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Mathematics

# Analysis of Performance Measure of Cost Function-Fuzzy Batch Arrival Queuing Model

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**ABSTRACT** The main aim of this Paper is analyzing Various Performance Measures interms of Crisp Values for the total expected cost of fuzzy batch arrival queuing model where the arrival rate, Service rate and batch size, Service cost and holding costs are all fuzzy numbers and also they Triangular Fuzzy numbers, Trapezoidal Fuzzy numbers. Here we convert the fuzzy inter-arrival rate, Service rate, batch size service cost are all in to Crisp Values for Using Ranking Function Method.

**KEYWORDS :** Fuzzy Sets, batch arrival queue, Fuzzy Ranking membership Function.

## 1. INTRODUCTION:

Queuing Method are effective method for Performance analysis computer and Telecommunication Systems, Manufacturing Production systems

A Queue is formed at any time when the demand for a Service exceeds the capacity to provide that service. Queuing models are effective method for Performance analysis of computer and telecommunication systems, manufacturing / Production Systems and Inventory control are discussed by many Researchers. In this paper a study of FM [K]/1 type models many type of batch arrival Queuing models has been investigated so far then [5,6] developed FM/FM/1 and FM/FM[k]/1 Fuzzy Systems. Let C.H, Hung H.L, Ke.J.C [7.8] developed parametric Programming Approach for Batch arrival Queues with vacction policies and Fuzzy parameters and also discussed. On a batch Arrival Queue with setup and uncertain Parameters patterns. Recently Rue and Roshine [10] dealt with the control policy on Batch arrival and Lilly Robert [11] developed the Profit analysis of Fuzzy M/Ek/1 queuing system by using non-Linear programming techniques. Then is this Paper we develop a method which is able to provide TEC for bulk arrival queues with Fuzzified exponential arrival rate, Service time, batch size, Service cost and holding cost Abbasbandy and Hajjari[24] introduced a new approach for ranking of trapezoidal Fuzzy numbers based on Left and right spread at some $\propto$ -levelsof trapezoidal Fuzzy numbers

## 2. Definition: Trapezoidal Fuzzy number:

A Triangular Fuzzy number $\tilde{X}(\mathrm{x})$ can be represented by Three Parameter by $\bar{X}(x_{1,}x_{2,}x_{3};1)$ with membership function $\mu_{\tilde{x}}(x)$ is

$$\text{given by}\mu_{\tilde{x}}(x) = \begin{cases} \frac{x-x_1}{x_2-x_1} & , 0 \le x \le x_1 \\ 1 & , x = x_2 \\ \frac{x-x_3}{x_2-x_3} & , x_2 \le x \le x_3 \\ 0 & , \quad Otherwise \end{cases}$$

## 3. Definition: Trapezoidal Fuzzy number:

A Triangular Fuzzy number $\tilde{X}(\mathrm{x})$ can be represented by Four Parameter $(x_{1,}x_{2,}x_{3,}x_4)$ with membership function $\mu_{\tilde{x}}(x)$ is given

$$\text{by}\mu_{\tilde{x}}(x) = \begin{cases} \frac{x-x_1}{x_2-x_1} & , x_1 \le x \le x_3 \\ 1 & , x_2 \le x \le x_3 \\ \frac{x-x_4}{x_3-x_4} & , x_3 \le x \le x_4 \\ 0 & , \quad Otherwise \end{cases}$$

## 4. Model Description Cost Measures of Fuzzy Batch Arrival Queue:

Consider a Fuzzy batch arrival Queuing model for finding the Total expect cost (TEC).It is assumed that the customers arrive at a batch arrival as a Poisson Process with Fuzzy rate $\tilde{\lambda}$, The service time as an exponential distribution with Fuzzy rate $\tilde{\mu}$, tne expected batch size with Fuzzy rate $\tilde{k}$, the service cost with Fuzzy rate $\widetilde{C_s}$ and the holding cost with Fuzzy rate $\widetilde{C_h}$ are known approximately and can be represented by convex Fuzzy Sets

