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A Note On (Q, L)-Fuzzy Subhemirings of A Hemiring

N. ANITHA	Assistant Professor in Mathematics, Periyar University PG, Extn, centre, Dharmapuri.
M. LATHA	Department of Mathematics, Karpagam University,Coimbatore

ABSTRACT *In this paper, we made an attempt to study the algebraic nature of an (Q, L)-fuzzy subhemirings of a hemi ring.*

KEYWORDS : L-fuzzy set, (Q, L)-fuzzy subhemiring, pseudo (Q, L)-fuzzy coset.

INTRODUCTION:

There are many concepts of universal algebras generalizing an associative ring (R; + ;.). Some of them in particular, nearrings and several kinds of semi rings have been proven very useful. Semirings (called also halfrings) are algebras (R; + ;.) share the same properties as a ring except that (R; +) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R; + ;.) is said to be a semiring (R; +) and (R ;.) are semigroups satisfying a.(b+c) =a.b+a.c and (b+c).a=b.a+c.a for all a,b and c in R. A semiring R is said to be additively commutative if a+b=b+a for all a,b an c in R. A semiring R may have an identity 1, defined by 1.a=a=a.1 and a zero 0, defined by 0+a=a=a+0 and a.0=0=0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh [14], several researchers explores on the generalization of the concept of fuzzy sets. The notion of anti left h-ideals in hemi ring was introduced by Akram.M and K.H.Dar [1].The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan [7].Osman Kanzanci, Sultan yamark and serife yilmaz in [6] have introduced the Notion of intuitionistic Q-fuzzification of N-subgroups (sub near-rings) in a near –ring and investigated some related properties. Solairaju.A and R.Nagarajan, have given a new structure in construction of Q-fuzzy groups and subgroups [10] and [11]. In this paper, we introduce the properties and some theorems in (Q,L)- fuzzy subhemirings of a hemiring.

1. PRELIMINARIES

1.1 Definition: Let X be a non-empty set and L= (L,≥) be a lattice with least element 0 and greastest element 1.

1.2 Definition: Let X be a non-empty set and Q be a non-empty set. A (Q, L)--fuzzy subset A of X is function $A: X \times Q \rightarrow L$

1.3 Definition:Let (R,+, .) be a hemiring. A (Q,L)-fuzzy subset A of R is said to be an (Q,L)-fuzzy subhemiring of R if it satisfies the following conditions:

(i) $\mu_A(x+y,q) \geq \left((\mu_A(x,q) \wedge \mu_A(y,q)\right)$

(ii) $\mu_A(xy,q) \geq \left((\mu_A(x,q) \wedge \mu_A(y,q)\right)$, for all x and y in R, and q in Q.

1.4 Definition: Let A and B be (Q, L)-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by AxB is defined as

AxB=$\{\langle((x,y),q),\ \mu_{AXB}(x,y),q)\rangle/$ for all x in G and y in H & q in Q$\}$,where $\mu_{AXB}(x,y),q) = \{(\mu_A(x,q)\wedge\mu_B(y,q)\}$.

1.5 Definition: Let A be a (Q,L)-fuzzy subset in a set S, the strongest relation (Q,L)-fuzzy relation on S, that is a (Q,L)- fuzzy relation on A is V given by $\mu_V((x,y),q) = \{(\mu_A(x,q)\wedge \mu_B(y,q)\}$ for all x and y in S and q in Q.

1.6 Definition: Let (R, +,) and $(R',+,\ .)$ be any two hemirings. Let $f: R \rightarrow R'$ be any function and A be an (Q, L)-fuzzy subhemiring in R, V be an (Q, L)-fuzzy subhemiring in $f(R) = R'$, defined by $\mu_V(y,q) = \sup_{x\in f^{-1}(y)}\mu_A(x,q)$ for all x in R and y in R' and q in Q. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.7 Definition: Let A be an (Q, L)-fuzzy subhemiring of a hemiring (R, +, .) and a in R, Then the pseudo (Q, L)-fuzzy coset $(aA)^p$ is defined by$\left(\left(a\mu_A\right)^P\right)(x,q) = p^{(a)}\mu_A(x,q)$, for every x in R, q in Q and for some p in P.

