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Modified Method For Solving Fully Fuzzy Transportation Problem

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ABSTRACT *The Transportation Problem is one of the earliest applications of linear programming problems. There are several methods in literature for finding the fuzzy optimal solution for fuzzy transportation problems. In this paper, we propose a new algorithm to find the fuzzy optimal solution for fully fuzzy transportation problem by representing all the parameters are trapezoidal fuzzy numbers. The proposed method is easy to understand and the numerical example shows the efficiency of the algorithm.*

KEYWORDS : Trapezoidal fuzzy numbers, Fuzzy Transportation Problem, Optimal Solution.

1. Introduction

The transportation problem is one of the earliest applications of the linear programming problems. The basic transportation problem was originally developed by Hitchcock[12]. In classical form, the transportation problem minimizes the cost of transporting a product which is available at some source and is required at various destinations. In general transportation problem is solved with the assumptions that the coefficients or cost parameters, availability and demand are specified in a precise way (i.e) in crisp environment. In real life, there are many diverse situations due to uncertainty in judgement, lack of evidence etc., sometimes it is not possible to get relevant precise data for all parameters. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy numbers introduced by Zadeh [29], may represents this data. So fuzzy decision making method is needed here.

In 1963, Dantzig used the simplex method to the transportation problem as the primal simplex transportation method. An initial basic feasible solution for the transportation problem can be obtained by using the north west corner rule, row minima, column minima, matrix minima or the vogel's approximation method. The modified distribution method is useful for finding the optimal solution for the transportation problems. Chanas and Cooper [7] developed the stepping stone method which provides an alternative way of determining the simplex-method information. The Linear interactive and Discrete Optimization (LINDO) [20] and many other commercial and academic packages are useful to find the solution of the transportation problems.

Zimmerman [31] showed that solutions obtained by fuzzy linear programming are always efficient. Subsequently, Zimmermann's fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems. Chanas et al. [4] presented a fuzzy linear programming model for solving transportation problems with crisp cost coefficients and fuzzy availability and demand values. Moreover, Chanas and Kuchta[6] designed an algorithm for solving the integer fuzzy transportation problem with fuzzy availability and demand volumes in the sense of maximizing the joint satisfaction of the fuzzy goal and constraints. The basic definitions [30] used throughout the work are as follows:

In this paper, we are discuss 1. Basic definition, 2. Fully Fuzzy Transportation Problem, Solution of Fuzzy Transportation Problem and Modified Distribution Method, 3. Examble.

Definition 1.1

A crisp set or a classical set A is defined as a collection of distinct and distinguishable objects. The objects are called elements of A. A crisp set A, defined on the universal set X, can also be represented by A = {(x, μ_A(x)): x $\in$X}

Where μ_A : X □{0,1} is called characteristic function defined by

$$\mu_A(x) = \begin{cases} 1, & \text{f x } \in \text{A} \\ 0 & \text{otherwise} \end{cases}$$

Definition 1.2

The characteristic function μ_A of a crisp set A□X assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range. i.e. $\mu_{\tilde{A}}$: X □{0,1} . The assigned value indicates the membership grade of the element in the set A.

Definition 1.3

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}) : x \in X\}$ defined by $\mu_{\tilde{A}}$ for each x□X is called a fuzzy set.

Definition 1.4

Let $\tilde{A}$ be a fuzzy set that ($\tilde{A}$)$_\alpha$ = {x□X: $\mu_{\tilde{A}} \geq \alpha$, 0□α1} is said to be an α-cut of $\tilde{A}$.

2. Fully fuzzy transportation problem

In crisp transportation problems, it is assumed that decision maker is sure about the precise values of transportation cost, availability and demand of the product. In real world applications, all these parameters of the transportation problems may not be known precisely due to controllable factors. To deal with such situations, fuzzy set theory is applied in literature to solve the transportation problems.

