IF : 4.547 | IC Value 80.26

VOLUME-6, ISSUE-7, JULY-2017 • ISSN No 2277 - 8160

Original Research Paper

Mathematics

$\alpha g\delta$-CLOSED SETS IN TOPOLOGICAL SPACE

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ABSTRACT

In this paper a new class of sets, namely $\alpha g\delta$-closed sets is introduced in topological spaces. We prove that this class lies between the class of δ-closed sets and the class of αg-closed sets. Also we find some basic properties of $\alpha g\delta$-closed sets.

KEYWORDS : generalized closed sets, δ-closure and $\delta\hat{g}$-closed set.

1 Introduction

Levine [5], Mashhour et al [8], Njastad [9] and Velicko [12] introduced semi-open sets, pre-open sets, α-open sets and δ-closed sets respectively. Levine [6] introduced generalized closed sets(briefly g-closed) sets and studied their properties. Bhattacharya and Lahiri [3], Arya and Nour [2], Maki et al [7] introduced semi-generalized closed (briefly sg-closed) sets, generalized semi-closed (briefly gs-closed) sets, generalized α-closed (briefly gα-closed) sets and α-generalized closed(briefly αg-closed) sets respectively. Veerakumar [13] introduced $\hat{g}$-closed sets in topological spaces. M. Lellis Thivagar, B. Meera Devi and E. Hatir [5] introduced $\delta\hat{g}$-closed sets in topological spaces. The purpose of this paper is to define a new class of closed sets called $\alpha g\delta$-closed sets and also we obtain some basic properties of $\alpha g\delta$-closed sets in topological spaces.

2 Preliminaries

Throughout this paper (X, τ)(or simply X) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. A subset A of a space (X,τ) is called a
(i) semi-open set [5] if A $\subseteq$ cl(int(A)).
(ii) pre-open set [8] if A $\subseteq$ int(cl(A)).
(iii) α-open set [9] if A $\subseteq$ int(cl(int(A))).
(iv) regular open set [10] if A = int(cl(A)).

The complement of a semi-open (resp. pre-open, α-open and regular open) set is called semi-closed (reps. pre-closed, α-closed and regular closed).

Definition 2.2. The δ-interior [12] of a subset A of X is the union of all regular open set of X contained in A and is denoted by $Int_\delta(A)$. The subset A is called δ-open [12] if A= $Int_\delta(A)$. The complement of a δ-open is called δ-closed. Alternatively, a set A$\subseteq$ (X,τ) is called δ-closed [12] if A = $cl_\delta(A)$, where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq Ø, U \in \tau$ and $x \in U\}$.

Definition 2.4. A subset A of (X,τ) is called
(i) generalized closed(briefly g-closed) set [6] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open set in ((X,τ).
(ii) semi-generalized closed (briefly sg-closed) set [3] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open set in (X,τ).
(iii) generalized semi-closed (briefly gs-closed) set [2] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open set in (X,τ).
(iv) α-generalized closed (briefly αg-closed) set [7] if αcl(A $\subseteq$ U whenever A $\subseteq$ U and U is open set in (X,τ).
(v) generalized α-closed (briefly gα-closed) set [7] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open set in (X,τ).
(vi) δ-generalized closed (briefly δg-closed) set [4] if clδ(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).
(vii) $\hat{g}$-closed set [11] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is a semi-open set in (X,τ).
(viii) α-$\hat{g}$-closed (briefly $\alpha\hat{g}$-closed) set [1] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\hat{g}$-open set in (X,τ).
(ix) $\delta\hat{g}$-closed [4] if clδ(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\hat{g}$-open set in (X,τ).

The complement of a g-closed (resp. sg-closed, gs-closed, αg-closed, gα-closed, δg-closed, $\hat{g}$-closed, $\alpha\hat{g}$-closed and $\delta\hat{g}$-closed) set is called g-open (reps. sg-open, gs-open, αg-open, gα-open, δg-open, $\hat{g}$-open, $\alpha\hat{g}$-open and $\delta\hat{g}$-open).

Theorem 2.4. Every open set is α-open.

3 $\alpha g\delta$-Closed sets

We introduce the following definition.

Definition 3.1. A subset A of a space (X,τ) is called $\alpha g\delta$-closed if $cl_\delta(A) \subseteq$ U whenever A $\subseteq$ U and U is α-open set in (X,τ).

Proposition 3.2. Every δ-closed set is $\alpha g\delta$-closed set.

Proof. Let A be an δ-closed and U be any α-open containing A in (X,τ). Since A is δ-closed, $cl_\delta(A)$ = A for every subset A of X. Therefore $cl_\delta(A) \subseteq$ U and hence A is $\alpha g\delta$-closed set.

Remark 3.3. The converse of the above theorem is not true in general as shown in the following example.

