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On Intuitionistic Fuzzy σ - Baire Spaces

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ABSTRACT In this paper the concepts of intuitionistic fuzzy σ-Baire spaces are introduced and characterizations of intuitionistic fuzzy σ-Baire spaces are studied and several examples are given.

KEYWORDS : Intuitionistic fuzzy F_σ set, Intuitionistic fuzzy G_δ set, Intuitionistic fuzzy nowhere dense set, Intuitionistic fuzzy σ-nowhere dense set, Intuitionistic fuzzy σ-first category, Intuitionistic fuzzy σ-second category, Intuitionistic fuzzy σ-residual set and Intuitionistic fuzzy σ-Baire space.

1. Introduction

The fuzzy concept has invaded almost all branches of mathematics ever since the introduction of fuzzy sets by L.A.Zadeh [12]. The theory of fuzzy topological spaces was introduced and developed by C.L.Chang [6] and since then various notions in classical topology have been extended to fuzzy topological space. The idea of "intuitionistic fuzzy set" was first published by Atanassov [1] and many works by the same author and his colleagues appeared in the literature [2,3,4]. Later, this concept was generalized to "intuitionistic L-fuzzy sets" by Atanassov and Stoeva [5]. The concept of σ −nowhere dense set was introduced and studied by Jiling Cao and Sina Greenwood [9] in 2000. The concept of fuzzy σ −Baire space is introduces and studied by G.Thangaraj and E.Poongothai[11]. In this paper, we introduce the concept of Intuitionistic fuzzy σ − nowhere dense set and Intuitionistic fuzzy σ −Baire spaces. We discuss several characterizations of those spaces and examples are given to illustrate the concepts introduced in this paper.

2. Preliminaries

Definition 2.1 [3] Let X be a non-empty set. An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \vartheta_A(x)\rangle : x \in X\}$, where $\mu_A(x): X \to [0,1]$ and $\vartheta_A(x): X \to [0,1]$ denote the membership and non-membership functions of A respectively, and $0 \le \mu_A(x) + \vartheta_A(x) \le 1$, for each $x \in X$.

Definition 2.2 [3] Let A and B be two IFSs of the non-empty set X such that

$$A = \{\langle x, \mu_A(x), \vartheta_A(x)\rangle : x \in X\},$$

$$B = \{\langle x, \mu_B(x), \vartheta_B(x)\rangle : x \in X\}.$$

We define the following basic operations on A and B.

(i) $A \subseteq B\ iff\ \mu_A(x) \le \mu_B(x)\ and\ \vartheta_A(x) \ge \vartheta_B(x), \forall x \in X$

(ii) $A \supseteq B\ iff\ \mu_A(x) \ge \mu_B(x)\ and\ \vartheta_A(x) \le \vartheta_B(x), \forall x \in X$

(iii) $A = B\ iff\ \mu_A(x) = \mu_B(x)\ and\ \vartheta_A(x) = \vartheta_B(x), \forall x \in X$

(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \vartheta_A(x) \wedge \vartheta_B(x)\rangle : x \in X\}$

(v) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \vartheta_A(x) \vee \vartheta_B(x)\rangle : x \in X\}$

(vi) $A^c = \{\langle x, \vartheta_A(x), \mu_A(x)\rangle : x \in X\}$.

Definition 2.3 [7] An Intuitionistic fuzzy topology (IFT) on X is a family T of IFSs in X satisfying the following axioms.

(i) $0,1 \in T$

(ii) $G_1 \cap G_2 \in T, for\ any\ G_1, G_2 \in T$

(iii) $\cup G_i \in T\ for\ any\ family\ \{G_i / i \in J\} \subseteq T$

In this case, the pair (X, T) is called an Intuitionistic fuzzy topological space (IFTS) and any IFS in T is known as Intuitionistic fuzzy open set (IFOS) in X.

