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# Discrete-Time N-Limited Nonstop Forwarding Queue with Batch Geometric Arrivals and Geometric Services


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**ABSTRACT** *We consider a Geo^X / Geo / 1 queue with N-limited nonstop forwarding. In this queueing system, when the server breaks down, up to N customers can be serviced during the repair time. The steady-state distribution of the number of customers is given by using matrix analytic approach.*

**KEYWORDS : geometric service, discrete-time queue, N -limited nonstop forwarding**

## INTRODUCTION

This paper analyses a discrete-time $N$-limited nonstop forwarding queue. This queueing system operates as follows: customers arrive according to a batch geometric process. The server starts immediately the repair process whenever the server breaks down. Despite the server breakdown, up to $N$ customers can be serviced during the repair time. Lee and Choi [1] analyzed a Geo/Geo/1 queue with $N$-limited nonstop forwarding. Lee [2] analyzed a Geo$^X$/ D / 1 queue with $N$ -limited nonstop forwarding. In this paper, we analyze a Geo$^X$ / Geo / 1 queue with geometric service times and $N$-limited nonstop forwarding.

## SYSTEM MODEL

We consider a Geo$^X$ / Geo / 1 queue in which the time axis is divided into fixed-length contiguous intervals, referred to as slots. Customers arrive according to a batch geometric process. The numbers of arrivals during the consecutive slots are assumed to be i.i.d. random variables with distribution $\{a_k, k = 0, 1, \dots\}$. It is assumed that the service of a customer can start only at a slot boundary. The service times of customers are assumed to follow geometric distribution with probability $q$. The system has a buffer of infinite capacity.

It is assumed that the server is subject to breakdowns. The server broken down starts immediately the repair process. The lifetime of the server is assumed to be geometrically distributed with probability $\alpha$. The repair times of servers follows a geometrical distribution with probability $\beta$. When the server breaks down, the system continues to forward the next $N$ customers. The inter-arrival times, the service times, the failure times, and the repair times are assumed to be mutually independent of each other.

Let $M(k)$ be the number of customers in the system at the beginning of slot $k$. Let $S(k)$ be the system state at the beginning of slot $k$: if the server is under repair and the system has forwarded $N - n$ customers after the server's breakdown, then the value $S(k)$ is $n$ for $n = 0, 1, \dots, N$; if the server is normal, the value $S(k)$ is $N + 1$. Then the process $\{(M(k), S(k)), k = 0, 1, \dots\}$ is a Markovian process with state space $\{0, 1, 2, \dots\} \times \{0, 1, \dots, N + 1\}$. We have the transition matrix $\boldsymbol{P}$ of the Markovian process $\{(M(k), S(k)), k = 0, 1, \dots\}$:

$$\boldsymbol{P} = \begin{pmatrix} a_0(\boldsymbol{C}+\boldsymbol{F}) & a_1(\boldsymbol{C}+\boldsymbol{F}) & a_2(\boldsymbol{C}+\boldsymbol{F}) & \cdots \\ a_0 q\boldsymbol{B} & a_1 q\boldsymbol{B} + a_0\boldsymbol{D} & a_2 q\boldsymbol{B} + a_1\boldsymbol{D} & \cdots \\ \boldsymbol{0} & a_0 q\boldsymbol{B} & a_1 q\boldsymbol{B} + a_0\boldsymbol{D} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

where $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{F}$ are $(N+2)\times(N+2)$ matrices given by

$$\boldsymbol{B} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1-\beta & 0 & 0 & \cdots & 0 & 0 & \beta \\ 0 & 1-\beta & 0 & \cdots & 0 & 0 & \beta \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \beta \\ 0 & 0 & 0 & \cdots & 1-\beta & 0 & \beta \\ 0 & 0 & 0 & \cdots & 0 & \alpha & 1-\alpha \end{pmatrix}$$

$$\boldsymbol{C} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1-\beta & 0 & \cdots & 0 & 0 & \beta \\ 0 & 0 & 1-\beta & \cdots & 0 & 0 & \beta \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1-\beta & 0 & \beta \\ 0 & 0 & 0 & \cdots & 0 & 1-\beta & \beta \\ 0 & 0 & 0 & \cdots & 0 & \alpha & 1-\alpha \end{pmatrix}$$

$$\boldsymbol{F} = \begin{pmatrix} 1-\beta & 0 & \cdots & 0 & \beta \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix}$$

and $\boldsymbol{D} = (1-q)\boldsymbol{C} + \boldsymbol{F}$.