Let $\emptyset_{\tilde{\lambda}}(u), \emptyset_{\tilde{\mu}}(v), \emptyset_{\tilde{k}}(w), \emptyset_{\widetilde{C_s}}(m)$ and $\emptyset_{\widetilde{C_h}}(u)$ denotes the membership function of $\tilde{\lambda}, \tilde{\mu}\tilde{k}, \widetilde{C_s}$ and $\widetilde{C_h}$ respectively. Then we have the following Sets,

$$\tilde{\lambda} = \{(u, \emptyset_{\tilde{\lambda}}(u))/u_{\in}U\}; \quad \tilde{\mu} = \{(v, \emptyset_{\tilde{\mu}}(v))/v_{\in}V\}; \quad \tilde{k} = \{(w, \emptyset_{\tilde{k}}(w))/w_{\in}W\}$$

$$\widetilde{C_s} = \{(m, \emptyset_{\widetilde{C_s}}(m))/m_{\in}M\}; \quad \widetilde{C_h} = \{(n, \emptyset_{\widetilde{C_h}}(n))/n_{\in}N\};$$

Where U, V, W, M, N are the script Universal Sets of the arrival, Service rate, batch size, Service cost and holding cost rates respectively.

Let F (u, v, w, k, m, n,) denote the system characteristic of interest. Since u, v, w, m, n, are Fuzzy numbers and F (u, v, w, m, n) also a Fuzzy numbers. Let A represents the membership function of the expected cost.

(ie) The Total expected cost (TEC) is A=F (u, v, w, m, n) = m+wn

$$\left[\frac{E(w)^2+E(w)}{2\{v-uE(w)\}}\right] \quad (1)$$

## 5. Algorithm Ranking Function method:

Let a convex Trapezoidal Fuzzy number $\tilde{X}(x)\tilde{X}(x)= \tilde{X}(x_1, x_2, x_3, x_4; w)$. Then the Ranking Index is defined by

$$\tilde{X}(x) = \int_0^w \left[\frac{L^{-1}(x)+R^{-1}(x)}{2}\right] dx; \text{Where } L^{-1}(x) = x_1 + \frac{(x_2 - x_1)}{w} x \text{ and} R^{-1}(x) = x_3 + \frac{(x_4 - x_3)}{w} x$$

$$\Rightarrow R(\tilde{x}) = \frac{1}{4}[w(x_1 + x_2 + x_3 + x_4)] \quad (2)$$

**(ii)** Let a convex Triangular Fuzzy number $\tilde{X}(x)= \tilde{X}(x_1, x_2, x_3,; w)$;

Then the Ranking Index is given by $R(\tilde{x}) = \int_0^w \frac{1}{2}[L^{-1}(x) + R^{-1}(x)]\, dx$; Where $L^{-1}(x) = x_1 + \frac{1}{w}[x_3 - x_1]x$ and

$$R^{-1}(x) = x_2 + \frac{1}{w}[x_3 - x_2]x$$

$$\Rightarrow R(\tilde{x}) = \frac{1}{4}[w(x_1 + 2x_2 + x_3)] \quad (3)$$

## 6. Numerical Example:

ConsiderTV service center systems in which TVsets are arrived by batches according to a Poisson process. In that situation all TV setexcept the service a processing TV set are needed to hold in service. So the holding cost exists. The owner of the center wishes to evaluate how much the total expected cost for whole service. It's evident that this system follows FM {k}/FM/1 and the total expected cost of the system can be derived by the proposed approach.

### 6.1 For Trapezoidal Fuzzy Number:

(i) When w=0.5 that is generalized fuzzy numbers will give moderate value1.

