Original Research Paper Mathematics

Domination D – Distance of a Graph

N.Arianayagam	Department of Mathematics, Government College of Engineering Tirunelveli-627 007,India
J.John	Department of Mathematics, Government College of Engineering Tirunelveli-627 007,India

ABSTRACT

For vertices u and v in a connected graph G, the detour distance D (u,v) is the length of the longest u- v path in G. A u – v path of length D (u,v) is called a u – v detour. It is known that the detour distance is a metric on the vertex set V (G). Chart and et al introduced the concept of detour distance by considering the length of the longest path between u and v. Kathiresan et al introduced the concept of superior distance and signal distance. In some of these distances only the length of various paths were considered. By considering the degrees of dominating set vertices present in the path and in addition, subtract the length of the path. In this article we introduced the concept of domination D – distance. We study some properties of this new distance

KEYWORDS :

Introduction

For a graph $G = (V, E)$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. We consider connected graphs with atleast two vertices. For basic definitions and terminologies we refer to [1, 4].

For vertices u and v in a connected graph G, the detour distance $D\,(u, v)$ is the length of the longest $u - v$ path in G. A $u - v$ path of length $D\,(u, v)$ is called a $u - v$ detour. It is known that the detour distance is a metric on the vertex set $V(G)$. The detour eccentricity $e_D(v)$ of a vertex v in G is the maximum detour distance from v to vertex of G. The detour radius, $\mathrm{rad}_D\, G$ of G is the minimum detour eccentricity among the vertices of G, while the detour diameter, $\mathrm{diam}_D\, G$ of G is the maximum detour eccentricity among the vertices of G. These concept were studied by chartrand et al [2].

A set $S \subseteq V\,(G)$ is called a dominating set of G if every vertex in $V\,(G) - S$ is adjacent to some vertex in S. The domination number $\gamma(G)$ of G is the minimum order of its dominating sets and any dominating set of order $\gamma(G)$ is called γ-set of G. These concept were by Kathiresan and Sumathi introduced the concept of signal distance in G. Reddy Babu et al introduced a $D -$ distance as follows : The D – distance $d^D\,(u, v)$ between two vertices u, v of a connected graph G is detoured as $d^D\,(u, v) = \min\{l^D\,(s)\}$ where the minimum is taken over all $u - v$ paths s in G. In other words $d^D\,(u, v) = min\{d\,(u, v) + deg\,(u) + deg\,(v) + \sum deg\,(w)\}$ where the sum runs over all intermediate vertices w in s and minimum is taken over all $u - v$ path in G.

In this article we introduce a new distance, which we call as Domination D –distance was considered. Here we, consider the degree of beginning and end vertices of $u - v$ path and also in addition to find the degree of the dominating vertices present in a path, from this all degree we subtract the detour distance. While we defining its length. Using this length we define the Dominating D −distance.

2. Dominating D −distance number of a graph.

Definition 2.1 If u, v are vertices of a connected graph G, the D −length of a $u - v$ path P is defined as $l^D(p) = \deg(u) + \deg(v) + \sum \deg(s) - D(u, v)$

where the sums runs over all the intermediate dominating set vertices in set s of p in a connected graph G.

Definition 2.2 The Dominating $D -$ distance ${\gamma_D}^D(u, v)$ between two vertices u, v of a connected graph G is defined as ${\gamma_D}^D(u, v) = \{l^D(p)\}$. In otherwords

${\gamma_D}^D(u, v) = \deg(u) + \deg(v) + \sum \deg(s) - D(u, v)$ where the sums runs over all dominating vertices in the set S in a connected graph G and the path p is taken over all the vertices lies in between $u - v$ paths in G & $u, v \notin S$. The dominating $D -$ distance number ${\gamma_D}^D(G)$ of G is $l(p) - 1 = \Delta(G)$.

Example 2.3

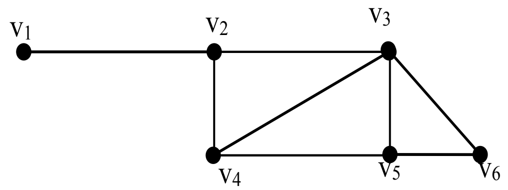

Figure 2.1

Volume-5, Issue-9, September - 2016 • ISSN No 2277 - 8160

For the graph G given in figure 2.1 $p = \{u = v_1, v_2, v_3, v_4, v_5, v_6 = v\}$ is the longest $u - v$ path in G. Then Dominating D −length of the path p, is $l^D(p) = 1 + 2 + 3 + 3 - 5 = 4$ .So that the Dominating D −distance as ${\gamma_D}^D(u, v)$ of G is $l(p) - 1 = 5 - 1 = 4 = \Delta(G)$.

Remarks 2.4

There are more than one longest path between u & v of G for the graph G is Figure 2.1. $p_1 = \{u = v_1, v_2, v_4, v_3, v_5, v_6, = v\}$ $p_2 = \{u = v_1, v_2, v_4, v_5, v_3, v_6, = v\}$ are the another $u - v$ paths of G.