The complement A^c of an IFOS A in an IFTS (X, T) is called an Intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4 [7] Let (X,T) be an IFTS and $A = \langle X, \mu_A, \vartheta_A \rangle$ be an IFS in X. Then the Intuitionistic fuzzy interior and an Intuitionistic fuzzy closure are defined by

$$int\,(A) = \cup \{G/G \text{ is an } IFOS \text{ in } X and\ G \subseteq A\},$$

$$cl\,(A) \ = \cap \{K/K \text{ is an } IFCS \text{ in } X and\ A \subseteq K\}.$$

Proposition 2.5 [7] Let (X,T) be any Intuitionistic fuzzy topological space. Let A be an IFS in (X,T). Then

(i) $1 - IFcl(A) \ = IF\ int\,(1 - A)$
(ii) $1 - IFint(A) = IF\ cl(1 - A).$

Definition 2.6 [8] An Intuitionistic Fuzzy Set A in an Intuitionistic fuzzy topological space (X,T) is called Intuitionistic fuzzy dense if there exists no Intuitionistic fuzzy closed set B in (X,T) such that $A \subset B \subset 1$.

Definition 2.7 [10] An Intuitionistic fuzzy set A in an Intuitionistic fuzzy topological spaces (X,T) is called an Intuitionistic fuzzy F_σ set in (X,T) if $A = \ \cup_{i=1}^{\infty} A_i$ where$1 - A_i \in T, \forall i.$

Definition2.8 [10] An Intuitionistic fuzzy set A in an Intuitionistic fuzzy topological space (X,T) is called an Intuitionistic fuzzy G_δ set in (X,T) if $A = \cap_{i=1}^{\infty} A_i$ where $A_i \in T, \forall i.$

3. Intuitionistic Fuzzy σ –nowhere dense sets

Definition 3.1 Let (X,T) be an Intuitionistic fuzzy topological space.An intuitionistic fuzzy set A in (X,T) is called an Intuitionistic fuzzy σ –nowhere dense set if A is an Intuitionistic fuzzy F_σ set such that $IFint(A) = 0$.

Example 3.2 Let $X = \{a, b, c\}$. Define the Intuitionistic fuzzy sets A,B,C and D as follows:

$$A = \langle x, \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.5}\right) \rangle$$

$$B = \langle x, \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}\right) \rangle$$

$$C = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.4}\right) \rangle \text{ and}$$

$$D = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7}\right) \rangle$$

Then $T = \{0, 1, A, B, C, D\}$ is an Intuitionistic fuzzy topologies on X. Thus (X, T) is an

Intuitionistic fuzzy topological spaces. Now consider the fuzzy set $G = \bar{A} \cup \bar{B} \cup C \cup D$ in (X,T) . Then G is an Intuitionistic fuzzy F_σ set in (X,T) and $int(G) = 0$ and hence G is an Intuitionistic fuzzy σ –nowhere dense set in (X,T).

Proposition 3.3 If A is an Intuitionistic fuzzy dense set in (X,T) such that $IFB \ \leq IF(1 - A)$, where B is an Intuitionistic fuzzy F_σ set in (X,T), then B is an Intuitionistic fuzzy σ –nowhere dense set in (X,T).

Proof Let A be an Intuitionistic fuzzy dense set in (X,T) such that $IFB \ \leq IF(1 - A)$ implies that $IFint\,(B) \leq IFint(1 - A) = 1 - IFcl(A) = \ 1 - 1 = 0$ and hence

$IFint(B) = 0.$ Therefore B is an Intuitionistic fuzzy σ –nowhere dense set in (X,T).

Proposition 3.4 If A is an Intuitionistic fuzzy F_σ set and Intuitionistic fuzzy nowhere dense set in (X,T), then A is an Intuitionistic fuzzy σ –nowhere dense set in (X,T).

Proof Now $IF\ A \ \leq IFcl(A)$ for any fuzzy set in (X,T). Then, $IFint(A) \leq IFintcl(A)$. Since A is an Intuitionistic fuzzy nowhere dense set in (X,T), $IFintcl(A) = 0$ and hence $IFint(A) = 0$ and A is an Intuitionistic fuzzy F_σ set implies that A is an Intuitionistic fuzzy σ –nowhere dense set in (X,T).

