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ON THE SIMPLE GROUP GENERATED BY STRUCTURE EQUATION $F^5+F^4+F^3+F^2+F=0$

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ABSTRACT *In this paper simple group generated by the sturcture equation , $F^5+F^4+F^3+F^2+F=0$ is studied. Properties of the Kernel, collection of tangent and normal vectors have also been discussed.*

KEYWORDS : Differnetiable manifold, projection operators, linear congruence, group and its normal sub groups, simple group, Kernel, tangent and normal vectors

1. INTRODUCTION:

Let M^n be a differentiable manifold of class C^∞ and F be a (1,1) tensor of class C^∞ defined on M^n, satisfying

(1.1) $F^5+F^4+F^3+F^2+F=0$

we define the operators l and m on M^n by

(1.2) $l=F^5, \qquad m=I-F^5$

where I denotes the identity operator.

Theorem (1.1): Let F, l and m satisfy (1,1) and (1.2) then

(1.3) (i) $F^s=F^r$ where $s\equiv r \pmod 5,\ r,s\geq 1$

(ii) $l+m=I,\ l^2=l,\ m^2=m,\ lm=ml=0$

$Fl=lF=F, \quad Fm=mF=0$

Proof (i): from (1.1), we get

(1.4) $F^5=-F^4-F^3-F^2-F$

$\Rightarrow F^6=-F^5-F^4-F^3-F^2$

$$\Rightarrow F^6 = F^4 + F^3 + F^2 + F - F^4 - F^3 - F^2$$

$$\Rightarrow F^6 = F$$

$$\Rightarrow F^7 = F^2, F^8 = F^3, F^9 = F^4, F^{10} = F^5, F^{11} = F^6 = F$$

$\Rightarrow F^s = F^r$ where $s \equiv r$ (mod 5)

(ii) From (1.2) and (1.3) (i), we get the required results.

Theorem (1.2): Let *F* and *m* satisfy (1.1) and (1.2) then the set

(1.5) $M_5 = \{m + F,\ m + F^2,\ m + F^3, m + F^4, m + F^5\}$

is a cyclic and simple group, under multiplicatrion of operators.

Proof: Using (1.3) We have the Cayley table for M_5

	$m+F$	$m+F^2$	$m+F^3$	$m+F^4$	$m+F^5$
$m+F$	$m+F^2$	$m+F^3$	$m+F^4$	$m+F^5$	$m+F$
$m+F^2$	$m+F^3$	$m+F^4$	$m+F^5$	$m+F$	$m+F^2$
$m+F^3$	$m+F^4$	$m+F^5$	$m+F$	$m+F^2$	$m+F^3$
$m+F^4$	$m+F^5$	$m+F$	$m+F^2$	$m+F^3$	$m+F^4$

$m+F^5$	$m+F$	$m+F^2$	$m+F^3$	$m+F^4$	$m+F^5$

From the table it is clear that

a) **Closure property:** The product of any two elements of M_5 is again in M_5 $\therefore M_5$ is closed under multiplication.

b) **Associative property:** Since the multiplication of operators is always associative $\therefore$ it hold for elements of M_5 also.

c) **Existence of Identity:** From the Table (1.6), it is clear that $m+F^5$ works as an identity element of M_5. from (1.2) it is also clear that $m+F^5=I$.

d) **Existence of inverse:** From the table (1.6), we have

(1.7) $$\left(m+F\right)^{-1}=m+F^4 \Rightarrow \left(m+F^4\right)^{-1}=m+F$$

$$\left(m+F^2\right)^{-1}=m+F^3 \Rightarrow \left(m+F^3\right)^{-1}=m+F^2$$

$$\left(m+F^5\right)^{-1}=m+F^5=I$$

Thus each element of M_5 has its multiplicative inverse in M_5.

In all M_5 is a group. Moreover

(1.8) 0 (M_5)=5 (a prime number).

Therefore, M_5 is cyclic, $M_5 = \langle m+F \rangle = \langle m+F^2 \rangle = \langle m+F^3 \rangle = \langle m+F^4 \rangle$. Also M_5 has no proper normal subgroup therefore, M_5 is simple.

Theorem (1.2): Let (1,1) tensors $p, q \in M_5$ where

(1.9) $p = m+F, \quad q = m+F^3$, then

(1.10) $p^3 = q, q^2 = p, p^5 = I = q^5 = p^2 q = q^3 p.$

Proof: From (1.2), (1.3) and (1.9), we get (1.10).

Theorem (1.3): Let (1,1) tensors $\alpha, \beta \in M_5$, where

(1.11) $\alpha = m+F^2, \quad \beta = m+F^4$, then

(1.12) $\alpha^2 = \beta, \beta^3 = \alpha, \ \alpha\beta^2 = I = \alpha^3 \beta$

Proof: From (1.2), (1.3), and (1.11), we get (1.12)

2. METRIC F-STRUCTURE:

Let g be the Riemannian metric satisfying

(2.1) $`F(X,Y) = g(FX,Y)$ is symmetric,

then

(2.2) $g(FX,Y) = g(X,FY)$

and $\{F,g\}$ is call ed a metric F-structure.

Theorem (2.1): Let F satisfies (1.1) then

(2.3) $g(F^5X,F^5Y) = g(X,Y) - `m(X,Y)$, where,

(2.4) $`m(X,Y) = g(mX,Y) = g(X,mY)$

Proof: Using (1.2), (1.3) (2.2), (2.4), we have

(**2**.5) $g(F^5X,F^5Y) = g(X,F^{10}Y)$

$$= g(X,F^5Y)$$

$$= g(X, lY)$$

$$= g(X, (I - m)Y)$$

$$= g(X, Y) - g(X, mY)$$

$$= g(X, Y) - `m(X, Y)$$

3. KERNEL, TANGENT AND NORMAL VECTORS: We define

(3.1) $Ker\ F = \{X : FX = 0\}$

(3.2) $Tan\ F = \{X : FX \parallel X\} = \{X : FX = \lambda X\}$

(3.3) $Nor\ F = \{X : g(X, FY) = 0, \forall\ Y\}$

Theorem (3.1): Let F satisfies (1.1), then

(3.4) $Ker\ F = Ker\ F^2 = \ldots = Ker\ F^5$

(3.5) $Tan\ F = Tan\ F^2 = \ldots = Tan\ F^5$

(3.6) $Nor\ F = Nor\ F^2 = \ldots = Nor\ F^5$

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