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On the Heat Transfer and Flow of Viscous Fluid Due to Exponentially Shrinking Sheet With Suction and Arbitrary Porous Media

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ABSTRACT

An analysis for the flow and heat transfer of viscous incompressible fluid over an exponentially shrinking sheet is carried out. The influence of suction and porous media of exponential form has been considered. The exponential form similarity transformations are used to derive the governing equations of the present problem. The perturbation method has been used to solve the similarity equations at large suction. The resulting equations have been solved by analytical method. The effect of porous medium is to enhance the skin-friction as well as rate of heat transfer.

KEYWORDS : Exponentially shrinking sheet, variable suction, porous media and heat transfer.

Introduction

Fluid flows through porous media over are known to have many applications of practical importance in physical and industrial processes. The fiber and granular insulations, thermal insulation of buildings, cores and designs of pebble bed nuclear reactors, winding structures for high power density elastic machines, food processing and storage, underground disposal of heavy water are some of the examples of these flows (Rehman et al. [1]). The fluid saturated porous materials with their transport characterizations are known to have profound applications in the petroleum and geothermal industries. Moreover, the underground crushed rocks saturated with liquid changes its position through the material under pressure gradient, causing the earthquakes to occur. Further, the boundary layer flows over stretching and shrinking surfaces have various applications in the engineering and technology. A new class of flow was observed by Wang [2] during his investigation for the behaviour of liquid film flow on an unsteady stretching sheet, and the base for the analysis of viscous fluids over shrinking surfaces was provided. Mahapatra and Nandy [3] found that by adding adequate suction or stagnation point flow the similar solution will exist. Miklavcic and Wang [4] obtained the existence and uniqueness conditions for the similarity solution of viscous fluid over shrinking surfaces and showed that the behaviour of fluid depends on the externally imposed mass suction. The exact solution for the MHD flow of Newtonian and non-Newtonian fluids due to shrinking sheet was investigated by Hayat et al. [5] and Fang and Zhang [6] respectively. Recently Bhattacharyya et al. [7] obtained the dual solutions for the boundary layer flow of Maxwell fluid over a shrinking sheet. The flow of micropolar fluid over a linear shrinking sheet has been investigated by Yacob and Ishak [8]. Turkyilmazoglu [9] obtained the dual and triple solutions for the hydromagnetic slip flow of non-Newtonian fluid over a shrinking surface.

In literature, many of the cases of shrinking sheet are considered with the assumption that the surface is having linear velocity and linear temperature distribution. But in reality, from physical point of view it must be nonlinear, either power law or exponential. There are some research papers available in literature discussing the flow and heat transfer over exponentially shrinking surfaces. Magyari and Keller [10] are assumed to be the first one to study the boundary flow over an exponentially stretching sheet. The boundary layer flow and heat transfer over an exponentially shrinking sheet was investigated by Bhattacharyya [11]. The stagnation point flow over an exponentially shrinking sheet was investigated by Bhattacharyya and Vajravelu [12]. Rohni et al. [13] presented the characteristics of the flow at the stagnation point over an exponentially shrinking sheet with mass suction.

However, the flow dynamics over an exponentially shrinking sheet is still open and more characteristics are yet to be investigated. Hence, the objective of the present paper is to investigate the effects of variable suction and porous media on the flow due to exponentially shrinking sheet.

Mathematical Formulation

Let us consider the steady two-dimensional boundary layer flow of a viscous, incompressible, fluid and heat transfer over an exponentially shrinking sheet. The porous medium is having the following form:

$K^* = K_0 \exp(-x/\ell)$ and K_0 is a constant.

The governing equations of the present problem are equations of continuity, motion and energy which may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K^*}u \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

The boundary conditions are given by

$$\left.\begin{aligned} &u = U_w(x), \quad v = v_w, \quad T = T_w(x) = T_\infty + T_0\exp(x/2\ell) \quad \text{at} \quad y = 0 \\ &u = 0, \quad T = 0 \quad \text{at} \quad y \to \infty \end{aligned}\right\} \qquad (4)$$

The shrinking sheet velocity U_w is given by $U_w(x) = -c\exp(x/\ell)$, where $c > 0$ is shrinking constant. Here ℓ, T_0, T_w and T_∞ are the characteristic length of the sheet, mean temperature, temperature of the sheet and ambient temperature of the fluid respectively.