Suppose the arrival rate, Service rate, batch size and Service cost and holding cost rate are Trapezoidal Fuzzy numbers respectively by

According to (2), the Ranking Index of $\tilde{\lambda}$ is R ($\tilde{\lambda}$) =R (2, 3, 4, 5, 0.5);

$$=\frac{1}{4}[0.5(2+3+4+5)] = 1.75 = u$$

Similarly we get other parameter as follow,

R ($\tilde{\mu}$) =R (3, 4, 5, 6; 0.5);$=\frac{1}{4}[0.5(3+4+5+6)] = 2.25 = v$

R ($\tilde{k}$) $=\frac{1}{4}[0.5(6+7+8+9)] = 3.75 = w$

R ($\widetilde{C_s}$) =R (2000, 3000, 4000, 5000; 0.5); $=\frac{1}{4}[0.5\{2000+3000+4000+5000\}]$=1750=m

R ($\widetilde{C_k}$) =R (40, 50, 60, 70; 0.5); $=\frac{1}{4}[0.5\{40+50+60+70\}] = 27.5$=n

Using the above values in (1), The Total Expected cost is,TEC = m+wn$\left[\frac{E(w)^2+E(w)}{2\{v-uE(w)\}}\right]$;

=1750+ (3.75) (27.5)$\left[\frac{(3.75)^2+(3.75)}{2\{2.25-1.75(3.75)\}}\right]$;

=1750+ (3.75) (27.5)$\left[\frac{17.8125}{-8625}\right]$; =1537.

**6.2** When w=1 that is normalized Fuzzy numbers will gives the Optimistic Value,

Suppose the arrival rate, Service rate, batch size rate, Service cost rate and holding cost rate are Trapezoidal Fuzzy numbers represented by

$\tilde{\lambda}$ = [2,3,4,5; 1] ; $\tilde{\mu}$ = [3,4,5,6; 1]; $\tilde{k}$ = [6,7,8,9; 1]; $\widetilde{C_s}$ = [2000,3000,4000; 5000; 1]; $\widetilde{C_k}$ = [40,50,60,70; 1] ;

From (2), the Ranking Index of $\tilde{\lambda}$ is R ($\tilde{\lambda}$)(ie) R ($\tilde{\lambda}$) = R (2, 3, 4, 5; 1)

$=\frac{1}{4}[\{2+3+4+5\}] = 3.50 = u$

$=\frac{1}{4}[14]$ =3.5

Similarly, we get other value,

R ($\tilde{\mu}$) =R (3, 4, 5, 6; 0.5);$=\frac{1}{4}[1\{3+4+5+6\}] = 4.50 = v$

$=\frac{1}{4}[18]$ =4.5

R ($\tilde{k}$) =R (6, 7, 8, 9; 1); R ($\tilde{k}$) $=\frac{1}{4}[1\{6+7+8+9\}] = 7.50 = w$

$= \frac{1}{4}[30]$ =7.5

R ($\widetilde{C_s}$) =R (2000, 3000, 4000, 5000; 1);

$= \frac{1}{4}[1\{2000+3000+4000+5000\}]$

$=\frac{1}{4}[14{,}000]$ =3500=m

R ($\widetilde{C_k}$) =R (40, 50, 60, 70; 1); $= \frac{1}{4}[1\{40+50+60+70\}]$

$= \frac{1}{4}(220) = 55$=n

Using the above values in (1), we get the total expected cost= m+wn$\left[\frac{E(w)^2+E(w)}{2\{v-uE(w)\}}\right]$;

=3500+ (7.50) (55)$\left[\frac{(7.50)^2+(7.50)}{2\{4.50-3.50(7.50)\}}\right]$;

=3500+ (7.50) (55)$\left[\frac{63.75}{-43.5}\right]$;

=3500+ (41.25)(-1.4655);

=3500-604.5258; =2895.48.