The balanced fuzzy transportation problems, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand can be formulated as follows:

$$Z = \min \sum_{i=1}^{m} \sum_{j=1}^{n} (\tilde{c}_{ij} \otimes \tilde{x}_{ij})$$

Subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, j = 1,2,...n$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_i, \text{i} = 1,2,...m$$

$$\sum_{j=1}^{n} \tilde{b}_j = \sum_{i=1}^{m} \tilde{a}_i \quad \text{Where}$$

m- total number of sources;

n-total number of destinations;

$\tilde{a}_i$ - The fuzzy availability of the product at i[th] source;

$\tilde{b}_j$ - The fuzzy demand of the product at j[th] destination;

$\tilde{c}_{ij}$ - The fuzzy transportation cost for one uit quantity of the product from i[th] source to j[th] destination;

$\tilde{x}_{ij}$ - The fuzzy quantity of the product that should be transported from i[th] source to j[th] destination;

$\sum_{i=1}^{m}\tilde{a}_i$ - Total fuzzy availability of the product;

$\sum_{j=1}^{n}\tilde{b}_j$ - Total fuzzy demand of the product;

$\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}\otimes\tilde{x}_{ij})$ - Total fuzzy transportation cost

Destination(j) Source (i)	1	2	----	N	Fuzzy availability
1	C_{11} X_{11}	C_{12} X_{12}	------	C_{1n} X_{1n}	a_1
2	C_{21} X_{21}	C_{22} X_{22}	------	$C2_n$ X_{2n}	a_2
. . .	. . .	. . .	--------	. . .	. . .
M	C_{m1} X_{m1}	C_{m2} X_{m2}	------	C_{mn} X_{mn}	a_m
	b_1	b_2	------	b_n	$\sum_{i=1}^{m} a_i \approx \sum_{j=1}^{n} b_j$

2.1 Solution of fully fuzzy transportation problems

The solution of FFTP can be obtained in two stages, namely fuzzy initial basic feasible solution and fuzzy optimal solution. For finding initial basic feasible solution of a fuzzy transportation problem there are numerous methods but fuzzy vogel's approximation method is preferred over the other methods. Since the initial fuzzy basic feasible solution obrained by this method is either optimal or very close to the optimal solution.

2.2 Fuzzy modified distribution method

The method is used to find the fuzzy optimal solution. The various steps of the method are

Step 1

Find out a set of numbers $\tilde{U}_i$ and $\tilde{V}_j$ for each row and column satisfying $\tilde{U}_i \oplus \tilde{V}_j = \tilde{C}_j$ for each basic cell.

Step 2

To start with we assign a zero trapezoidal fuzzy number to any row or column having maximum number of allocations. If the maximum number of allocation is more than one, then choose any one arbitrary.

Step 3

Find out for each nonbasic cell the net evaluation value $\tilde{U}_i \oplus \tilde{V}_j = \tilde{C}_j$ this step gives the optimality conclusion.

Case (1) If $\Re(\tilde{C}_{ij} = \tilde{U}_i \oplus \tilde{V}_j) > 0$, □ i, j then the solution is optimal and a unique solution exists.

Case (2) if $\Re(\tilde{C}_{ij} = \tilde{U}_i \oplus \tilde{V}_j) \geq 0$, then the solution is fuzzy optimal, but an alternate solution exists

Case(3) If $\Re(\tilde{C}_{ij} = \tilde{U}_i \oplus \tilde{V}_j) < 0$ for at least one i, j then the solution is not fuzzy optimal.

In this case we go to next step, to improve the total fuzzy transportation cost.

Step 4

Select the nonbasic cell having the most negative of $\Re(\tilde{C}_{ij} = \tilde{U}_i \oplus \tilde{V}_j)$ from this cell draw a closed path horizontally and vertically to the nearest basic cell with the restriction that the corner of the closed path mush not lie in any nonbasic cell. Assign sign + and – alternately and find the fuzzy minimum allocation from the cell having negative sign. This allocation should be added to the allocation having negative sign.

Step 5

The above step yields a better solution by making one (or more) basic cell as nonbasic cell and one nonbasic cell as basic cell. For this new set of fuzzy basic feasible solution repeat from Step-1, until a fuzzy optimal solution is obtained.

