P3), (P2), (P4), (P1) and (P5) as follows:

$$(P3): \text{Minimize } Z_3 = \sum_{j=1}^n \sum_{i=1}^m \text{third value of } (c_{ij}^2, c_{ij}^3, c_{ij}^4)(c_{ij}^1, c_{ij}^3, c_{ij}^5) \otimes (x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5)$$

subject to

Constraints in the decomposition problem in which at least one decision variable of the (P3) occurs and all decision variables are non-negative.

$$(P2): \text{Minimize } Z_2 = \sum_{j=1}^n \sum_{i=1}^m \text{second value of } (c_{ij}^2, c_{ij}^3, c_{ij}^4)(c_{ij}^1, c_{ij}^3, c_{ij}^5) \otimes (x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5)$$

subject to

$$Z_2 \leq Z_3^\circ$$

Constraints in the decomposition problem in which at least one decision variable of the (P2) occurs and are not used in (P3);

all variables in the constraints and objective function in (P2) must satisfy the fuzzy triangular intuitionistic bounded constraints; replacing all values of the decision variables which are obtained in (P3) and all decision variables are non-negative;

Where Z_3° is the optimal objective value of the problem (P3);

$$(P4): \text{Minimize } Z_4 = \sum_{j=1}^n \sum_{i=1}^m \text{fourth value of } (c_{ij}^2, c_{ij}^3, c_{ij}^4)(c_{ij}^1, c_{ij}^3, c_{ij}^5) \otimes (x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5)$$

subject to

$$Z_4 \geq Z_3^\circ$$

Constraints in the decomposition problem in which at least one decision variable of the (P4) occurs and are not used in (P3) and (P2);

all variables in the constraints and objective function in (P4) must satisfy the fuzzy triangular intuitionistic bounded constraints ;
replacing all values of the decision variables which are obtained in (P3) and all decision variables are non-negative;

Where Z_3° is the optimal objective value of the problem (P3);

$$(P1): \text{Minimize } Z_1 = \sum_{j=1}^n \sum_{i=1}^m \text{first value of } (c_{ij}^2, c_{ij}^3, c_{ij}^4)(c_{ij}^1, c_{ij}^3, c_{ij}^5) \otimes (x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5)$$

subject to

$$Z_1 \leq Z_2^\circ$$

Constraints in the decomposition problem in which at least one decision variable of the (P1) occurs and are not used in (P2), (P3) and (P4);

all variables in the constraints and objective function in (P1) must satisfy the fuzzy triangular intuitionistic bounded constraints ;
replacing all values of the decision variables which are obtained in (P2), (P3) and (P4) and all decision variables are non-negative;

where Z_2° is the optimal objective value of the problem (P2) and

$$(P5): \text{Minimize } Z_5 = \sum_{j=1}^n \sum_{i=1}^m \text{fifth value of } (c_{ij}^2, c_{ij}^3, c_{ij}^4)(c_{ij}^1, c_{ij}^3, c_{ij}^5) \otimes (x_{ij}^2, x_{ij}^3, x_{ij}^4)(x_{ij}^1, x_{ij}^3, x_{ij}^5)$$

subject to

$$Z_5 \geq Z_4^\circ$$

Constraints in the decomposition problem in which at least one decision variable of the (P5) occurs and are not used in (P1), (P2), (P3) and (P4);

all variables in the constraints and objective function in (P5) must satisfy the fuzzy triangular intuitionistic bounded constraints ;
replacing all values of the decision variables which are obtained in (P1), (P2), (P3) and (P4), and all decision variables are non-negative;

where Z_4° is the optimal objective value of the problem (P4).

Now, we propose a new algorithm namely, breaking method for solving fully IFTP with mixed IFC. The proposed method proceeds as follows.

STEP 1: Construct (P3), (P2), (P4), (P1) and (P5) problems from the given the fully IFTP.

STEP 2: Using existing linear programming technique, solve the problem (P3), then the problems (P2) and (P4), then the problems (P1) and (P5) in the order only and obtain the values of all real decision variables $x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^1$ and x_{ij}^5 for $i=1,2,\dots,m; j=1,2,\dots,n$ and the values of all objectives Z_3, Z_2, Z_4, Z_1 and Z_5 . Let the decision variables values be $x_{ij}^{2^\circ}, x_{ij}^{3^\circ}, x_{ij}^{4^\circ}, x_{ij}^{1^\circ}$ and $x_{ij}^{5^\circ}$ for $i=1,2,\dots,m; j=1,2,\dots,n$ and the objective values be $Z_3^\circ, Z_2^\circ, Z_4^\circ, Z_1^\circ$ and Z_5° .