2. PROPERTIES OF (Q,L)-FUZZY SUBHEMIRINGS OF A HEMIRING

2.1 Theorem: Union of any two (Q,L)-fuzzy subhemiring of a hemiring R is an (Q,L)-fuzzy subhemiring of R.

Proof: Let A and B be any two (Q, L)-fuzzy subhemirings of a hemiring R and x and y in R. Let A =$\{\langle(x,q),\mu_A(x,q)\rangle/\ x \in R\ \&q \in Q\}$ and B =$\{\langle(x,q),\mu_B(x,q)\rangle/\ x \in R\ \&q \in Q\}$ and also Let $C = AUB = \{\langle(x,q),\mu_C(x,q)\rangle\ /\ x \in R\ \&\ q \in Q\}$, where $\{(\mu_A\ (x,q)\ \wedge\ \mu_B(x,q)\} =\ \mu_C(x,q)$,

Now , $\mu_C(x+y,q) \geq\ \left\{(\mu_A(x,q) \wedge \mu_A(y,q)\right) \wedge ((\mu_B(x,q) \wedge \mu_B(y,q))\} \geq \left(\mu_C(x,q) \wedge \mu_C(y,q)\right)$.

Therefore,$\mu_C(x+y,q) \geq\ \left(\mu_C(x,q) \wedge \mu_C(y,q)\right)$, for all x and y in R and q in Q. And $\mu_C(\ xy\ ,q) \geq \left\{\left(\mu_A(x,q)\ \wedge\ \mu_A(y,q)\right) \wedge\ \left((\mu_B(x,q)\ \wedge\ \mu_B(y,q)\right)\right\} \geq \left(\mu_C\ (x,q)\ \wedge\ \mu_C\ (y,q)\right)$.

Therefore, $\mu_C(xy, q) \geq \left(\mu_C(x, q) \wedge \mu_C(y, q)\right)$, for all x and y in R and q in Q. Therefore C is an (Q, L)-fuzzy subhemiring of a hemiring R.

2.2 Theorem: The Union of a family of (Q,L)-fuzzy subhemiring of hemiring R is an (Q,L)-fuzzy subhemiring of R.

Proof: It is trivial.

2.3 Theorem: If A and B are two (Q,L)-fuzzy subhemirings of the hemirings R_1 and R_2 respectively,then product AXB is an (Q,L)-fuzzy subhemiring of $R_1 X R_2$.

Proof: Let A and B be two (Q,L)- fuzzy subhemirings of the hemirings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then $((x_1, y_1), q)$ and $((x_2, y_2), q)$ are in $R_1 X R_2$.Now, $\mu_{AXB}[((x_1, y_1) + (x_2, y_2)), q)] \geq \left\{ (\mu_A(x_1, q) \wedge \mu_A(x_2, q)\right) \wedge \left(\left(\mu_B(y_1, q) \wedge \mu_B(y_2, q)\right)\right\} \geq \left(\mu_{AXB}(x_1, y_1), q) \wedge \mu_{AXB}(x_2, y_2), q)\right)$ .Therefore,$\mu_{AXB}[((x_1, y_1) + (x_2, y_2)), q)] \geq \left(\mu_{AXB}(x_1, y_1), q) \wedge \mu_{AXB}(x_2, y_2), q)\right)$. Also, μ_{AXB} [$((x_1, y_1)(x_2, y_2))$, q)] $\geq \left\{(\mu_A(x_1, q) \wedge \mu_A(x_2, q)\right) \wedge ((\mu_B(y_1, q) \wedge \mu_B(y_2, q))\} \geq \left(\mu_{AXB}(x_1, y_1), q) \wedge \mu_{AXB}(x_2, y_2), q)\right)$. Therefore,$\mu_{AXB}[((x_1, y_1)(x_2, y_2)), q)] \geq \left(\mu_{AXB}(x_1, y_1), q) \wedge \mu_{AXB}(x_2, y_2), q)\right)$.Hence AXB is an (Q,L)-fuzzy subhemiring of a hemiring $R_1 X R_2$.

2.4 Theorem: Let A be a (Q, L)-fuzzy subset of a hemiring R and V be the strongest fuzzy relation of R. Then A is an (Q, L)-fuzzy subhemiring of R if and only if V is an (Q,L)-fuzzy subhemiring of RxR.

