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Research Paper

STATISTICS

Progressive Type-II censored Sampling for Estimating Scale Parameter

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ABSTRACT *In this paper, assuming that sample is obtained through progressive Type II right censoring scheme, we obtain Minimum Risk Equivariant(MRE) estimator for the parameter of the uniform model in three situations based on the recent results of Leo Alexander andChandrasekar(1999a) obtained for Type-II right censored sample . The paper is organized as follows: Section 2 deals with the problem of equivariant estimation under Squared error loss function, Absolute error loss function and Linex loss function (Varian, 1975).*

KEYWORDS : Equivariant estimation,Location-scale model,Progressive Censored sampling, QA-MRE andUniform model

1. Introduction

In life-testing experiments, the common practice is to terminate the experiment when certain number of items have failed (Type-II censoring) or certain stipulated time has elapsed (Type-I censoring). Progressive censoring involves removing certain fixed number of surviving units at each failure which is an extended version of Type-II censoring scheme. As pointed out by Balakrishan and Sandhu(1996), the scheme of progressive censoring is an attractive feature as it saves both cost and time for the experimenter. Lehmann and Casella(1998) discussed the marginal Equivariant estimation of the parameters of location, scale and location-scale models. Viveros and Balakrishnan(1994) developed exact conditional inference based on progressive Type-II censored sample. Edwin Prabakaran and Chandrasekar (1994) developed simultaneous Equivariant estimation approach and illustrated with suitable examples.

1.1 Preliminaries

Let N denote the total number randomly selected items put to test simultaneously and n designate the number of samples specimens which fail. Thus the number of completely determined life spans is n. At the time of the i-th failure, r_i surviving units are randomly withdrawn from the test, i=1,2,…,n. Clearly, $r_n = N - n - \sum_{i=1}^{n-1} r_i$.

Let $X_{i:N}, i = 1,2,...,n$,denote the failure times of the completely observed times. Then, the joint probability density function (pdf) of $(X_{1:N}, X_{2:N},..., X_{n:N})$ is

$$g_\theta(x_1,x_2,...,x_n) = \prod_{i=1}^{n}(N - \sum_{j=1}^{i-1} r_j - i + 1) \times f_\theta(x_i)\{1 - F_\theta(x_i)\}^{r_i} \quad ...(1.1)$$

Here, f_θ *and* F_θ denote the common pdf and distribution function of the N items under life-test. Further, $r_1, r_2,..., r_n$ are assumed to be pre-fixed by the experimenter.

2. Scale model

In this the common pdf is taken to be

$$f_\tau(x)=\begin{cases}1/\tau\ , & 0\le x\le\tau\ ;\ \tau>0\\ 0\ , & otherwise\end{cases}$$

Thus (1.1) reduces to

$$g_\tau(x_1,...,x_n)=\{\prod_{i=1}^{n}(N-\sum_{j=1}^{i-1}r_j-i+1)\}$$
$$(1/\tau^n)\prod_{i=1}^{n}\{1-x_i/\tau\}^{r_i}\ ,\ ...(2.1)$$
$$0\le x_1\le x_n\le\tau\ ;\tau>0.$$

Thus the joint distribution of $\mathbf{X}=(X_{1:N},...,X_{n:N})$ belongs to a scale family with the scale parameter τ. We are interested in estimating τ^m, m fixed, by considering three loss functions, namelySquared error loss function, Absolute error loss function and Linexloss function, the MRE estimator τ^m is derived.

Case (i): If the loss function is of the form

$$\gamma(\delta/\tau^m)=(\delta/\tau^m-1)^2,$$

then $w^*=\dfrac{E_1(\delta_0{}^2\mid\mathbf{z})}{E_1(\delta_0\mid\mathbf{z})}$.

Clearly $\delta_0(\mathbf{X})=X^m_{n:N}$ is scale equivariant estimator but not a complete sufficient statistic. Since we are interested in the evaluation of conditional distribution under $\tau=1$, we take $\tau=1$ in (2.1). In order to find w^*, consider transformation

$$Z_n=X_{n:N}\quad and$$
$$Z_i=X_{i:N}/X_{n:N}\ ,\quad i=1,2,...,n-1.$$

Then
$$X_{n:N}=Z_n\ \ and$$
$$X_{i:N}=Z_i\,Z_n\ ,\quad i=1,2,...,n-1.$$

The Jacobian of the transformation is given by $J=Z_n^{\,n-1}$.. Thus the joint pdf of $(Z_1,Z_2,...,Z_n)$ is given by

$$h(z_1,...,z_n)=\ \{\prod_{i=1}^{n}(N-\sum_{j=1}^{i-1}r_j-i+1)\}$$
$$z_n^{\,n-1}(1-z_n)^{r_n}\prod_{i=1}^{n-1}(1-z_nz_i)^{r_{in}},$$
$$0<z_1<...<z_n<1, 0<z_n<1.$$