Where

$$\tilde{U}_i = (u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}) \ \&\ \tilde{V}_j = (v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)})$$

3. Numerical example

Suppose there are three sources and four destination. Let $\tilde{C}_{ij}$ be the fuzzy transportation cost for unit quantity of the product from ith source to jth destination, $\tilde{a}_i$ be the fuzzy availability at i[th] source and $\tilde{b}_j$ be the fuzzy demand at jth destination are shown in table (1). Find the fuzzy quality of the product transported from each source to various destinations. So that the total fuzzy transportation cost is minimum.

Destination (j) → Source (i) ↓	1	2	3	4	Fuzzy availability
1	(-2,0,2,8)	(-2,0,2,8)	(-2,0,2,8)	(-1,0,14)	(0,2,4,6)
2	(4,8,12,16)	(4,7,9,12)	(2,4,6,8)	(1,3,5,7)	(2,4,9,13)
3	(2,4,9,13)	(0,6,8,10)	(0,6,8,10)	(4,7,9,12)	(2,4,6,8)
Fuzzy Demand	(1,3,5,7)	(0,2,4,6)	(1,3,5,7)	(1,3,5,7)	

Solution

$\sum_{i=1}^{m} a_i = (4,10,19,27)$ &

$\sum_{j=1}^{n} b_j = (3,11,19,27)$

Since

$$\sum_{i=1}^{m} a_i = 15 = \sum_{j=1}^{n} b_j$$

So $\sum_{i=1}^{m} a_i \approx \sum_{j=1}^{n} b_j$ the problem is balanced fuzzy transportation problem using the steps, discussed in 2.2. The obtained fuzzy initial basic feasible solution is shown in table.

Destination (j) → Source (i) ↓	1	2	3	4	Fuzzy availability
1	(-2,0,2,8) (0,2,4,6)	(-2,0,2,8)	(-2,0,2,8)	(-1,0,1,4)	(0,2,4,6)
2	(4,8,12,16)	(4,7,9,12)	(2,4,6,8) (-5,-1,6,12)	(1,3,5,7) (1,3,5,7)	(2,4,9,13)
3	(2,4,9,13) (-5,-1,3,7)	(0,6,8,10) (0,2,4,6)	(0,6,8,10) (-11,-3,6,12)	(4,7,9,12)	(2,4,6,8)
Fuzzy Demand	(1,3,5,7)	(0,2,4,6)	(1,3,5,7)	(1,3,5,7)	

Since the number of basic cells are m+n-1=6, so the solution is non-degenerate fuzzy basic feasible solution .

$$\tilde{C}_{11}\otimes\tilde{X}_{11}\oplus\tilde{C}_{23}\otimes\tilde{X}_{23}\oplus\tilde{C}_{24}\otimes\tilde{X}_{24}\oplus\tilde{C}_{31}\otimes\tilde{X}_{31}\oplus\tilde{C}_{32}\otimes\tilde{X}_{32}\oplus\tilde{C}_{33}\otimes\tilde{X}_{33}$$
$$=(-2,0,2,8)\otimes(0,2,4,6)\oplus(2,4,6,8)\otimes(-5,-1,6,12)\oplus(1,3,5,7)\otimes(1,3,5,7)\oplus(2,4,9,13)\otimes$$
$$(-5,-1,3,7)\oplus(0,6,8,10)\otimes(0,2,4,6)\oplus(0,6,8,10)\otimes(-11,-3,6,12)\approx(-226,-18,176,464)$$

Where $\tilde{C}_{11}, \tilde{C}_{23}, \tilde{C}_{24}, \tilde{C}_{31}, \tilde{C}_{32}, \tilde{C}_{33}$ are fuzzy cost coefficient

$\tilde{X}_{11}, \tilde{X}_{23}, \tilde{X}_{24}, \tilde{X}_{31}, \tilde{X}_{32}, \tilde{X}_{33}$ are fuzzy allocations.

Applying the fuzzy modified distribution method, determine a set of numbers $\tilde{U}_i \ \&\ \tilde{V}_j$ for each row and column such that $\tilde{U}_i \oplus \tilde{V}_j = \tilde{C}_j$

for each basic cell.