Proof: Suppose that A is an (Q,L)-fuzzy subhemiring of a hemiring R. Then for any $X = (x_1, x_2)$ and $Y = (y_1, y_2)$ are in R x R. We have $\mu_V(XY, q) \geq \{(\mu_A(x_1 y_1, q) \wedge \mu_A(x_2 y_2, q)\} \geq \left\{\left(\mu_A(x_1, q) \wedge \mu_A(y_1, q)\right) \wedge ((\mu_A(x_2, q) \wedge \mu_A(y_2, q)\right)\right\} \geq (\mu_V((x_1, x_2), q) \wedge \mu_V((y_1, y_2), q) \geq (\mu_V(X, q) \wedge \mu_V(Y, q)$, for all X and Y in R x R and q in Q. Therefore, $\mu_V(XY, q) \geq \left(\mu_V(X, q) \wedge \mu_V(Y, q)\right)$, for all X and Y in R x R and q in Q. This proves that V is an (Q,L)-fuzzy subhemiring of a hemiring of R x R. Conversely assume that V is an (Q,L) - fuzzy subhemiring of a hemiring of R x R , then for any $X = (x_1, x_2)$ and $Y = (y_1, y_2)$ are in R x R . we have $\{(\mu_A(x_1 + y_1, q) \wedge \mu_A(x_2+y_2, q)) = \mu_V(X + Y, q) \geq (\mu_V(X, q) \wedge \mu_V(Y, q) = (\mu_V((x_1, x_2), q) \wedge \mu_V((y_1, y_2), q) = (\mu_A(x_1, q) \wedge \mu_A(y_1, q)) \wedge ((\mu_A(x_2, q) \wedge \mu_A(y_2, q))$. If $x_2 = 0, y_2 = 0$ we get $\mu_A(x_1 + y_1, q) \geq (\mu_A(x_1, q) \wedge \mu_A(y_1, q))$ for all x_1 and y_1 in R . An□, $\{(\mu_A(x_1 y_1, q) \wedge \mu_A(x_2 y_2, q)\} = \mu_V(XY, q) \geq (\mu_V(X, q) \wedge \mu_V(Y, q)) = (\mu_V((x_1, □_2), q)) \wedge \mu_V((y_1, y_2), q) = (\{\mu_A(x_1, q) \wedge \mu_A(x_2, q)\} \wedge \{\mu_A(y_1, q) \wedge \mu_A(y_2, q)\})$.

If $x_2 = 0, y_2 = 0$ we get $\mu_A(x_1 y_1, q) \geq (\mu_A(x_1, q) \wedge \mu_A(y_1, q))$ for all x_1 and y_1 in R.Therefore A is an (Q,L)-fuzzy subhemiring of R.

2.5 Theorem: If A is an (Q,L)-fuzzy subhemiring of a hemiring (R, +, .) if and only if $\mu_A(x + y, q) \geq (\mu_A(x, q) \wedge \mu_A(y, q)$, $\mu_A(xy, q) \geq (\mu_A(x, q) \wedge \mu_A(y, q)$ for all x and y in R.

Proof: It is trivial.

2.6 Theorem: If A is an (Q,L)-fuzzy subhemiring of a hemiring (R, +, .),then $H = \{x/x \in R : \mu_A(x, q) = 0\}$ is either empty or is a subhemiring of R.

Proof: It is trivial.

2.7 Theorem: Let A is an (Q,L)-fuzzy subhemiring of a hemiring (R, +, .) .If $\mu_A(x + y, q) = 1$, then either $\mu_A(x, q) = 1$ or $\mu_A(y, q) = 1$, for all x and y in R.

Proof: It is trivial.

2.8 Theorem: Let A is an (Q,L)-fuzzy subhemiring of a hemiring (R, +, .),then the pseudo (Q,L)-fuzzy coset $(aA)^P$ is (Q,L)-fuzzy subhemiring of a hemiring R, for every a in R.

Proof: Let A is an (Q,L)-fuzzy subhemiring of a hemiring R. For every x and y in R, we have $((a\,\mu_A)^p)(x + y, q) \geq p(a)(\mu_A(x, q) \wedge \mu_A(y, q)) \in (p(a)\mu_A(x, q) \wedge p(a)\mu_A(y, q) = ((a\,\mu_A)^p(x, q) \wedge (a\,\mu_A)^p(y, q))$. Therefore,$((a\,\mu_A)^p)(x + y, q) \geq ((a\,\mu_A)^p(x, q) \wedge (a\,\mu_A)^p(y, q))$.