## STABILITY CONDITION OF THE SYSTEM

A necessary and sufficient condition to ensure the existence for the stationary probability vector of the process $\{(M(k), S(k))\}$ is provided. Let $\boldsymbol{\gamma} \equiv (\gamma_0, \gamma_1, \ldots, \gamma_{N+1})$ be the stationary probability vector of $\boldsymbol{A} \equiv a_0 q\boldsymbol{B} + \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\infty}(a_i q\boldsymbol{B} + a_{i-1}\boldsymbol{D}) = q\boldsymbol{B} + \boldsymbol{D}$ , i.e., $\boldsymbol{\gamma A} = \boldsymbol{\gamma}$ and $\boldsymbol{\gamma e} = 1$, where $\boldsymbol{e}$ is the column vector, all of the elements equal to 1. Solving the linear equations, we get

$$\gamma_0 = \frac{\alpha}{\alpha+\beta}\left[\frac{q(1-\beta)}{1-(1-q)(1-\beta)}\right]^N,$$

$$\gamma_i = \frac{\alpha}{\alpha+\beta}\frac{\beta}{q(1-\beta)}\left[\frac{q(1-\beta)}{1-(1-q)(1-\beta)}\right]^{N-i+1}, i = 1, 2, \ldots, N,$$

$$\gamma_{N+1} = \frac{\beta}{\alpha+\beta}.$$

The equilibrium condition of the system is

$$\boldsymbol{\gamma}\sum_{\boldsymbol{i}=\boldsymbol{1}}^{\infty}(a_i q\boldsymbol{B} + a_{i-1}\boldsymbol{D})\boldsymbol{e} < a_0 q\boldsymbol{\gamma B e},$$

which is given by

$$\sum_{\boldsymbol{i}=\boldsymbol{1}}^{\infty} i a_i < q\left[1 - \frac{\alpha}{\alpha+\beta}\left\{\frac{q(1-\beta)}{1-(1-q)(1-\beta)}\right\}^N\right],$$

the left-hand-side of which is the mean number of customers arriving in a slot and the right-hand-side of which is the probability that the service is available.

## STEADY STATE ANALYSIS

Under the above stability condition, we can compute the steady state distribution $\boldsymbol{\pi} \equiv (\boldsymbol{\pi_0}, \boldsymbol{\pi_1}, \ldots)$ of the Markov chain, where

$$\pi_{i,j} \equiv \lim_{k\to\infty} \mathrm{P}\{M(k) = i, S(k) = j\},$$

$$\boldsymbol{\pi_i} \equiv (\pi_{i,0}, \pi_{i,1}, \ldots, \pi_{i,N+1}).$$

Note that the steady state distribution $\boldsymbol{\pi}$ is the solution to the linear system $\boldsymbol{\pi P} = \boldsymbol{\pi}$ and $\boldsymbol{\pi e} = 1$.

For the solution of M/G/1-type processes, several algorithms exist. These algorithms start with the computation of matrix $\boldsymbol{G}$, which is the solution of the following equation:

$$\boldsymbol{G} \equiv a_0 q\boldsymbol{B} + \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\infty}(a_i q\boldsymbol{B} + a_{i-1}\boldsymbol{D})\boldsymbol{G}^i.$$

The matrix $\boldsymbol{G}$ is obtained by solving iteratively the above equation. The stationary probability vector $\boldsymbol{\pi}$ is computed recursively using Ramaswami's recursive formula [3].

## CONCLUSIONS

We considered a $N$-limited nonstop forwarding queue. We gave the stability condition of $\mathrm{Geo}^X$ / Geo / 1 queue with $N$-limited nonstop forwarding queue and the steady-state distribution of the system state and the number of customers by matrix analytic approach.

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