Also, the joint pdf of $(Z_1,Z_2,...,Z_{n-1})$ is given by

$$h_1(z_1,...,z_{n-1})=\{\prod_{i=1}^{n}(N-\sum_{j=1}^{i-1}r_j-i+1)\}$$
$$\int_0^1 z_n^{\,n-1}(1-z_n)^{r_n}\prod_{i=1}^{n-1}(1-z_nz_i)^{r_i}\ dz_n.$$

Thus the conditional pdf of Z_n given $(Z_1,Z_2,...,Z_{n-1})$ is given by

$$h_2(z_n\mid z_1,...,z_{n-1})=\{z_n^{\,n-1}(1-z_n)^{r_n}\prod_{i=1}^{n-1}(1-z_nz_i)^{r_i}\}/$$
$$\{\int_0^1 z_n^{\,n-1}(1-z_n)^{r_n}\prod_{i=1}^{n-1}(1-z_nz_i)^{r_i}\ dz_n\}...(2.2)$$

Now $w^* = \dfrac{E_1(\delta_0^{\,2} \mid \mathbf{z})}{E_1(\delta_0 \mid \mathbf{z})}$.

Here

$$E_1(\delta_0^{\,2} \mid \mathbf{z}) = \{\int_0^1 z_n^{\,2m+n-1}(1-z_n)^{r_n}\prod_{i=1}^{n-1}(1-z_n z_i)^{r_i}\,dz_n\}/$$
$$\{\int_0^1 z_n^{\,n-1}(1-z_n)^{r_n}\prod_{i=1}^{n-1}(1-z_n z_i)^{r_i}\,dz_n\} \quad ...(2.3)$$

and

$$E_1(\delta_0 \mid \mathbf{z})$$
$$= \{\int_0^1 z_n^{\,m+n-1}(1-z_n)^{r_n}\prod_{i=1}^{n-1}(1-z_n z_i)^{r_i}\,dz_n\}/$$
$$\{\int_0^1 z_n^{\,n-1}(1-z_n)^{r_n}\prod_{i=1}^{n-1}(1-z_n z_i)^{r_i}\,dz_n\}, \quad ...(2.4)$$

in view of (2.2)

Therefore the MRE estimator of τ^m is given by

$$\delta^*(\mathbf{X})$$
$$= X_{n:N}^m\{\int_0^1 z_n^{\,m+n-1}(1-z_n)^{r_n}\prod_{i=1}^{n-1}(1-z_n z_i)^{r_i}\,dz_n\}/$$
$$\{\int_0^1 z_n^{\,2m+n-1}(1-z_n)^{r_n}\prod_{i=1}^{n-1}(1-z_n z_i)^{r_i}\,dz_n\} \quad ...(2.5)$$

in view of (2.3) and (2.4).

If $N = 3, n = 2, r_1 = 1, r_2 = 0$, then the MRE estimator of τ^m, in view of (2.5), is given by

$$\delta^*(\mathbf{X}) = X_{2:3}^m \frac{\int_0^1 z_2^{\,m+1}(1-z_2 z_1)\,dz_2}{\int_0^1 z_2^{\,2m+1}(1-z_2 z_1)\,dz_2}$$

$$= X_{2:3}^m \frac{\left\{\int_0^1 z_2^{\,m+1}dz_2 - z_1\int_0^1 z_2^{\,m+2}dz_2\right\}}{\left\{\int_0^1 z_2^{\,2m+1}dz_2 - z_1\int_0^1 z_2^{\,2m+2}dz_2\right\}}$$

$$= X_{2:3}^m \;\{(\frac{(m+2)(X_{2:3}-X_{1:3})+X_{2:3}}{(2m+2)(X_{2:3}-X_{1:3})+X_{2:3}})\times (\frac{(2m+2)(2m+3)}{(m+2)(m+3)})\}.$$