Definition 3.5[8] Let (X,T) be an IFTS. Then (X,T) is called an Intuitionistic fuzzy open hereditarily irresolvable space if, $IFint(IFcl(A)) \neq 0$, then $IFint(A) \neq 0$ for any Intuitionistic fuzzy set A in (X,T).

Proposition 3.6 If (X,T) is an Intuitionistic fuzzy open hereditarily irresolvable space, any Intuitionistic fuzzy σ −nowhere dense set in (X,T) is an Intuitionistic fuzzy nowhere dense set in (X,T).

Proof. Let A be an Intuitionistic fuzzy σ −nowhere dense set in an Intuitionistic fuzzy open hereditarily irresolvable space (X,T). Then A is an Intuitionistic fuzzy F_σ set in (X,T) such that $IFint(A) = 0$. Since (X,T) is an Intuitionistic fuzzy open hereditarily irresolvable space, $IFint(A) = 0$ implies that $IFint(IFcl(A)) = 0$. Hence A is an Intuitionistic fuzzy nowhere dense set in (X,T).

Definition 3.7 An IFS A in an IFTS (X,T) is called Intuitionistic fuzzy $\sigma -$first category if $A = \cup_{i=1}^{\infty} A_i$, where $A_i's$ are Intuitionistic fuzzy σ −nowhere dense set in (X,T). Any other Intuitionistic fuzzy set in (X,T) said to be of σ −second category.

Theorem 3.8[8] Let (X,T) be an Intuitionistic fuzzy topological space. If (X,T) is called an Intuitionistic fuzzy open hereditarily irresolvable space, then $IF\ int(A) = 0$ for any non-zero Intuitionistic fuzzy set A in (X,T)implies $IFint\big(IFcl(A)\big) = 0$.

Definition 3.9 Let A be an Intuitionistic fuzzy $\sigma -$first category set in (X,T). Then $1 - A$ is called an Intuitionistic fuzzy σ −residual set in (X,T).

Definition 3.10 An IFTS is called an Intuitionistic fuzzy σ −first category space if $1 = \cup_{i=1}^{\infty} A_i$, where $A_i's$ are Intuitionistic fuzzy σ −nowhere dense set in (X,T). (X,T) is called Intuitionistic fuzzy σ −second category space if it is not an Intuitionistic fuzzy σ −first category space.

4. Intuitionistic fuzzy σ −Baire Space

Definition 4.1 Let (X,T) be an Intuitionistic fuzzy topological space. Then (X,T) is called an Intuitionisitc fuzzy σ −Baire Space if $IF\ int\ (\cup_{i=1}^{\infty} A_i) = 0$ where $A_i's$ are Intuitionistic fuzzy σ −nowhere dense sets in (X,T).

Example 4.2 Let $X = \{a,b,c\}$. Define the Intuitionistic fuzzy sets A,B,C and D as follows:

$$A = \langle x, \left(\frac{a}{0.8}, \frac{b}{0.6}, \frac{c}{0.5}\right), \left(\frac{a}{0.4}, \frac{b}{0.2}, \frac{c}{0.3}\right)\rangle$$

$$B = \langle x, \left(\frac{a}{0.6}, \frac{b}{0.9}, \frac{c}{0.6}\right), \left(\frac{a}{0.3}, \frac{b}{0.1}, \frac{c}{0.2}\right)\rangle$$

$$C = \langle x, \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.7}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right)\rangle \text{ and}$$

$$D = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3}\right), \left(\frac{a}{0.8}, \frac{b}{0.7}, \frac{c}{0.8}\right)\rangle$$

Then $T = \{0,1,A,B,C,D\}$ is an Intuitionistic fuzzy topologies on X. Thus (X,T) is an

Intuitionistic fuzzy topological spaces. Now consider the fuzzy set

$G = \{A \cap B \cap (A \cup B)\}$, $H = \{A \cap B \cap (A \cap B)\}, I = \{A \cap B \cap (A \cap B)A(A \cup B)\}$;

Where, G,H and I are Intuitionistic fuzzy σ −nowhere dense sets in (X,T) . Also $IF\ int(G \cup H \cup I) = 0$. Hence (X,T) is an Intuitionistic fuzzy σ −Baire Space.