We introduce the following similarity variables

$$\psi = \sqrt{2\nu\ell c}\, f(\omega)\exp(x/2\ell), \; T = T_\infty + (T_w - T_\infty)\theta(\omega) \qquad (5)$$

where ω is the similarity variable defined by

$$\omega = y\sqrt{\frac{c}{2\nu\ell}}\exp(x/2\ell) \qquad (6)$$

and ψ is the stream function which is defined in the classical form as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$. Thus we have the following expressions as

$$u = c\exp(x/\ell)f'(\omega) \text{ and } v = -\sqrt{\frac{\nu c}{2\ell}}\exp(x/2\ell)[f(\omega) + \omega f'(\omega)] \qquad (7)$$

where prime denotes differentiation with respect to ω. This suggests that, we can assume

$$v_w(x) = -\sqrt{\frac{\nu c}{2\ell}}\exp(x/2\ell)S' \qquad (8)$$

where $S > 0$ is the dimensionless suction parameter.

Using equation (5) to (7) in equations (2) and (3), we obtain the following ordinary differential equations

$$f''' + ff'' - 2f'^2 - 2\frac{1}{Da}f' = 0 \qquad (9)$$

$$\theta'' + P(f\theta' - f'\theta) = 0 \qquad (10)$$

The boundary conditions transform to

$$\left.\begin{array}{ll} f(\omega) = f_w, \;\; f'(\omega) = -1, \;\; \theta(\omega) = 1 & at\ \omega = 0 \\ f'(\omega) \to 0, \;\; \theta(\omega) \to 0 & as\ \omega \to 0 \end{array}\right\} \qquad (11)$$

The physical parameters of interest in the present problem, the skin friction coefficient C_f and the Nusselt number Nu, are defined by

$$C_f = \frac{\mu}{\rho U_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0} \qquad (12)$$

$$Nu = \frac{\ell}{T_w - T_\infty}\left(-\frac{\partial T}{\partial y}\right)_{y=0} \qquad (13)$$

Substituting (5) to (7) into above two equations, we get the following expressions of skin friction and Nusselt number:

$$C_f\sqrt{2\mathrm{Re}}\exp(x/2\ell) = f''(0) \qquad (14)$$

$$\sqrt{2/\mathrm{Re}}\exp(-x/2\ell)Nu = -\theta'(0) \qquad (15)$$

The non-dimensional parameters introduced in the above equations are:

$Da = \frac{K_0 c}{\nu\ell}$ (porous medium permeability), $P = \frac{\mu C_p}{k}$ (Prandtl number) and $\mathrm{Re} = \frac{c\ell}{\nu}$ (Reynolds number).

Method of solution

The differential equations (9) and (10) under the boundary conditions (11) are solved using the series expansion method as suggested by Singh and Dikshit [14].

$$\Omega = \omega S, \;\; f(\omega) = SF(\Omega) \text{ and } \theta(\omega) = \theta(\Omega) \qquad (16)$$

The equations (9) to (11) becomes

$$F''' + FF'' - 2F'^2 - \frac{2\epsilon}{Da}F' = 0 \qquad (17)$$

$$G'' + P(FG' - F'G) = 0 \qquad (18)$$

$$\left.\begin{array}{ll} F(\Omega) = 1, \;\; F'(\Omega) = -\frac{1}{S^2} = \epsilon, \;\; G(\Omega) = 1 & at\ \Omega = 0 \\ F'(\Omega) \to 0, \;\; G(\Omega) \to 0 & as\ \Omega \to 0 \end{array}\right\} \qquad (19)$$

where prime denotes the differentiation with respect to Ω.

