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Research Paper Mathematics

Blocking Probability of Loss System with two Jobs Running on Different Server Groups

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ABSTRACT *This paper considers a queue with two different jobs running on different multi-server groups. The queueing system supports the multi-type prioritized customers. To increase the acceptance for high priority customers, it is assumed that the system operates under a server reservation policy. Our mathematical analysis uses multi-dimensional operations of vectors or matrices to deal with multiple types of customers and the two jobs in one time. We estimate call blocking probabilities.*

KEYWORDS : blocking probability, loss system, multi-server, multiple jobs

INTRODUCTION

This paper considers a queueing system with two different jobs running on different multiple server groups. The system supports the multi-type prioritized customers. To support more customers and to increase the acceptance for high priority customers, it is assumed that the system operates under a server reservation policy. Our mathematical analysis uses multi-dimensional operations of random vectors or matrices to deal with multiple types of customers and the two different jobs in one time. We approximate the distribution of the service units in use. This leads to the blocking probabilities

QUEUEING MODEL

There are two jobs, A and B, and $S_{\max}$ servers, among which S_{sw} are allocated on job A and the others on job B. Each server processes $C_{\max}$ service units at the same time. At the same time, $C_{\max}\times S_{sw}$ and $C_{\max}\times(S_{\max}-S_{sw})$ service units can be used for job A and B, respectively. We assume that there exist I customer types. Type-i customers will require u_i and d_i service units on job A and B, respectively. The arrival processes of high and low priority type-i customers follow Poisson processes with rate λ_i^H and λ_i^L, respectively. All customer arrival processes are independent of each other. The service times of type-i customers have an exponential distribution with rate μ_i. All service times are independent of each other and of the arrival processes. It is assumed that R_u service units for job A and R_d units for job B are reserved for high priority customers, i.e., a low priority customer of type-i will be blocked if the number of service units in use for job A is greater than $C_{\max}\times S_{sw}-R_u-u_i$ or the number of service units in use for job B is greater than $C_{\max}\times(S_{\max}-S_{sw})-R_d-d_i$.

ANALYSIS

A random vector to represent the system state is denoted by $N\equiv(N_1,N_2,\dots,N_I)$, where N_i represents the number of type-i customers. The flow balance equations of the system states can be formulated and evaluated by applying Gauss-Siedel iteration. However, this approach is inadequate for realistic parameter sets having large state space. An alternative method is therefore necessary. We assume the system is in equilibrium state. Let $\rho_i^H\equiv\lambda_i^H/\mu_i$ and $\rho_i^L\equiv\lambda_i^L/\mu_i$. Let $J=N\times B$ with

$$B=\begin{pmatrix} u_1 & u_2 & \cdots & u_I \\ d_1 & d_2 & \cdots & d_I \end{pmatrix}'$$

and let $q(j_1,j_2)\equiv P(J=(j_1,j_2))$.

CASE WITHOUT RESERVATION POLICY

Let $n\equiv(n_1,n_2,\dots,n_I)$ be the system state. The state probability is given by the product form

$$P(N=n)=\frac{1}{D}\prod_{i=1}^{I}\frac{(\rho_i^H+\rho_i^L)^{n_i}}{n_i!}, \tag{1}$$

where D is the normalizing constant. Although we can use (1) to calculate the blocking probabilities, we introduce efficient recursion for the occupancy distribution. We can obtain

$$(\rho_i^H+\rho_i^L)q(j_1-u_i,j_2-d_i)=E[N_i\,|\,J=(j_1,j_2)]q(j_1,j_2), \tag{2}$$

where $q(j_1,j_2)=0$ for $j_1<0$ or $j_2<0$. From (2),

$$\sum_{i=1}^{I}(u_i+d_i)(\rho_i^H+\rho_i^L)q(j_1-u_i,j_2-d_i)=(j_1+j_2)q(j_1,j_2). \quad (3)$$

The unnormalized probability mass function $q(.,.)$ can be computed from (3) with initial value $q(0,0)=1$. After normalization of $q(.,.)$, the blocking probability B_i of type-i customers is obtained by

$$B_i=1-\sum_{j_1=0}^{C_{\max}\times S_{sw}-u_i}\sum_{j_2=0}^{C_{\max}\times(S_{\max}-S_{sw})-d_i}q(j_1,j_2). \quad (4)$$

The total blocking probability P_B is computed as

$$P_B=\sum_{i=1}^{I}(\lambda_i^H+\lambda_i^L)B_i\Big/\sum_{j=1}^{I}(\lambda_j^H+\lambda_j^L).$$

CASE WITH RESERVATION POLICY

In case with channel reservation policy, we can obtain

$$\rho_i(j_1,j_2)q(j_1-u_i,j_2-d_i)=E[N_i\mid J=(j_1,j_2)]q(j_1,j_2), \quad (5)$$

where

$$\rho_i(j_1,j_2)=\begin{cases}\rho_i^H+\rho_i^L, & j_1\le C_{\max}\times S_{sw}-R_u,\ j_2\le C_{\max}\times(S_{\max}-S_{sw})-R_d,\\ \rho_i^H, & otherwise.\end{cases}$$

Multiplying (5) by u_i+d_i and summing them over i,

$$\sum_{i=1}^{I}(u_i+d_i)\rho_i(j_1,j_2)q(j_1-u_i,j_2-d_i)=(j_1+j_2)q(j_1,j_2). \quad (6)$$

The function $q(.,.)$ can be computed from (6) with initial value $q(0,0)=1$ Let B_i^H and B_i^L be the blocking probabilities of high and low priority type- i customers, respectively. After normalization of $q(.,.)$, the probabilities B_i^H and B_i^L can be computed as

$$B_i^H=1-\sum_{j_1=0}^{C_{\max}\times S_{sw}-u_i}\sum_{j_2=0}^{C_{\max}\times(S_{\max}-S_{sw})-d_i}q(j_1,j_2),$$

$$B_i^L=1-\sum_{j_1=0}^{C_{\max}\times S_{sw}-R_u-u_i}\sum_{j_2=0}^{C_{\max}\times(S_{\max}-S_{sw})-R_d-d_i}q(j_1,j_2).$$

The total blocking probabilities P_B^H and P_B^L of high and low priority customers, respectively, are

$$P_B^H=\sum_{i=1}^{I}\lambda_i^H B_i\Big/\sum_{j=1}^{I}\lambda_j^H,$$

$$P_B^L=\sum_{i=1}^{I}\lambda_i^L B_i\Big/\sum_{j=1}^{I}\lambda_j^L.$$

CONCLUSIONS

We considered a queue with two different jobs running on different multi-server groups. The system supports the multi-type prioritized customers and operates under a server reservation policy. Using multi-dimensional operations of vectors or matrices, we estimated call blocking probabilities.

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