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ABELIAN PROPERTIES OF SOLVABLE GROUPS AND ITS IRREDUCIBLE CHARACTER

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ABSTRACT *- Our main aim to obtain whether there exists any groups where e4 – e3 < |G| < e4 + e3. Also, there is no concrete proof that the such group cannot exist. From assumptions of theorem, it is know that if such a group does exist, then all the normal subgroups of G must be nonabelian. we examine whether the bound |G| ≤ e4 – e3 can be proved when G is a simple group. Durfee and Jensen proved that if G has a nontrivial, abelian normal sub-group, then G has a normal subgroup N so that (G,N) is a p-Gagola pair for some prime p. Thus, if there exists a group G with e4 – e3 > |G|, then d > e2 – e and all the nontrivial, normal subgroups of G are nonabelian.*

KEYWORDS : Solvable group, Abelian, Nonabelian, Irreducible, Normal subgroup

INTRODUCTION

Let G be a finite nonabelian group d being the degree of some nonlinear irreducible character degree of G. we derive theorem when d is the maximal irreducible character degree of G.

It is known that d divides |G|, so there is an integer e such that |G| = d(d+e). Since d^2 < |G|, where e is a positive integer. Berkovich has shown that e = 1 if and only if G is a 2-transitive Frobenius group. It is known that there are 2-transitive groups of arbitrarily large order, and so, d may be arbitrarily large. when e > 1. But N. snyder has proved that |G| ≤ (2e)! under the restriction e > 1. He showed that if e = 2, then |G| ≤ 8 and if e = 3, then |G| ≤ 54, and in both of these cases, there exist examples of these orders; Thus the bounds given are best possible for e = 2 and 3. Isaacs has shown that |G| ≤ Be^6 for some universal constant B and in many cases that |G| ≤ $e^6 + e^4$. Durfee and Jensen have proved that |G| ≤ $e^6 – e^4$ without application of non abelian simple group. When G is solvable and either e is a prime or e is divisible by at least two distinct primes, they prove that |G| ≤ $e^4 – e^3$. Hence, the only possibility that G is solvable and |G| > $e^4 – e^3$ is when e is a prime power that is not prime. We thus find that when e = 2 and e = 3, the expression $e^4 – e^3$ yields the bound.

Isaacs has shown that there exists a solvable group G for every prime power q of order $q^3(q – 1)$ where d = q(q – 1). In case e = q, so d = e^2 – e and |G| = $e^4 – e^3$. On the other hand, there are no known groups G where |G| > $e^4 – e^3$, which implies that |G| ≤ $e^4 – e_3$ is the correct bound.

Gagola studied groups that have an irreducible character that vanish on all but two conjugacy classes. He proved that if G has a character x∈Irr(G) so that x vanishes on all but two conjugacy classes of G. x is called a Gagola character. Gagola proved that such Gagola characters are unique. Also, he proved that G has a unique minimal normal subgroup N. He proved that N is an elementary abelian p-group for some prime p. (G, N) is a p-Gagola pair if G has a Gagola character and N is the unique minimal normal subgroup of G and is a p-group.

Theorem 1

Let (G, N) be a p-Gagola pair some prime p. If P is a Sylow p-subgroup of G, then d ≤ e^2 – e and $|N|^2$ ≤ |P : N| = $|G : N|_p$.

Proof:

There are three cases: -

(i) when p is odd. (ii) when p = 2 and G is solvable, and for (iii) when p = 2 and G is nonsolvable. we prove the case i) when p = 2 and G is solvable. In particular, when p = 2 and G is solvable, for which it is desirable to derive properties of Suzuki 2-groups. The full automorphism group of a Suzuki 2-group is obtained. We prove it into there steps by taking its different values for p. Hence, theorem is proved.

Since |G| = d(d + e), to prove |G| ≤ e4 – e3 it is sufficient to prove that d ≤ e2 – e and to prove |G| ≤ e4 + e3, it is sufficient to prove that d ≤ e2. Let us state some basic concepts introduced by S. Jenson. If ψ, ∈ Irr(G), then dominates if ψ = (1)ψ. It is known that solvable groups have a nontrivial, abelian normal subgroup.

Theorem 2

Let G be a Frobenius complement that is a Z-group. If p is a prime that divides |G|, then G has a unique subgroup of order p.

Proof:

It is known that p divides only one of |G| or |G : G|. If p divides |G|, then since G is cyclic, then G has a unique subgroup of order p. Since p does not divide |G : G| and G is normal in G, which shows that the unique subgroup of G having order p.

Let us suppose that p divides |G : G|. We know that G/G is cyclic, so there is a unique subgroup X/G having order p. It is knonw that X contains all the subgroups of G having order p, so it suffices to prove that X has a unique subgroup of order p. Let P be a subgroup of X having order p. Let Q be any subgroup of G having prime order, say |Q| = q. Then PQ is a subgroup of order pq, and by either Satz an application of the results due to, we find that PQ is cyclic. Hence, Q centralizes P. It follows that every subgroup of G of prime order centralizes P, and thus, CG(P) contains every subgroup of G having prime order. Since G is abelian and has order coprime to p, Fitting's theorem is applied so that G = [G, P]×CG(P). Since CG(P) contains all the subgroups of prime order, we obtain G = CG(P).

Theorem 3

Let G be a solvable Frobenius complement. If p is a prime divisor of |G| and P is a subgroup of G of order p, then either:

(i) P is normal in G; or

(ii) p = 3, 9 does not divide |G|, and a Sylow 2-subgroup of G is quaternion of order 8.

Proof

If p = 2, then the result is true by either Satz. Also, if |G| is odd, then it is true. Therefore, we may assume that |G| is even and p is odd. By a theorem of Zassenhaus G has a normal subgroup N so that N is a Z-group and G/N is isomorphic to a subgroup of S4. Thus, if p > 3 or 9 divides |G|, then p divides |N|. Since subgroups of Frobenius complements are Frobenius complements, by suitable application of assertions of lemma (2.1.8), we find that P is characteristic in N, and so P is characteristic in G.

Thus, it may be assume that p = 3 and 9 does not divide |G|. Let H be a Hall 2-complement of G containing P. Thus, H is a Frobenius complement with odd order. Let Q be a Sylow 2-subgroup of G so that QP is a subgroup of G. Since P is a subgroup of H with prime order, P is normal in H. If P is central in PQ, then P is normal in G since G = HQ. Thus, P is not central in PQ, and so, the center of PQ is a 2-group. It is know that either P is normal in PQ and hence G, or the Fitting subgroup of PQ is isomorphic to the quaternions. This implies that Q is quaternion of order 8. Hence, theorem is proved.

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