Proposition 4.3 Let (X,T) be an Intuitionistic fuzzy topological space. Then the following are equivalent.

(i) (X,T) is an Intuitionistic fuzzy σ −Baire Space.
(ii) $IFint(A) = 0$, for every Intuitionistic fuzzy σ −first category set A in (X,T).
(iii) $IFcl(B) = 1$, for every Intuitionistic fuzzy σ −residual set B in (X,T).

Proof$(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$

Let A be an Intuitionistic fuzzy σ −first category set in $(X,T), then\ A = \cup_{i=1}^{\infty} A_i$, where $A_i's$ are Intuitionistic fuzzy σ −nowhere dense sets in (X,T). Now $IFint(A) = IFint(\cup_{i=1}^{\infty} A_i) = 0$. Since (X,T) is an Intuitionistic fuzzy σ −Baire Space. Therefore $IFint(A) = 0$ for any Intuitionistic fuzzy σ −first category set A in (X,T).

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{iii})$

Let B be an Intuitionistic fuzzy σ −residual set in (X,T). Then $(1-B)$ is an Intuitionistic fuzzy σ −first category set A in (X,T).By hypothesis, $IFint(1-B)=0$ which implies that $1-IFcl(B)=0$. Hence $IFcl(B)=1$ for any Intuitionistic fuzzy σ −residual set A in (X,T).

$(\boldsymbol{iii}) \Rightarrow (\boldsymbol{i})$

Let A be an Intuitionistic fuzzy σ −first category set in (X,T), $then\ A = \cup_{i=1}^{\infty} A_i$, where $A_i's$ are Intuitionistic fuzzy σ −nowhere dense sets in (X,T). Now A is an Intuitionistic fuzzy σ −first category set implies that $1-A$ is an Intuitionistic fuzzy σ −residual set in (X,T). By hypothesis, we have $IFCl(1-A)=1$, which implies that $1-IFint(A)=1$. Hence $IFint(A)=0$. That is, $IFint\ (\cup_{i=1}^{\infty} A_i)=0$, where $A_i's$ are Intuitionistic fuzzy σ −nowhere dense sets in (X,T). Hence (X,T) is an Intuitionistic fuzzy σ −Baire Space.

Theorem 4.4 [10] In an IFTS (X,T),an IFS A is an Intuitionistic fuzzy σ −nowhere dense sets in (X,T), in and only if $1-A$ is an Intuitionistic fuzzy dense and Intuitionistic fuzzy G_δ- set in (X,T).

Proposition4.5 If $IFcl\left(\cap_{i=1}^{\infty}(A_i)\right)=1$, where $A_i's$ are Intuitionistic fuzzy dense and Intuitionistic fuzzy G_δ- sets in (X,T), then (X,T) is an Intuitionistic fuzzy σ −Baire Space.

Proof. Now $IFcl\left(\cap_{i=1}^{\infty}(A_i)\right)=1$, implies that $1-IFcl\left(\cap_{i=1}^{\infty}(A_i)\right)=0$. Then we have $IFint\left(1-\cap_{i=1}^{\infty}(A_i)\right)=0$, which implies that $IFint\left(\cup_{i=1}^{\infty}(1-A_i)\right)=0$. Let $B_i=1-A_i$. Then $IFint\left(\cup_{i=1}^{\infty}(B_i)\right)=0$. Since $A_i's$ are Intuitionistic fuzzy dense and Intuitionistic fuzzy G_δ- sets in (X,T), by theorem 4.4, $1-A_i$ is an Intuitionistic fuzzy σ −nowhere dense set in (X,T). Hence $IFint\left(\cup_{i=1}^{\infty}(B_i)\right)=0$, where $B_i's$ are Intuitionistic fuzzy σ −nowhere dense sest in (X,T). Therefore (X,T) is an Intuitionistic fuzzy σ −Baire Space.

Proposition 4.6 If the Intuitionistic fuzzy topological space (X,T) is an Intuitionistic fuzzy σ −Baire Space, then (X,T) is an Intuitionistic fuzzy σ −second category space.

