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Geo/Geo/1 Queue for Graceful Restart Mechanism

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ABSTRACT *Graceful restart is designed to minimize packet loss when a management system fails to function. A system with graceful restart can continue to forward some packets even in the event of a processor failure. We consider a Geo/Geo/1 queue with graceful restart. The steady-state distribution of the number of customers is given by matrix geometric analysis.*

KEYWORDS : graceful restart, Geo/Geo/1 queue, queue size distribution

INTRODUCTION

Queueing systems with servers subject to breakdowns and repairs have been studied extensively [1, 2]. Recently, Falin [3], Lee [4], Choudhury and Ke [5], and others considered the unreliable queues wherein, when a server fails, customers in the system should wait for the server to be repaired without being served. However, there are other practical situations: when a server breakdown occurs, the system continues to forward some customers without stopping immediately. For example, Stateful Switchover and Graceful Restart are designed to minimize packet loss during a management system failure by maintaining L2 and L3 forwarding, respectively, where the system continues forwarding some packets in the event of a processor failure [6, 7].

Motivated by this factor, Lee [8] considered a discrete-time queue with deterministic service times and graceful restart, where the service continues instead of being stopped completely even in the case that the server is defective. In this paper, we analyze the Geo/Geo/1 queue with geometric service times and graceful restart.

SYSTEM MODEL

We consider a Geo/Geo/1 queue in which the time axis is divided into fixed-length contiguous intervals, referred to as slots. Customers arrive according to a geometric process. Let p be the probability that a customer enters the system during a slot. It is assumed that the service of a customer can start only at a slot boundary. The service times of customers are assumed to follow geometric distribution with parameter q. The system has a buffer of infinite capacity. Customers are served in FCFS order. The exact location of arrival instants within the slot length are not specified here. It is even irrelevant as long as the system is observed at slot boundaries only.

It is assumed that the server is subject to breakdowns. The server broken down starts immediately the repair process. The lifetime of the server is assumed to be geometrically distributed with parameter α, where $1-\alpha$ is the probability that a failure does not occur in a slot. The repair times of servers follows a geometrical distribution with parameter β, where $1-\beta$ is the probability that a failure will not be concluded in a slot. When the server breaks down, the system continues to forward the next N customers. The interarrival times, the service times, the failure times, and the repair times are assumed to be mutually independent of each other.

Let $M(k)$ be the number of customers in the system at the beginning of slot k. Let $S(k)$ be the server state at the beginning of slot k:

$$S(k) \equiv \begin{cases} n & \begin{array}{l}\text{if the server is under repair and} \\ \quad \text{the system has forwarded} \\ \quad N-n \text{ customers after the} \\ \quad \text{server's breakdown,}\end{array} \\ N+1 & \text{if the server is normal.} \end{cases}$$

Then $\{(M(k), S(k))\}$ is a Markovian process with state space $\{0,1,2\} \times \{0,1,\dots,N+1\}$. We have the transition matrix of $\{(M(k), S(k))\}$:

$$\boldsymbol{P} = \begin{pmatrix} (1-p)(\boldsymbol{C}+\boldsymbol{F}) & p(\boldsymbol{C}+\boldsymbol{F}) & \boldsymbol{0} & \boldsymbol{0} & \cdots \\ (1-p)q\boldsymbol{B} & \boldsymbol{D} & p(1-q)\boldsymbol{C}+p\boldsymbol{F} & \boldsymbol{0} & \cdots \\ \boldsymbol{0} & (1-p)q\boldsymbol{B} & \boldsymbol{D} & p(1-q)\boldsymbol{C}+p\boldsymbol{F} & \cdots \\ \boldsymbol{0} & \boldsymbol{0} & (1-p)q\boldsymbol{B} & \boldsymbol{D} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{F}$ are $(N+2)\times(N+2)$ matrices given by

$$\boldsymbol{B} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1-\beta & 0 & 0 & \cdots & 0 & 0 & \beta \\ 0 & 1-\beta & 0 & \cdots & 0 & 0 & \beta \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \beta \\ 0 & 0 & 0 & \cdots & 1-\beta & 0 & \beta \\ 0 & 0 & 0 & \cdots & 0 & \alpha & 1-\alpha \end{pmatrix}$$

$$\boldsymbol{C} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1-\beta & 0 & \cdots & 0 & 0 & \beta \\ 0 & 0 & 1-\beta & \cdots & 0 & 0 & \beta \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1-\beta & 0 & \beta \\ 0 & 0 & 0 & \cdots & 0 & 1-\beta & \beta \\ 0 & 0 & 0 & \cdots & 0 & \alpha & 1-\alpha \end{pmatrix}$$

$$\boldsymbol{F} = \begin{pmatrix} 1-\beta & 0 & \cdots & 0 & \beta \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix}$$

and $\boldsymbol{D} = pq\boldsymbol{B} + (1-p)(1-q)\boldsymbol{C} + (1-p)\boldsymbol{F}$.