Since 3rd row has maximum numbers of allocations, take $\widetilde{U}_3 = (-2,-1,1,2)$

The remaining $\widetilde{U}_i$ & $\widetilde{V}_j$ can be obtained as given below

$$\widetilde{C}_{31} \approx \widetilde{U}_3 \oplus \widetilde{V}_1 \qquad \therefore \widetilde{V}_1 \approx (0,3,10,15)$$

$$\widetilde{C}_{32} \approx \widetilde{U}_3 \oplus \widetilde{V}_2 \qquad \therefore \widetilde{V}_2 \approx (-2,5,9,12)$$

$$\widetilde{C}_{33} \approx \widetilde{U}_3 \oplus \widetilde{V}_3 \qquad \therefore \widetilde{V}_3 \approx (-2,5,9,12)$$

$$\widetilde{C}_{11} \approx \widetilde{U}_1 \oplus \widetilde{V}_1 \qquad \therefore \widetilde{U}_1 \approx (-17,-10,-1,8)$$

$$\widetilde{C}_{23} \approx \widetilde{U}_2 \oplus \widetilde{V}_3 \qquad \therefore \widetilde{U}_2 \approx (-10,-5,1,10)$$

$$\widetilde{C}_{24} \approx \widetilde{U}_2 \oplus \widetilde{V}_4 \qquad \therefore \widetilde{V}_2 \approx (-9,2,10,17)$$

t evaluation $\widetilde{U}_i \oplus \widetilde{V}_j = \widetilde{C}_j$ is calculated and shown in Table (3)

Fuzzy Optimal Solution

Destination (j) → Source (i) ↓	1	2	3	4	Fuzzy availability
1	(-2,0,2,8) (0,2,4,6)	(-2,0,2,8) *(-22,-8,7,27)	(-2,0,2,8) *(-22,-8,7,27)	(-1,0,1,4) *(-26,-9,9,30)	(0,2,4,6)
2	(4,8,12,16) *(-21,-3,14,26)	(4,7,9,12) *(-18,-3,9,24)	(2,4,6,8) (-5,-1,6,12)	(1,3,5,7) (1,3,5,7)	(2,4,9,13)
3	(2,4,9,13) (-5,-1,3,7)	(0,6,8,10) (0,2,4,6)	(0,6,8,10) (-11,-3,6,12)	(4,7,9,12) *(-15,-4,8,23)	(2,4,6,8)
Fuzzy Demand	(1,3,5,7)	(0,2,4,6)	(1,3,5,7)	(1,3,5,7)	

∴ The fuzzy optimal solution in terms of trapezoidal fuzzy number is

$$\widetilde{X}_{11} = (0,2,4,6),\ \widetilde{X}_{23} = (-5,-1,6,12),\ \widetilde{X}_{24} = (1,3,5,7),$$

$$\widetilde{X}_{31} = (-5,-1,3,7),\ \widetilde{X}_{32} = (0,2,4,6),\ \widetilde{X}_{33} = (-11,-3,6,12)$$

Hence, the minimum total fuzzy transportation cost is

$$\widetilde{C}_{11} \otimes \widetilde{X}_{11} \oplus \widetilde{C}_{23} \otimes \widetilde{X}_{23} \oplus \widetilde{C}_{24} \otimes \widetilde{X}_{24} \oplus \widetilde{C}_{31} \otimes \widetilde{X}_{31} \oplus \widetilde{C}_{32} \otimes \widetilde{X}_{32} \oplus \widetilde{C}_{33} \otimes \widetilde{X}_{33}$$
$$= (-2,0,2,8) \otimes (0,2,4,6) \oplus (2,4,6,8) \otimes (-5,-1,6,12) \oplus (1,3,5,7) \otimes (1,3,5,7) \oplus (2,4,9,13) \otimes$$
$$(-5,-1,3,7) \oplus (0,6,8,10) \otimes (0,2,4,6) \oplus (0,6,8,10) \otimes (-11,-3,6,12) \approx (-226,-18,176,464)$$

4. Conclusion

In this paper, an existing method for solving a FFTP in which all the parameters are represented by trapezoidal fuzzy numbers, is presented. To illustrate the existing method, a numerical example is solved and obtained results are discussed.

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