Now, $((a\,\mu_A)^p)(xy\,,q) \geq p(a)\left(\mu_A(x,q) \wedge \mu_A(y,q)\right) = \left(p(a)\mu_A(x,q) \wedge p(a)\mu_A(y,q) = \right.$ $((a\,\mu_A)^p(x,q) \wedge (a\,\mu_A)^p(y,q)\Big)$. Therefore,$((a\,\mu_A)^p)(xy\,,q) \geq ((a\,\mu_A)^p(x,q) \wedge (a\,\mu_A)^p(y,q))$. Hence $(aA)^P$ is an (Q,L)-fuzzy subhemiring of a hemiring R .

2.9 Theorem: Let (R, +,.) and (R′, +, .) be any two hemirings. The homomorphic image of an (Q, L)-fuzzy subhemiring of R is an (Q, L)-fuzzy subhemiring of R′.

Proof: Let $f: R \to R'$ be a homomorphism.Then,$f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let V= f(A), where A is an (Q,L) fuzzy subhemiring of R. Now, for $f(x)$, $f(y)$ in R', $\mu_V((f(x) + (f(y)), q)) \geq \mu_A(x+y, q) \geq (\mu_A(x,q) \wedge \mu_A(y,q)$, which implies that $\mu_V((f(x) + (f(y)), q)) \geq (\mu_V(f(x), q) \wedge (\mu_V(f(y), q)$. Again, $\mu_V\,(\,(f(x)\,f(y), q)) \geq \mu_A(xy, q) \geq (\mu_A(x,q) \wedge \mu_A(y,q)$,which implies that $\mu_V((f(x)(f(y)), q)) \geq (\mu_V\left((f(x), q) \wedge \mu_V(f(y), q)\right)$. Hence V is an (Q,L)-fuzzy subhemiring of hemiring R′.

2.10 Theorem: Let (R, +,.) and (R′, +,.) be any two hemirings. The homomorphic preimage of an (Q, L)-fuzzy subhemiring of R′ is an (Q,L)-fuzzy subhemiring of R.

Proof: Let $f: R \to R'$ be a homomorphism.Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let V= f(A) where V is an (Q,L) fuzzy subhemiring of R′.Now, for x, y in R, $\mu_A\left((x+y, q)\right) = \mu_V((f(x) + f(y)), q) \geq (\mu_V(f(x), q) \wedge \mu_V(f(y), q)$ $= (\mu_A(x,q) \wedge \mu_A(y,q)$, which implies that $\mu_A\left((x+y, q)\right) \geq (\mu_A(x,q) \wedge \mu_A(y,q)$. Again, $\mu_A\left((xy, q)\right) = \mu_V((f(x)f(y)\,, q) \geq (\mu_V(f(x)\,, q) \wedge \mu_V(f(y), q) = (\mu_A(x,q) \wedge \mu_A(y,q)$, which implies that $\mu_A\left((xy, q)\right) \geq (\mu_A(x,q) \wedge \mu_A(y,q)$. Hence A is an (Q, L)-fuzzy subhemiring of hemiring R.

2.11 Theorem: Let (R, +, .) and (R′, +, .) be any two hemirings. The anti-homomorphic image of an (Q,L)-fuzzy subhemiring of R is an (Q,L)-fuzzy subhemiring of □′.

Proof: Let $f: R \to R'$ be a anti-homomorphism.Then,$f(x+y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R. Let V= f(A) where A is an (Q,L)- fuzzy subhemiring of R.

Now, for $f(x), f(y)$ in R', $\mu_V((f(x)+f(y),q)) \geq \mu_A(y+x,q) \geq (\mu_A(y,q) \wedge \mu_A(x,q) = (\mu_A(x,q) \wedge \mu_A(y,q)$, which implies that $\mu_V((f(x)+f(y),q)) \geq (\mu_V\left((f(x),q) \wedge \mu_V(f(y),q)\right)$. Again, $\mu_V\left((f(x)f(y),\ q)\right) \geq \mu_A(yx,\ q) \geq (\mu_A(y,q) \wedge \mu_A(x,q) = (\mu_A(x,\ q) \wedge \mu_A(y,\ q)$ which implies that $\mu_V((f(x)f(y),q)) \geq (\mu_V\left((f(x),q) \wedge \mu_V(f(y),q)\right)$. Hence V is an (Q,L)-fuzzy subhemiring of hemiring R'.