### 6.3 For Triangular Fuzzy number:

(i) When w=0.5, that is normalized Fuzzy numbers will gives the moderate Value

Suppose the arrival rate, Service rate, batch size rate, Service cost rate and holding cost rate are Triangular Fuzzy numbers represented by,

$\tilde{\lambda}$ = [3,4,5; 0.5] ; $\tilde{\mu}$ = [4,5,6,7; 0.5]; $\tilde{k}$ = [7,8,9,10; 0.5] ; $\widetilde{C_s}$ = [3000,4000,6000; 0.5]; $\widetilde{C_k}$ = [50,60,80,0.5] ;

From (2), the Ranking Index of $\tilde{\lambda}$ is R ($\tilde{\lambda}$) = R (3, 4, 6; 0.5),

R ($\tilde{\lambda}$) = R (3, 4, 6; 0.5); $=\frac{1}{4}[0.5\{3+10+6\}] = 2.375 = u$

Similarly, the other values are,

R ($\tilde{\mu}$) =R (4, 5, 7; 0.5); $=\frac{1}{4}[0.5\{4+10+7\}] = 2.625 = v$

R ($\tilde{k}$) = (7, 8, 10; 0.5); $=\frac{1}{4}[0.5\{7+16+10\}] = 4.125 = w$

R ($\widetilde{C_s}$) =R (3000, 4000, 6000; 0.5) $=\frac{1}{4}[0.5\{3000+8000+6000\}]$ =2125=m

R ($\widetilde{C_k}$) =R (50, 60, 80; 0.5); $= \frac{1}{4}[0.5\{50+120+80\}]$ =31.25=n

Using the above values in (1),The Total expected cost = m+wn$\left[\frac{E(w)^2+E(w)}{2\{v-uE(w)\}}\right]$;

=2125+ (4.125) (31.25)$\left[\frac{(4.125)^2+(4.125)}{2\{2.625-2.375(4.125)\}}\right]$;

=2125+ (4.125) (31.25)$\left[\frac{21.1406}{-14.3438}\right]$;

=2125+ (128.9063) (-1.4738); =1935

**(ii)** When w=1, that is normalized Fuzzy numbers will gives the Optimality Value 1,

Suppose the arrival rate, Service rate, batch size, Service cost rate and holding cost rate by Fuzzy Triangular numbers represented as

$\tilde{\lambda}$ = [3,4,6; 1] ; $\tilde{\mu}$ = [4,6,7; 1] $\tilde{k}$ = [7,8,10; 1] ; $\widetilde{C_s}$ = [3000,4000; 6000; 1]; $\widetilde{C_k}$ = [50,60,80; 1]

According From (2), the Ranking Index of $\tilde{\lambda}$ is R ($\tilde{\lambda}$)R ($\tilde{\lambda}$) = R (3, 4, 6; 1);

$$=\frac{1}{4}[1\{3+8+6\}] = 4.25 = u$$

Proceeding similarly, we get other values,

R ($\tilde{\mu}$) =R (4,5,7; 1); $=\frac{1}{4}[1\{4+10+7\}] = 5.25 = v$

R ($\tilde{k}$) = R (7, 8, 10; 1); $=\frac{1}{4}[1\{7+16+10\}] = 8.25 = w$

R ($\widetilde{C_s}$) =R (3000, 4000, 5000; 1); $=\frac{1}{4}[1\{3000+8000+5000\}]$=4000=m

R ($\widetilde{C_k}$) =R (50, 60, 80; 1); $= \frac{1}{4}[1\{50+120+80\}]$ =6250=n

Using the above values in (1),The Total expected cost = m+wn$\left[\frac{E(w)^2+E(w)}{2\{v-uE(w)\}}\right]$;

=4000+ (8.25) (62.50)$\left[\frac{(8.25)^2+(8.25)}{2\{5.25-4.25(8.25)\}}\right]$;

=4000+ (8.25) (62.50)$\left[\frac{76.3125}{-59.625}\right]$;

=4000+ (515.625) (-1.2799);

=3340.05.

### 7. Conclusion:

In this paper Fuzzy set theory has been approved to a batch arrival queuing model. Cost measures of fuzzy Batch arrival queuing model have been used in operations and service mechanism for evaluating system performance. Further the Fuzzy problem has been transformed into crisp problem using ranking function method. Since the performance measures such as the total expected cost are crisp values. We conclude that the solution of fuzzy problems can be obtained by ranking function method very effectively.

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