STEP 3: The optimal IF solution to the given IFT problems is $\tilde{x}_{ij}^{\circ I} = (x_{ij}^{\circ 2}, x_{ij}^{\circ 3}, x_{ij}^{\circ 4})(x_{ij}^{\circ 1}, x_{ij}^{\circ 3}, x_{ij}^{\circ 5})$, for $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$ and the total minimum IF objective value is $\tilde{Z}^{\circ I} = (Z_2^{\circ}, Z_3^{\circ}, Z_4^{\circ})(Z_1^{\circ}, Z_3^{\circ}, Z_5^{\circ})$.

Now, the proposed method for solving fully IFTP with mixed IFC is illustrated by the following numerical examples.

Example 1 Consider the following IFTP with mixed IFC:

Supply

(2, 2, 3)(1, 2, 4)	(4, 5, 6)(3, 5, 7)	(3, 4, 5)(2, 4, 6)	$\approx (2, 5, 8)(1, 5, 9)$
(5, 6, 7)(4, 6, 8)	(2, 3, 4)(1, 3, 5)	(1, 1, 2)(0, 1, 3)	$\succeq (3, 6, 9)(2, 6, 10)$
(7, 8, 9)(6, 8, 10)	(8, 9, 10)(7, 9, 11)	(1, 2, 3)(0, 2, 4)	$\preceq (6, 9, 12)(4, 9, 14)$

Demand $\approx (4, 8, 12)(1, 8, 15)$ $\succeq (8, 10, 12)(5, 10, 15)$ $\preceq (2, 5, 8)(1, 5, 9)$

Now, by solving the problems (P3), (P2), (P4), (P1) and (P5) using the proposed method, we obtain the following optimal solutions of the crisp problems (P3), (P2), (P4), (P1) and (P5) respectively:

(P3): $x_{11}^{\circ 3} = 5$, $x_{21}^{\circ 3} = 3$, $x_{22}^{\circ 3} = 10$ and Minimize $Z_3^{\circ} = 58$.

(P2): $x_{11}^{\circ 2} = 2$, $x_{21}^{\circ 2} = 2$, $x_{22}^{\circ 2} = 8$ and Minimize $Z_2^{\circ} = 30$.

(P4): $x_{21}^{\circ 4} = 4$, $x_{22}^{\circ 4} = 12$ and Minimize $Z_4^{\circ} = 100$.

(P1): $x_{11}^{\circ 1} = 1$, $x_{21}^{\circ 1} = 0$, $x_{22}^{\circ 1} = 5$ and Minimize $Z_1^{\circ} = 6$.

(P5): $x_{11}^{\circ 5} = 9$, $x_{21}^{\circ 5} = 6$, $x_{22}^{\circ 5} = 15$ and Minimize $Z_5^{\circ} = 159$.

Thus, the optimal IF solution of the given IFTP is $\tilde{x}_{11}^{\circ I} \approx (2, 5, 8)(1, 5, 9)$, $\tilde{x}_{21}^{\circ I} \approx (2, 3, 4)(0, 3, 6)$, $\tilde{x}_{22}^{\circ I} \approx (8, 10, 12)(5, 10, 15)$ and the total minimum IFT cost is $(30, 58, 100)(6, 58, 159)$.

REMARK 2 The proposed algorithm can also be used for finding an optimal MFL solution for fully IFTP with equality IFC.

CONCLUSION

In this paper a new algorithm namely breaking method is developed, which is based on the crisp LP problems to find an optimal MFL intuitionistic fuzzy solution to IFTP with mixed IFC. The main advantage of the proposed method is that, it does not require any method to find an initial basic feasible solution and the methods to find an optimal solution for transportation problem. With help of existing computer software, IFTP can be solved since it is based only on crisp LP problem. Fuzzy ranking functions were not used and there is no restriction on the elements of the cost matrix in the proposed method. Also, the optimal MFL intuitionistic fuzzy solution does not contain any negative part; the solution is meaningful and can be applied in real life situations. So, the proposed method can serve managers by providing one of the best MFL solutions to a variety of distribution problems. Thus, the breaking method provides an applicable intuitionistic fuzzy optimal solution which helps the decision makers while they are handling real life IFTP having mixed IFC.

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