Remark 2.5

Objective that for any two vertices u, v of G we have $D\,(u, v) > {\gamma_D}^D(u, v)$. The equality holds iff u, v are identical.

Remark 2.6

For any graph G the Dominating D-distance ${\gamma_D}^D(u, v)$ between two vertices u and v of G equal to maximum degree of vertices in that graph G and also it is equal to $D\,(u, v) - 1$ so that ${\gamma_D}^D(u, v) = \Delta\,(G) = v\,(G) - 2$ for the graph G in Figure 2.1 ${\gamma_D}^D(u, v) = 4$ and degree of the vertex $V_3 = 4$. Which is the maximum degree in that graph G.

Example 2.7

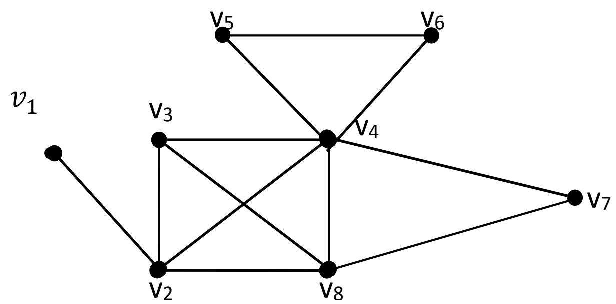

Figure 2.2

For the graph G in Figure 2.2. $p_1 = \{u = v_1, v_2, v_3, v_8, v_7, v_4, v_5, v_6 = v\}$

$p_1 = \{u = v_1, v_2, v_3, v_8, v_7, v_4, v_6, v_5 = v\}$ are the two longest path between u & v in G.

The Dominating D – length of the path P1 $l^D(P_1) = 1 + 2 + 4 + 6 - 7 = 6$ and in path P2 $l^D(P_2) = 1 + 2 + 4 + 6 - 7 = 6$. And for the graph G in Figure 2.2 $\Delta(G) = deg(v_4) = 6$ and $D(u,v) = 7$.

Therefore $\gamma_D{}^D(u,v) = \Delta(G) = D(u,v) - 1 = V(G) - 2 = 6$.

Theorem 2.8

If G is any connected graph then the $\gamma_D{}^D$ - distance is a metric on the set of vertices of G.

Proof :

Let G be a connected graph $u, v \in V(G)$. Then it is clear by definition; that $\gamma_D{}^D(u,v) \geq 0$ and $\gamma_D{}^D(u,v) = 0 \Leftrightarrow$ u = v. Also we have $\gamma_D{}^D(u,v) = \gamma_D{}^D(v,u)$ .This it remains to show that D^D- satisfies the triangle inequality.

Let $u, v, W \in V(G)$. Let P and Q be $u - w$ and $w - v$ are two detour paths in G respectively such that $\gamma_D{}^D(u,w) = l^D(p)$ and $\gamma_D{}^D(w,v) = l^D(q)$. Let $R = P \, U \, Q$ be the detour $u - v$ path obtained by joining P and Q at W. Then $\gamma_D{}^D(u,w) + \gamma_D{}^D(w,v)$

$= l^D(p) + l^D(q)$

$= l^D(p \cup q)$

$= l^D(R) = V(G) - 2 = D(u,v) - 1$

$= \gamma_D{}^D(u,v)$.

This the triangle inequality holds and hence D^D is a metric on the vertex set $V(G)$.

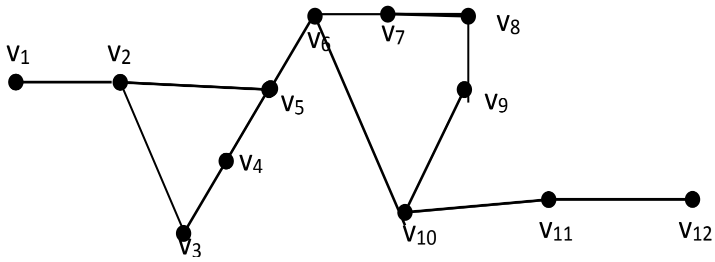

Figure 2.3

Remark 2.9 For the graph given in Figure 2.3 $P = v_1 - v_6$ and $Q = v_6 - v_{12}$ from that $R = P \cup Q$ is a detour path in G .Since $w = v_6$ is the cut vetex of G

Theorem 2.9

In a connected a graph G, two district vertices u, v are adjacent iff $\gamma_D{}^D(u, v) = \deg(u) + \deg(v) - 1$.

Proof :

If u and $v \in v(G)$ are adjacent $d(u) = d(v) = D(u, v) = 1$ and hence $\gamma_D{}^D(u, v) = \deg(u) + \deg(v) + \sum \deg(s) - D(u, v) = \gamma_D{}^D(u, v) = \deg(u) + \deg(v) - 1$ conversely, if $\gamma_D{}^D(u, v) = \deg(u) + \deg(v) - 1$ then, by definition of Dominating D – distance we get

$\gamma_D{}^D(u, v) = \deg(u) + \deg(v) + \sum \deg(s) - D(u, v) = 1 + 1 + 0 - 1 = 1$. Hence $\gamma_D{}^D(u, v) = 1$ and $\sum$ deg (S)=0.This implies u and v are adjacent.

