Volume-5, Issue-8, August - 2016 • ISSN No 2277 - 8160 IF : 3.62 | IC Value 70.36

Original Research Paper Mathematics

On the Ternary Quadratic Equation $x^2 + 4y^2 = 40z^2$

Dr.R.Anbuselvi	Associate Professor, Department of Mathematics, A.D.M. College for women, Nagapattinam, India
K.S.Araththi	Research scholar,Department of Mathematics, A.D.M. College for women, Nagapattinam, India

ABSTRACT *The ternary homogenous quadratic Diophantine equation representing cone given by is analyzed for finding its non-zero distinct integral solutions. Seven different patterns of integer solutions are presented. A few interesting relations between the solutions and special number patterns are given.*

KEYWORDS : Ternary quadratic, homogenous quadratic, integer solutions

I. INTRODUCTION

The Ternary Quadratic Diophantine Equation offers an unlimited field for research because of their variety [1-2]. For an extensive review of various problems, one may refer [3-10]. This communication concerns with yet another interesting Ternary Quadratic equation $x^2 + 4y^2 = 40z^2$ representing a homogenous cone for determining its infinitely many non-zero integral solutions. Also a few interesting relations among the solutions have been presented..

Notations used :

$t_{m,n}$= Polygonal number of rank n with size m.
obl_n = oblong number of rank n

II.METHOD OF ANALYSIS

The ternary Quadratic equation to be solved for its non-zero integer solution is

$x^2+4y^2=40z^2$ (1)

We present below different patterns of non-zero distinct integer solutions to (1)

Equation (1) can be written as

$(x+6z(x-6z) = 4(z+y)(z-y)$ which may be written in the form of ratio as

$$\frac{x+6z}{z+y} = \frac{4(z-y)}{x-6z} = \frac{a}{b} \quad (2)$$

This is equivalent to the following two equations

$bx + z(6b-a) - ay = 0$

$z(4b+6a) - 4by - ax = 0$

On employing the method of cross multiplication, we get

$$\left.\begin{aligned} x(a,b) &= 6a^2 - 24b^2 + 8ab \\ y(a,b) &= -a^2 + 4b^2 + 12ab \\ z(a,b) &= a^2 + 4b^2 \end{aligned}\right\} \quad (3)$$

Properties

(i) $x(1,n) + 6z(1,n) \equiv 0 \pmod 2$
(ii) $400y(n,-1) - 100x(1,n) \equiv 0 \pmod{10}$
(iii) $z(1,-n) + y(n,1) + t_{3,5} \equiv 0 \pmod 8$
(iv) $x(1,n) - y(n.1) + 23n^2 \equiv 0 \pmod 2$
(v) $y(n,n) + z(n,1) - t_{4,3} \equiv 0 \pmod 2$

Thus (3) represent the non- zero distinct integer solution of equation (1) in two parameters

It is observed that by rewriting (2) suitably, one may arrive at the following three patterns of solutions to (1).

Pattern II :

$x(a,b) = 24b^2 - 6a^2 + 8ab$
$y(a,b) = 4b^2 - a^2 - 12ab$
$z(a,b) = a^2 + 4b^2$

Properties:

(i) $4x(-n,1) + 6y(1,-n) \equiv 0 \pmod{10}$
(ii) $x(-n,1) + z(n,-1) + obl_n - t_{3,7} \equiv 0 \pmod 3$
(iii) $y(1,-n) + z(-1,n) - t_{3,1} \equiv 0 \pmod 6$
(iv) $x(1,n) - y(n,1) + t_{5,5} \equiv 0 \pmod 2$
(v) $y(1,n) - z(1,n) + 12n \equiv 0 \pmod 2$

Pattern III :

$x(a,b) = 24a^2 - 6b^2 + 8ab$
$y(a,b) = 4a^2 - b^2 - 12ab$
$z(a,b) = 4a^2 + b^2$

Properties:

(i) $x(1,n) + y(1,n) \equiv 0 \pmod 5$
(ii) $z(n,1) + 4z(n,-1) + t_{3,2} \equiv 0 \pmod 4$
(iii) $x(1,-n) + z(n,-1) + 5n^2 \equiv 0 \pmod 2$

Volume-5, Issue-8, August - 2016 • ISSN No 2277 - 8160

(iv) $x(m,1) - y(m,1) + t_{3,5} \equiv 0 \pmod 4$

(v) $y(1,n) + z(1,n) + 12n \equiv 0 \pmod 5$

Pattern IV :