2.12 Theorem: Let (R, +,.) and (R', +, .) be any two hemirings. The anti-homomorphic preimage of an (Q, L)-fuzzy subhemiring of R' is an (Q,L)-fuzzy subhemiring of R.

Proof: Let $V = f(A)$ where V is an $(Q,L)-$ fuzzysubhemiring of R'. Let x and y in R.Then $\mu_A\left((x+y,q)\right) = \mu_V((f(x)+f(y)),q) \geq (\mu_V(f(y),q) \wedge \mu_V(f(x),q) = (\mu_A(x,q) \wedge \mu_A(y,q)$, which implies that $\mu_A\left((x+y,q)\right) \geq (\mu_A(x\ ,\ q) \wedge \mu_A(y,\ q)$. Again, $\mu_A\left((x\,y,q)\right) = \mu_V(f(x)\,f(y),q) \geq (\mu_V(f(y),q) \wedge \mu_V(f(x),q) = (\mu_A(x,q) \wedge \mu_A(y,q)$, which implies that $\mu_A\left((xy,q)\right) \geq (\mu_A(x,q) \wedge \mu_A(y,q)$. Hence A is an (Q, L)-fuzzy subhemiring of hemiring R.

In the following Theorem∘ is the composition operation of functions:

2.13 Theorem: Let A be an (Q, L)-fuzzy subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H. Then A∘ f is an (Q, L)-fuzzy subhemiring of R.

Proof: Let x and y in R. Then we have, $(\mu_A \circ f)\left((x+y,\ q)\right) = \mu_A((f(x)+(f(y)),q) \geq (\mu_A(f(x),\ q) \wedge \mu_A(f(y),q) \geq \left(\mu_A \circ f\right)(x,q) \wedge \left(\mu_A \circ f\right)(y,q)$.which implies that $\left(\mu_A \circ f\right)\left((x+\ y,\ q)\right) \geq \left(\mu_A \circ f\right)(x,\ q) \wedge \left(\mu_A \circ f\right)(y,q)$. And, $(\mu_A \circ f)\left((xy,q)\right) = \mu_A((f(x)f(y)),q) \geq (\mu_A(f(x),\ q) \wedge \mu_A(f(y),\ q) \geq \left(\mu_A \circ f\right)(x,q) \wedge \left(\mu_A \circ f\right)(y,q)$, which implies that $\left(\mu_A \circ f\right)\left((xy,q)\right) \geq \left(\mu_A \circ f\right)(x,q) \wedge \left(\mu_A \circ f\right)(y,q)$.Therefore A∘f is an (Q, L)-fuzzy subhemiring of hemiring R.

2.14 Theorem: Let A be an (Q, L)-fuzzy subhemiring of hemiring H and f is an anti- isomorphism from a hemiring R onto H. Then A∘ f is an (Q,L)-fuzzy subhemiring of R.

Proof:Let x and y in R. Then we have, $(\mu_A \circ f)((x+y, q)) = \mu_A((f(y)+f(x)), q) \geq (\mu_A(f(x), q) \wedge \mu_A(f(y), q) \geq (\mu_A \circ f)(x, q) \wedge (\mu_A \circ f)(y, q)$. which implies that $(\mu_A \circ f)((x+y, q)) \geq ((\mu_A \circ f)(x, q) \wedge (\mu_A \circ f)(y, q))$. And, $(\mu_A \circ f)((xy, q))$ $= \mu_A((f(y)\, f(x)), q) \geq (\mu_A(f(x), q) \wedge \mu_A(f(y), q) \geq (\mu_A \circ f)(x, q) \wedge (\mu_A \circ f)(y, q)$, which implies that $(\mu_A \circ f)((xy, q)) \geq (\mu_A \circ f)(x, q) \wedge (\mu_A \circ f)(y, q)$. Therefore A∘f is an (Q, L)-fuzzy subhemiring of hemiring R.

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