Moreover, the Pitman form the MRE estimator τ^m with respect to the squared error loss function is obtained as follows. Therefore the Pitman estimator of τ^m is given by

$$\delta^*(\mathbf{X}) = \frac{\int_0^{1/x_{n:N}} v^{m+n-1}\prod_{i=1}^{n}(1-vx_i)^{r_i}\,dv}{\int_0^{1/x_{n:N}} v^{2m+n-1}\prod_{i=1}^{n}(1-vx_i)^{r_i}\,dv} \quad ...(2.6)$$

Taking $vx_{n:N} = z_n$ in (2.6) the above estimator reduces to

$$\delta^*(\mathbf{X}) = \frac{X_{n:N}^{n+2m-1}\int_0^1 z_n^{\,m+n-1}(1-z_n x_1/x_n)^{r_1}...(1-z_n)^{r_n}\,dz_n}{X_{n:N}^{n+m-1}\int_0^1 z_n^{\,2m+n-1}(1-z_n x_1/x_n)^{r_1}...(1-z_n)^{r_n}\,dz_n}$$

$$= X_{n:N}^m \frac{\int_0^1 z_n^{\,m+n-1}(1-z_n)^{r_n}\prod_{i=1}^{n-1}(1-z_n z_i)^{r_i}\,dz_n}{\int_0^1 z_n^{\,2m+n-1}(1-z_n)^{r_n}\prod_{i=1}^{n-1}(1-z_n z_i)^{r_i}\,dz_n},$$

since $z_i = x_{i:N}/x_{n:N}$, $i = 1,2,...,n-1$.

The above estimator coincides with the one given in (2.5).

Remark 2.1. If $r_k = 0$, $k = 1,2,...,n-1$ *and* $r_n = N-n$, then the estimator in (2.5) reduces to

$$\delta^*(\mathbf{X}) = \frac{\Gamma(m+n)\Gamma(2m+N+1)}{\Gamma(2m+n)\Gamma(m+N+1)} X_{n:N}^m,$$

which is same as the one under Type II right censored case (Leo Alexander ,2000).

Case (ii): If the loss function is of the form

$$\gamma(\delta/\tau) = |\delta-\tau|/\tau,$$

then $c = w^*$ satisfies the following

equation

$$N\int_0^c x_{n:N}^N \, dx_{n:N} = N\int_c^1 x_{n:N}^N \, dx_{n:N} ,$$

since $\delta_0 = X_{n:N}$ has the pdf given as

$$f(x_{n:N}) = N\, x_{n:N}^{N-1} ,\ 0 < x_{n:N} < 1 .$$

Thus $w^* = (1/2)^{1/(N-1)}$.

Therefore the MRE estimator of τ is given by

$$\delta^*(\mathbf{X}) = 2^{1/(N-1)} X_{n:N} .$$

Case (iii): Consider the scale invariant Linex loss function (Varian, 1975)

$$L(\tau;\delta) = e^{a(\delta/\tau-1)} - a(\delta/\tau-1) - 1 , \quad \tau > 0.$$

Take $\delta_0 = X_{n:N}$. In order to find w^* consider

consider

$$R(\delta \mid \mathbf{z}) = e^{-a} E_1(e^{a/w\delta_0} \mid \mathbf{z}) - a/w\, E_1(\delta_0 \mid \mathbf{z}) + a - 1$$

$$= e^{-a} \frac{\int_0^1 e^{a/wz_n} z_n^{\,n-1} (1-z_n)^{r_n} \prod_{i=1}^{n-1} (1 - z_n z_i)^{r_i} \, dz_n}{\int_0^1 z_n^{\,n-1} (1-z_n)^{r_n} \prod_{i=1}^{n-1} (1 - z_n z_i)^{r_i} \, dz_n}$$

$$-(a/w) \frac{\int_0^1 z_n^{\,n} (1-z_n)^{r_n} \prod_{i=1}^{n-1} (1 - z_n z_i)^{r_i} \, dz_n}{\int_0^1 z_n^{\,n-1} (1-z_n)^{r_n} \prod_{i=1}^{n-1} (1 - z_n z_i)^{r_i} \, dz_n}$$

$$+ a - 1,$$

in view of (2.4).

Thus w^* is to be obtained as the value of w by minimizing $R(\delta \mid y)$. Therefore the MRE estimator of τ is given by

$$\delta^*(\mathbf{X}) = \frac{\delta_0(\mathbf{X})}{w^*} = \frac{X_{n:N}}{w^*}.$$

Remark 2.2.

If $r_k = 0$, $k = 1,2,...,n-1$ and $r_n = N - n$, then the above estimator reduces to one obtained under Linex loss function of Leo Alexander (2000) .

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