Proof. Let (X,T) be an Intuitionistic fuzzy σ − Baire Space. Then $IFint\left(\cup_{i=1}^{\infty}(A_i)\right)=0$, where $A_i's$ are Intuitionistic fuzzy σ − nowhere dense sets in (X,T). Then $IF\left(\cup_{i=1}^{\infty}(A_i)\right)\neq 1_x$. [Otherwise, $IF\left(\cup_{i=1}^{\infty}(A_i)\right)=1_x$ implies that $IFint\left(\cup_{i=1}^{\infty}(A_i)\right)=IF\ int 1_x=1_x$ which implies that 0=1, a contradiction]. Hence (X,T) is an Intuitionistic fuzzy σ −second category space.

Proposition 4.7 If the Intuitionistic fuzzy topological space (X,T) is an Intuitionistic fuzzy σ −Baire space and Intuitionistic fuzzy open hereditarily irresolvable space, then (X,T) is an Intuitionistic fuzzy Baire Space.

Proof. Let (X,T) be an Intuitionistic fuzzy σ −Baire Space and IF open hereditarily irresolvable space. Then, $IFint\left(\cup_{i=1}^{\infty}(A_i)\right)=0$, where $A_i's$ are Intuitionistic fuzzy σ −nowhere dense sets in (X,T). By prop.3.6, $A_i's$ are Intuitionistic fuzzy nowhere dense sets in (X,T). Hence, $IFint\left(\cup_{i=1}^{\infty}(A_i)\right)=0$, where $A_i's$ are Intuitionistic fuzzy nowhere dense sets in (X,T). Therefore (X,T) is an Intuitionistic fuzzy Baire Space.

Proposition 4.8. If the IFTS (X,T) is an IFBaire space and if the Intuitionistic fuzzy nowhere dense sets in (X,T) are Intuitionistic fuzzy F_σ sets in (X,T), then (X,T) is an Intuitionistic fuzzy σ −Baire Space.

Proof. Let (X,T) be an Intuitionistic fuzzy Baire space such that every Intuitionistic fuzzy nowhere dense set A_i is an Intuitionistic fuzzy F_σ set in (X,T). Then $IFint\left(\cup_{i=1}^{\infty}(A_i)\right)=0$, where $A_i's$ are Intuitionistic fuzzy nowhere dense sets in (X,T). By prop. 3.4, A_i is an Intuitionistic fuzzy σ −nowhere dense set in (X,T). Hence $IFint\left(\cup_{i=1}^{\infty}(A_i)\right)=0$, where $A_i's$ are Intuitionistic fuzzy σ −nowhere dense sets in (X,T). Therefore (X,T) is an Intuitionistic fuzzy σ −Baire Space.

Proposition 4.9. Let (X,T) be an Intuitionistic fuzzy topological space. If $IF\left(\cap_{i=1}^{\infty}(A_i)\right)\neq 0$, where $A_i's$ are Intuitionistic fuzzy dense and Intuitionistic fuzzy G_δ- sets in (X,T), then (X,T) is an Intuitionistic fuzzy σ −second category space.

Proof. Now $IF\left(\cap_{i=1}^{\infty}(A_i)\right)\neq 0$ implies that $IF1-\left(\cap_{i=1}^{\infty}(A_i)\right)\neq 1-0=0$. Then we have $IF\left(\cup_{i=1}^{\infty}(1-A_i)\right)\neq 1$. Since A_i is an Intuitionistic fuzzy dense and Intuitionistic fuzzy G_δ- set in (X,T), by prop 4.3, $IF(1-A_i)$ is an Intuitionistic fuzzy σ −nowhere dense set in (X,T). Hence $IF\left(\cup_{i=1}^{\infty}(1-A_i)\right)\neq 1$, where $IF(1-A_i)'s$ are Intuitionistic fuzzy σ −nowhere dense sets in (X,T). Hence (X,T) is not an Intuitionistic fuzzy $\sigma-first$ category space. Therefore (X,T) is an Intuitionistic fuzzy σ −second category space.

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