Example 3.4. Let X = {a,b,c} with topology τ = {Ø, {b}, X}. Then the set {a,c} is $\alpha g\delta$-closed but not δ-closed in (X,τ).

Proposition 3.5. Every $\alpha g\delta$-closed set is g-closed.

Proof. Let A be an $\alpha g\delta$-closed and U be any open set containing A in (X,τ). Since every open set is α-open and A is $\alpha g\delta$-closed, $cl_\delta(A) \subseteq$ U for every subset A of X. Since cl(A) $\subseteq cl_\delta(A) \subseteq$ U, cl(A) $\subseteq$ U and hence A is g-closed.

Remark 3.6. An g-closed set need not be $\alpha g\delta$-closed set as shown in the following example.

Example 3.7. Let X = {a,b,c} with topology τ = {Ø, {a}, X}. Then the set {b} is g-closed but not $\alpha g\delta$-closed.

Proposition 3.8. Every $\alpha g\delta$-closed set is sg-closed.

Proof. Let A be an $\alpha g\delta$-closed and U be any open set containing A in (X,τ). Since every open set is α-open, $cl_\delta(A) \subseteq$ U for every subset A of X. Since every open set is semi-open and scl(A) $\subseteq cl_\delta(A) \subseteq$ U, scl(A) $\subseteq$ U and hence A is sg-closed.

Remark 3.9. A sg-closed set need not be $\alpha g\delta$-closed as shown in the following example.

Example 3.10. Let X = {a,b,c} with topology τ = {Ø, {a}, {c}, {a,c}, X}. Then the set {c} is sg-closed but not $\alpha g\delta$-closed.

Proposition 3.11. Every $\alpha g\delta$-closed set is gs-closed.

Proof. It is true that scl(A) $\subseteq cl_\delta(A)$ for every subset A of X.

Remark 3.12. A gs-closed set need not be αgδ-closed as shown in the following example.

Example 3.13. Let X = {a,b,c} with topology τ = {Ø, {b}, {a,b}, {b,c}, X}.

Then the set {a} is gs-closed but not αgδ-closed.

Proposition 3.14. Every αgδ-closed set is αg-closed.

Proof. It is true that αcl(A) ⊆ $cl_\delta(A)$ for every subset A of X.

Remark 3.15. A αg-closed set need not be αgδ-closed as shown in the following example.

Example 3.16. Let X = {a,b,c} with topology τ = {Ø, {c}, {a,c}, X}. Then the set {b,c} is αg-closed but not αgδ-closed.

Proposition 3.17. Every αgδ-closed set is gα-closed.

Proof. Let A be an αgδ-closed set and U be any open set containing A in (X,τ). Since every open set is α-open, $cl_\delta(A)$ ⊆ U for every subset A of X. Since αcl(A) ⊆ $cl_\delta(A)$ ⊆ U, αcl(A) ⊆ U and hence A is gα-closed.

Remark 3.18. A gα-closed set need not be αgδ-closed as shown in the following example.

Example 3.19. Let X = {a,b,c} with topology τ = {Ø, {a}, {a,b}, {a,c}, X}.Then the set {c} is gα-closed but not αgδ-closed.

Proposition 3.20. Every αgδ-closed is δg-closed.

Proof. Let A be an αgδ-closed set and U be any open set containing A in (X,τ). Since every open set is α-open, $cl_\delta(A)$ ⊆ U whenever A ⊆ U and U is α–open. Therefore $cl_\delta(A)$ ⊆ U and U is open. Hence A is δg-closed.

Remark 3.21. A δg-closed set need not be αgδ-closed as shown in the following example.

Example 3.22. Let X = {a,b,c} with topology τ = {Ø, {c}, X}. Then the set {a,c} is δg-closed but not αgδ-closed.

Remark 3.23. The class of αgδ-closed sets is properly placed between the classes of δ-closed and αg-closed sets.

Proposition 3.24. Every αgδ-closed set is ĝ-closed.

Proof. Let A be an αgδ-closed and U be any open set containing A. Since A is αgδ-closed, $cl_\delta(A)$ ⊆ U for every subset A of X. Since every open set is semi-open and cl(A) ⊆ $cl_\delta(A)$ ⊆ U, cl(A) ⊆ U and hence A is ĝ-closed.

Remark 3.25. A ĝ-closed set need not be αgδ-closed as shown in the following example.

Example 3.26. Let X= {a,b,c} with topology τ = {Ø, {a}, {a,b}, X}. Then the set {c} is ĝ-closed but not αgδ-closed.

Remark 3.27. The following example shows that αgδ-closeness is independent from αĝ-closeness, δĝ-closeness, α-closeness and semi-closeness.

Example 3.28. Let X = {a,b,c} with topology τ = {Ø, {a}, {b,c}, X}. Then the set {a,c} is αgδ-closed but neither αĝ-closed nor δĝ-closed and {b} is αgδ-closed but neither α-closed nor semi-closed.