For large suction, S assumes large positive values so that ϵ is small. Therefore, F and G can be expanded in terms of small perturbation quantity ϵ as

$$F = F_0 + \epsilon F_1 + \epsilon^2 F_2 + \epsilon^3 F_3 + \ldots \qquad (20)$$

$$G = G_0 + \epsilon G_1 + \epsilon^2 G_2 + \epsilon^3 G_3 + \ldots \qquad (21)$$

Substituting (20) and (21) into (17),(18) and (19), we obtain the following sets of ordinary differential equations along with the corresponding boundary conditions :

Zeroth Order $O(1)$:

$$F_0''' + F_0F_0'' - 2F_0'^2 = 0 \qquad (22)$$

$$G_0'' + \Pr(F_0G_0' - F_0'G_0) = 0 \qquad (23)$$

$$\left.\begin{array}{l} F_0(0) = 1, F_0'(0) = 0, \;\; F_0'(\infty) = 0 \\ G_0(0) = 1, \;\; G_0(\infty) = 0 \end{array}\right\} \qquad (24)$$

First-Order $O(\epsilon)$:

$$F_1''' + F_0F_1'' + F_1F_0'' - (2/Da)F_0' = 0 \qquad (25)$$

$$G_1'' + \Pr(F_1G_0' + F_0G_1' - F_0'G_1 - F_1'G_0) = 0 \qquad (26)$$

$$\left.\begin{array}{l} F_1(0) = 0, F_1'(0) = 1, \;\; F_1'(\infty) = 0 \\ G_1(0) = 0, \;\; G_1(\infty) = 0 \end{array}\right\} \qquad (27)$$

Second-Order $O\left(\epsilon^2\right)$:

$$F_2''' + F_2F_0'' + F_0F_2'' + F_1F_1'' - 4F_0'F_2' - 2F_1'^2 - (2/Da)F_1' = 0 \qquad (28)$$

$$G_2'' + \Pr(F_0G_2' + F_1G_1' + F_2G_0' - F_0'G_2 - F_1'G_1 - F_2'G_0) = 0 \qquad (29)$$

$$\left.\begin{array}{l} F_2(0) = 0, \;\; F_2'(0) = 0, \;\; F_2'(\infty) = 0 \\ G_2(0) = 0, \;\; G_2(\infty) = 0 \end{array}\right\} \qquad (30)$$

Third-Order $O\left(\epsilon^3\right)$:

$$F_3''' + F_0F_3'' + F_1F_2'' + F_2F_1'' + F_3F_0'' - 4F_1'F_2' - (2/Da)F_2' = 0 \qquad (31)$$

$$G_3'' + \Pr(F_0G_3' + F_1G_2' + F_2G_1' + F_3G_0' - F_0'G_3 - F_1'G_2 - F_2'G_1 - F_3'G_0) = 0 \qquad (32)$$

$$\left.\begin{array}{l} F_3(0) = 0, \;\; F_3'(0) = 1, \;\; F_3'(\infty) = 0 \\ G_3(0) = 0, \;\; G_3(\infty) = 0 \end{array}\right\} \qquad (33)$$

The obtained solutions of the above equations under the corresponding boundary conditions are:

$$F_0(\Omega) = 1 \qquad (34)$$

$$F_1(\Omega) = 1 - e^{-\Omega} \qquad (35)$$

$$F_2(\Omega) = -\frac{5+(8/Da)}{4} + \frac{3+(4/Da)}{2}e^{(-\Omega)} - \frac{1}{4}e^{(-2\Omega)} + (1+(2/Da))\Omega e^{(-\Omega)} \qquad (36)$$

$$F_3(\Omega) = B_{18} + (B_{19} - B_{14}\Omega)e^{(-\Omega)} + \frac{B_{15}}{4}e^{(-2\Omega)} - \frac{1}{24}e^{(-3\Omega)} - \frac{B_{16}}{2}\left(\Omega^2 + 4\Omega\right)e^{(-\Omega)} + B_{17}\Omega e^{(-2\Omega)} \qquad (37)$$

$$G_0(\Omega) = e^{(-\Pr\Omega)} \qquad (38)$$

$$G_1(\Omega) = \frac{P(P-1)}{P+1}\left(1 - e^{(-\Omega)}\right)e^{(-P\Omega)} - P\Omega e^{(-P\Omega)} \qquad (39)$$

$$G_2(\Omega) = B_{12}e^{(-P\Omega)} + B_9\Omega e^{(-P\Omega)} - B_{10}e^{(-(2+P)\Omega)} - B_{11}e^{(-(1+P)\Omega)} \qquad (40)$$

The velocity and temperature profiles can be calculated from the following expressions

$$f'(\omega) = -F_1' + \epsilon F_2' - \epsilon^2 F_3, \qquad (41)$$

$$\theta(\omega) = G_0 + \epsilon G_1 + \epsilon^2 G_2. \qquad (42)$$

In order to obtain more accurate results for velocity and temperature profiles, we have evaluated the expression up to the third order.