Hence the theorem.

Corollary 2.10

In a connected graph G, the two dominating vertices p & q are adjacent and having same degree than $\gamma_D{}^D(u, v) = \deg(p) + \deg(q) - 2$.

Example 2.11

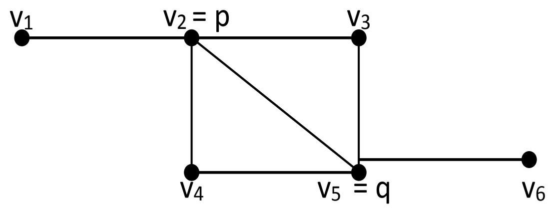

Figure 2.4

For a graph G in Figure 2.4 $\gamma_D{}^D(u,v) = d\,(v_1) + d\,(v_6) + deg\,(v_2) + deg\,(v_5) - D\,(u,v)$= 1 + 1 + 4 + 4 – 4 = 6. And also $\gamma_D{}^D(u,v) = \deg(v_2) = \deg(v_5) - 2$ =4+4-2=6.Therefore $\gamma_D{}^D(u,v) = \deg\,(\mathrm{p}) + \deg\,(\mathrm{q}) - 2$ where p & q are dominating vertices of G in Figure 2.3.

Remark 2.12

For a graph G given in Figure 2.5 $\gamma_D{}^D(u,v) = 7$ but $deg\,(P) + deg\,(q) - 2 = 8.$ Since the degree $p = v_2$& $q = v_4$ are different in Figure 2.4.

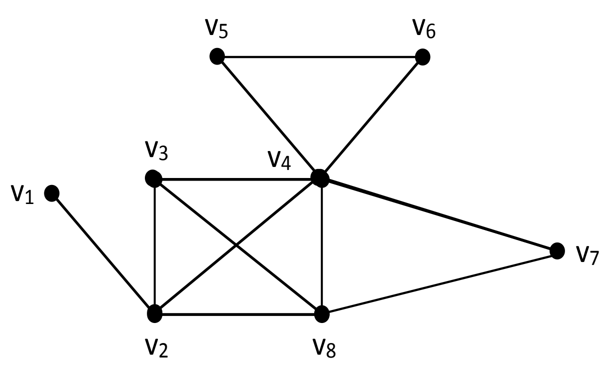

Figure 2.5

Some result on the Dominating D – Distance number of a graph.

Observation 2.13

For the star $G = K_{1,p-1}$, $\gamma_D{}^D(u,v) = p - 1(p \geq 3)$.

Volume-5, Issue-9, September - 2016 • ISSN No 2277 - 8160

Observation 2.14

For the connected graph $G = K_p\,(p \geq 2)$, ${\gamma_D}^D(k_p) = 2n - 2$.

"Real time Application of Domination D- Distance of a graph

Let v_1, v_2, v_3…….. be the vertices of the graph and the connected line between the vertices are edges of the graph. The number of edges incident to the vertices are called the degree of a vertex. Highest degree of vertex is can be identified by using Domination D- distance of a graph .The edges are considered as the different roads and this application of Domination D - Distance of a graph is implemented to identify a place for providing a most populated area and the vertex which has highest degree can be identified which helps in providing a traffic police booth to regulate the traffic. Generally traffic police booth is placed in the place where we have roads in all the direction .Heavily populated road can be identified when there are number of roads intersect each other. Traffic can to be regulated by using this theorem. This application can be utilized in different areas .One major example is briefed as we are marching towards the Smart city . The same can be applied by a business man to incorporate his business by identifying the people polling places. Where the shopping complex can be constructed using this method . Same way banking sector can use this method to install its ATM machine .The ultimate result of this method is to identify the heavily dominating point ."

References

1. F.Buckley and F. Harary, Distance in Graph, Addition – Wesley, reading M.A. 1990.
2. G. Chartrand, H. Escuadro and B. Zang. The Detour Distance in Graph, J. Combin-math. Combin computer., 53 (2005), 75-94.
3. G. Chartrand, N. Johns, and P. Zang, The Detour number of Graph, Utill, Math.64 (2003), 97-113.
4. Haynes TW, Hedetniemi S.T and slater P.J, Fundamentals of Domination in graphs, Marces Decker, Inc., New York 1998.
5. K.M. Kathiresan and G. Marimuthu, Superior distance in graphs, J. Combin, Com pvt., 61(2007) 73-80.
6. K.M. Kathiresan and R. Sumathi, A study on signal distance in graphs, Algebia graph theory and their applications, Norasa publishing house Pvt. Ltd., (2010) 50-54.
7. J.John and N. Arianayagam The detour domination number of a graph (submitted).
8. J.John and N. Arianayagam The Upper detour domination number of a graph (submitted).
9. J.John and N. Arianayagam The forcing detour domination number of a graph (submitted).