EQUILIBRIUM CONDITION OF THE SYSTEM

A necessary and sufficient condition to ensure the existence for the stationary probability vector of the process $\{(M(k), S(k))\}$ is provided. Let $\boldsymbol{\gamma} \equiv (\gamma_0,\ \gamma_1, \ldots,\ \gamma_{N+1})$ be the stationary probability vector of

$$\boldsymbol{A} \equiv (1-p)q\boldsymbol{B} + \boldsymbol{D} + p(1-q)\boldsymbol{C} + p\boldsymbol{F}$$

$$= q\boldsymbol{B} + (1-q)\boldsymbol{C} + \boldsymbol{F},$$

i.e., $\boldsymbol{\gamma A} = \boldsymbol{\gamma}$ and $\boldsymbol{\gamma e} = 1$, where $\boldsymbol{e}$ is the column vector, all of the elements equal to 1. Solving the linear equations, we get

$$\gamma_0 = \frac{\alpha}{\alpha+\beta}\left[\frac{q(1-\beta)}{1-(1-q)(1-\beta)}\right]^N,$$

$$\gamma_i = \frac{\alpha}{\alpha+\beta}\frac{\beta}{q(1-\beta)}\left[\frac{q(1-\beta)}{1-(1-q)(1-\beta)}\right]^{N-i+1}, i = 1,2,\ldots,N,$$

$$\gamma_{N+1} = \frac{\beta}{\alpha+\beta}.$$

The equilibrium condition [9] of the system is

$$p(1-q)\boldsymbol{\gamma C e} + p\boldsymbol{\gamma F e} < (1-p)q\boldsymbol{\gamma B e},$$

which is given by

$$p < q\left[1 - \frac{\alpha}{\alpha+\beta}\left\{\frac{q(1-\beta)}{1-(1-q)(1-\beta)}\right\}^N\right].$$

STEADY STATE ANALYSIS

Under the above equilibrium condition, we can compute the steady state distribution $\boldsymbol{\pi} \equiv (\boldsymbol{\pi_0},\ \boldsymbol{\pi_1}, \ldots)$ of the QBD Markov chain, where

$$\pi_{i,j} \equiv \lim_{k\to\infty} \mathrm{P}\{M(k) = i, S(k) = j\},$$

$$\boldsymbol{\pi_i} \equiv \left(\pi_{i,0},\ \pi_{i,1}, \ldots, \pi_{i,N+1}\right).$$

Note that the steady state distribution $\boldsymbol{\pi}$ is the solution to the linear system $\boldsymbol{\pi P} = \boldsymbol{\pi}$ and $\boldsymbol{\pi e} = 1$.

We need to find the so-called $\boldsymbol{R}$-matrix. The (i,j)-th entry of this matrix contains the expected number of visits to state j of level $i+1$, before returning to level i again. It can be found as the smallest nonnegative solution to the matrix equation

After having computed the matrix

$$\boldsymbol{R_0} = p(\boldsymbol{C} + \boldsymbol{F})[\boldsymbol{I} - \boldsymbol{D} - (1-p)q\boldsymbol{RB}]^{-1},$$

the steady state distribution $\boldsymbol{\pi}$ can be obtained from

$$\boldsymbol{\pi_0} \equiv (1-p)\boldsymbol{\pi_0}(\boldsymbol{C} + \boldsymbol{F} + q\boldsymbol{R_0 B}),$$

$$\boldsymbol{\pi_1} \equiv \boldsymbol{\pi_0 R_0},$$

$$\boldsymbol{\pi_i} \equiv \boldsymbol{\pi_{i-1} R},\ \ i > 1,$$

with $\boldsymbol{\pi_0 e} + \boldsymbol{\pi_0 R_0}(\boldsymbol{I} - \boldsymbol{R})^{-1}$.

CONCLUSIONS

Graceful restart is designed to minimize packet loss when a management system fails to function. In the graceful restart mechanism, the system can continue to forward some packets in the event of a processor failure. We considered the queueing system with graceful restart. We gave the equilibrium condition of Geo/Geo/1 queue with graceful restart and the steady-state distribution of the system state and the number of customers by matrix geometric analysis.

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