Results and Discussion

The parameters entering into the present problem are suction, porous medium permeability and Prandtl number. It is, therefore necessary to enquire the effects of the variations of each of them while the others are kept constant. The suction and porous medium permeability do not enter directly into the energy equation, but their effects come through the momentum equation solution. The Prandtl number enters directly in the energy equation whereas it has no effect on the momentum equation.

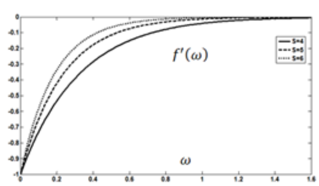

Figure 1: Velocity profiles with $Da = 1$

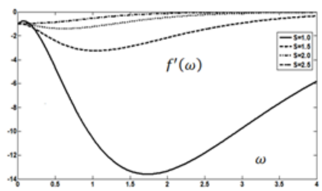

Figure 2: Velocity profiles with $Da = 1$

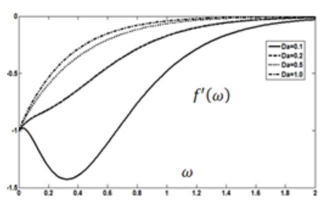

Figure 3: Velocity profiles with $S = 4$

The influences of suction parameter have been shown in Figures 1 and 2. These Figures clearly demonstrates that the velocity profiles are increased with the suction parameter. It is also revealed here that the momentum boundary layer thickness becomes thinner for $S \geq 2.4$ (app.), whereas it becomes thicker for $S \leq 2.4$ (app.).

The Figures 3 and 4 are plotted to verify the variations in the velocity profiles with the porous medium permeability along with the suitable combination of suction. It is clear from these Figures that both the velocity profiles and boundary layer thickness increases when $Da \geq$ ü (app.), whereas the opposite phenomenon is observed for $Da \leq 0.2$ (app.), that is velocity increases and boundary layer thickness decreases.

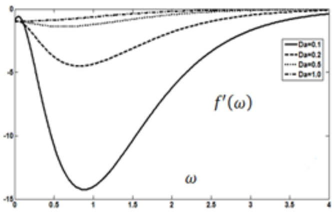

Figure 4: Velocity profiles with $S = 2$

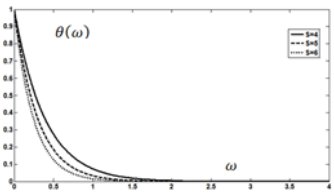

Figure 5: Temperature profiles with $Da = 0.5$ and $P = 0.71$

The effect of suction parameter, porous medium permeability and Prandtl number on temperature profiles have been illustrated in Figures 5, 6 and 7 respectively. In all these Figures both the temperature profiles and the thermal boundary layer thickness decreases except for the lower range of the permeability of the porous medium. In general, the thermal boundary layer thickness becomes thinner with the increase in Prandtl number. This is due to the physical fact that the increasing Prandtl number decreases the thermal conductivity of the fluid, hence causes a reduction in the thermal boundary layer thickness. The temperature distribution is quite interesting. The wall flux reduces with a flatter temperature near the sheet and the temperature drops fast to the ambient temperature. A higher dropping slope is observed for a higher value of Prandtl number in the fluid at a distance from the sheet.