$x(a,b) = -24a^2 + 6b^2 + 8ab$

$y(a,b) = 4a^2 - b^2 + 12ab$

$z(a,b) = b^2 + 4a^2$

Properties:

(i) $x(n,-1) + y(n,-1) + 20obl_n \equiv 0 \pmod 5$

(ii) $x(n,-1) + 6y(n,-1) + 80n = 0$

(iii) $y(1,n) + z(1,n) \equiv 0 \pmod 4$

(iv) $x(m,-1) + y(-1,m) - t_{4,3} \equiv 0 \pmod 5$

(v) $y(1,n) - z(n,1) - t_{3,5} \equiv 0 \pmod 5$

Pattern V :

Introducing the transformation

$x = 6\alpha \;\; y = x + 40T \;; z = x + 4T$ (4)

in (1), we get

$36\alpha^2 + 4(x+40T)^2 = 40(x+4T)^2$

$36\alpha^2 = 36x^2 - 5760T^2$

$\alpha^2 = x^2 - 160T^2$ (5)

which is satisfied by

$T = T(a,b) = 2ab$

$x = x(a,b) = 160a^2 + b^2$

$\alpha = \alpha(a,b) = 160a^2 - b^2$ (6)

Substituting (6) in (4) , non zero distinct integral solutions of (1) in two parameters is given by

$x = x(a,b) = 6\alpha = 960a^2 - 6b^2$

$y = y(a,b) = 960a^2 - 6\,b^2 + 8ab$

$z = z(a,b) = 960a^2 - 6b^2 + 8ab$

Properties:

(i) $x(n,-1) - y(n,-1) \equiv 0 \pmod 2$

(ii) $y(1,-n) - z(1,-n) \equiv 0 \pmod 8$

(iii) $x(1,-n) - z(1,-n) - obl_n + n^2 \equiv 0 \pmod 7$

(iv) $x(1,n) - y(1.n) \equiv 0 \pmod 5$

(v) $y(n,1) - z(n,1) + 16n = 0$

Pattern VI :

Assume $z = z(a,b) = a^2 + 4b^2$ (7)

Also write 40 as

$40 = (6 + i\sqrt{4})(6 - i\sqrt{4})$ (8)

Substitute (7) and (8) in (1) and employing the method of factorization, define

$x + i\sqrt{4y} = (6 + i\sqrt{4})(a + i\sqrt{4b})^2$ (9)

Equating the real and imaginary parts, we get

x = x(a,b) = $6a^2 - 24b^2 - 8ab$

y = y(a,b) = $a^2 - 4b^2 + 12ab$

z = z(a,b) = $a^2 + 4b^2$

Properties:

(i) $x(1,n) - y(1,n) + 20obl_n \equiv 0 \pmod 5$

(ii) $y(1,n) + z(1,n) \equiv 0 \pmod 2$

(iii) $x(1,n) - 12z(1,n) + t_{3,2} \equiv 0 \pmod 8$

(iv) $x(m,-1) - y(1,-m) + t_{8,7} \equiv 0 (\mathrm{mod})$

Pattern VII :

Write (1) as

$40z^2 - x^2 = 4y^2$------(9)

Write 4 as $4 = (\sqrt{40} + 6)(\sqrt{40} - 6)$ (10)

Assume $y = y(a,b) = 40a^2 - b^2$ (11)

Using (10) and (11) in (9) and employing the method of factorization, define

$\sqrt{40z} + x) = (\sqrt{40} + 6)(\sqrt{40a} + b)^2$ (12)

Equating rational and irrational parts in (12), we get

$$\left.\begin{aligned} x &= x(a,b) = 240a^2 + 6b^2 + 80ab \\ y &= y(a,b) = 40a^2 - b^2 \\ z &= z(a,b) = 40a^2 + b^2 + 12ab \end{aligned}\right\} \quad (13)$$

Thus (11) & (13) represent non – zero distinct integral solutions of (1) in two parameters.

Properties:

(i) $x(1,n)-y(1,n)+7n^2 \equiv 0 \pmod{10}$

(ii) $y(1,n)-z(1,n)+12n \equiv 0 \pmod{2}$

(iii) $x(1,n)-z(1,n)+12n \equiv 0 \pmod{2}$

(iv) $x(1,n)+z(1,n)-t_{5,4} \equiv 0 \pmod{4}$

III.CONCLUSION

In this paper, we have presented seven different patterns of non-zero distinct integer solutions to the ternary quadratic Diophantine equation $x^2+4y^2=40z^2$ representing a cone. To conclude one may search for other patterns of non-zero distinct integer solutions satisfying the cone under consideration.

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