Example 3.29. Let X = {a,b,c} with topology τ = {Ø, {b}, X}. Then the set {b,c} is αĝ-closed and δĝ-closed but not αgδ-closed.

Also the another example, Let X = {a,b,c} with topology τ = {Ø, {a}, {a,b}, X}. Then the set {b} is α-closed and semi-closed but not αgδ-closed.

Remark 3.30. The following diagram shows that the relationships of αgδ-closed sets with other known existing sets. A → B represents A implies B but not conversely

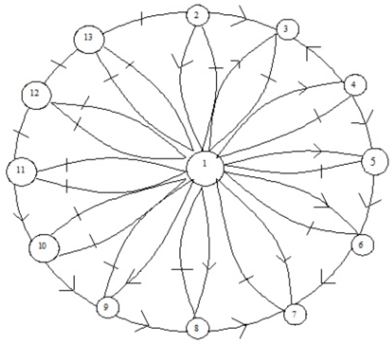

Fig. 1

1. αgδ-closed 2. δ-closed 3. δg-closed 4. ĝ-closed 5. g-closed 6. αg-closed 7. gs-closed 8. sg-closed 9. gα-closed 10. αĝ-closed 11.α-closed 12. δĝ-closed 13. closed

4 Properties

Theorem 4.1. The finite union of αgδ-closed sets is αgδ-closed.

Proof. Let {A_i / i = 1, 2,…,n} be a finite class of αgδ-closed subsets of a space (X,τ). Then for each α-open set U_i in X containing A_i, $cl_\delta(A_i)$ ⊆ U_i, i ϵ { 1,2,…,n}. Hence $\cup_i A_i$ ⊆ $\cup_i U_i$ = V. Since arbitrary union of α-open set in (X,τ) is also α-open set in (X,τ), V is open in (X,τ). Also $\cup_i cl_\delta(A_i) = cl_\delta(\cup_i A_i)$ ⊆ V. Therefore $\cup_i A_i$ is αgδ-closed in (X,τ).

Remark 4.2. Intersection of any two αgδ-closed sets in (X,τ) need not be αgδ-closed as shown in the following example.

Example 4.3. Let X= {a,b,c,d} with topology τ = {Ø, {a}, {b,c}, X}. Then the sets {a,b,d} and {b,c,d} are αgδ-closed sets but their intersection {b,d} is not αgδ-closed.

Proposition 4.4. Let A be an αgδ-closed set of (X,τ). Then $cl_\delta(A)$−A does not contain a nonempty α-closed set.

Proof. Suppose that A is αgδ-closed, let F be α-closed set contained in $cl_\delta(A)$−A. Now F^c is α-open set of (X,τ) such that A ⊆ F^c. Since A is αgδ-closed set of (X,τ), $cl_\delta(A)$ ⊆ F^c. Thus F ⊆ $(cl_\delta(A))^c$. Also F ⊆ $(cl_\delta(A))^c$∩$(cl_\delta(A))$ = Ø. Hence F = Ø.

Proposition 4.5. If A is α-open and αgδ-closed subset of (X,τ), then A is an δ-closed subset of (X,τ).

Proof. Since A is α-open and αgδ-closed, $cl_\delta(A)$ ⊆ A. Hence A is δ-closed.

Theorem 4.6. The intersection of a αgδ-closed set and a δ-closed set is always αgδ-closed.

Proof. Let A be αgδ-closed and let F be δ-closed. If U is an α-open set containing A∩F ⊆ U, then A ⊆ U ∪F^c and so $cl_\delta(A)$ ⊆ U ∪F^c. Now $cl_\delta(A∩F)$ ⊆ $cl_\delta(A)$∩F ⊆ U. Hence A ∩ F is αgδ-closed.

Proposition 4.7. If A is an αgδ-closed set in a space (X,τ) and A ⊆ B ⊆ $cl_\delta(A)$, then B is also a αgδ-closed.

Proof. Let U be an α-open set of (X,τ) such that B ⊆ U. Then A ⊆ U. Since A is αgδ-closed set, $cl_\delta(A)$ ⊆ U. Also since B ⊆ $cl_\delta(A)$, $cl_\delta(B)$ ⊆ $cl_\delta(cl_\delta(A))$ = $cl_\delta(A)$. Hence $cl_\delta(B)$ ⊆ U. Therefore B is also a αgδ-closed set.

Proposition 4.8. Let A be αgδ-closed subset of (X,τ). Then A is δ-closed iff $cl_\delta(A)$−A is α-closed.

Proof. Necessity: Let A be a δ-closed subset of X. Then cl(A)−A = Ø which is α-closed.

Sufficiency: Since A is αgδ-closed, by proposition 4.4, $cl_\delta(A)$−A does not contained a nonempty α-closed set. But $cl_\delta(A)$−A = Ø. That is $cl_\delta(A)$ = A. Hence A is δ-closed.

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