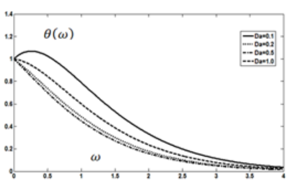

Figure 6: Temperature profiles with $S = 2$ and $P = 0.71$

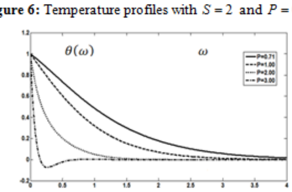

Figure 7: Temperature profiles with $Da = 0.5$ and $S = 2$

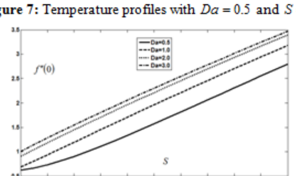

Figure 8: Skin-friction coefficient

The Figures 8 and 9 elucidates the effect of porous medium permeability on skin-friction and rate of heat transfer respectively with respect to the increasing suction parameter. Both the skin-friction and rate of heat transfer are increased with the increasing porous medium permeability. The range also increases with the increasing suction.

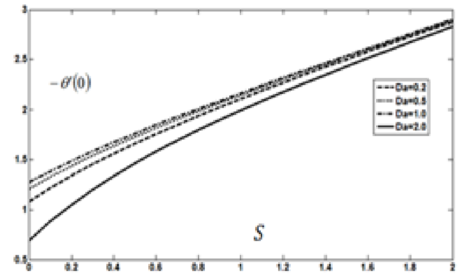

Figure 9: Rate of heat transfer coefficient with $P = 0.71$

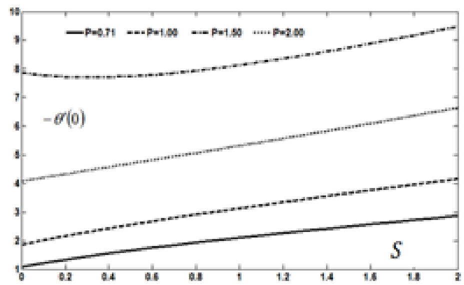

Figure 10: Rate of heat transfer coefficient with $Da = 0.5$

The figure 10 is plotted to reveal the effect of Prandtl number on the rate of heat transfer. From this Figure we note that the rate of heat transfer is significantly increased with the increasing Prandtl number.

Conclusions

The effects of suction parameter, porous medium and Prandtl number have been analyzed on the flow and heat transfer of viscous incompressible fluid over shrinking sheet. The similarity solutions are obtained in closed form by the perturbation technique. The conclusions of the study are noted as: the velocity profiles are increased with the suction parameter and porous medium permeability, however, the opposite effect is observed for temperature profiles; and the skin friction and the rate of heat transfer are significantly enhanced with the permeability of the porous medium with respect to the suction parameter; and both the velocity and temperature profiles approach the far field boundary conditions exponentially.

Appendix

$$B_1 = \frac{P(P-1)}{P+1}, \quad B_2 = \frac{P\left(P^2+1\right)}{P+1}, \quad B_3 = -\frac{5+(8/Da)}{4}, \quad B_4 = \frac{3+(4/Da)}{2},$$

$$B_5 = 1+(2/Da), \quad B_6 = \frac{1}{2}+P^2-P-B_2-B_4-B_1,$$

$$B_7 = -\frac{1}{2}+B_1+P\left(\frac{1}{4}+B_2(1-P)\right), \quad B_8 = (B_5+P)(1-P),$$

$$B_9 = B_2+P(1-B_3+P), \quad B_{10} = \frac{PB_7}{4+3P}, \quad B_{11} = \frac{PB_6}{(1+P)}+\frac{P^2(3+2P)B_8}{(1+P)^2},$$

$$B_{12} = B_{10}+B_{11}, \quad B_{13} = B_9-PB_{12} \quad D_1 = \frac{1}{2}-B_5, \quad B_{14} = D_1-B_3+1/Da,$$

$$B_{15} = 1-D_1-B_4-1/Da, \quad B_{16} = B_5^2, \quad B_{17} = \frac{B_{16}}{2},$$

$$B_{18} = -\frac{1}{12}+B_{14}+\frac{B_{15}}{4}+2B_{16}-B_{17}, \quad B_{19} = \frac{1}{8}-B_{14}-\frac{B_{15}}{2}-2B_{16}+B_{17},$$

$$B_{20} = B_{19}+B_{14}+2B_{16}, \quad B_{21} = \frac{B_{15}}{2}-B_{17}, \quad \text{and } B_{22} = B_{14}+B_{16}.$$

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