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Original Research Paper Mathematics

ON A DIOPHANTINE PROBLEM

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ABSTRACT This paper aims at determining two non-zero distinct integers N_1 and N_2 such that $N_1-N_2=2\alpha$, $N_1N_2=k\alpha^3$

KEYWORDS : Diophantine problem , Integer triples , System of equations.

Introduction

Number theory, called the Queen of Mathematics, is a broad and diverse part of Mathematics that developed from the study of the integers. The foundations for Number theory as a discipline were laid by the Greek mathematician Pythagoras and his disciples (known as Pythagoreans)**.** One of the oldest branches of mathematics itself, is the Diophantine equations since its origins can be found in texts of the ancient Babylonians, Chinese, Egyptians, Greeks and so on[7-8]. Diophantine problems were first introduced by Diophantus of Alexandria who studied this topic in the third century AD and he was one of the first Mathematicians to introduce symbolism to Algebra. The theory of Diophantine equations is a treasure house in which the search for many hidden relation and properties among numbers form a treasure hunt. In fact, Diophantine problems dominated most of the celebrated unsolved mathematical problems. Certain Diophantine problems come from physical problems or from immediate Mathematical generalizations and others come from geometry in a variety of ways. Certain Diophantine problems are neither trivial nor difficult to analyze [1-6]. Also one may refer [9-15].

In this communication , we attempt for obtaining three non-zero distinct integers N_1 and N_2 such that $N_1-N_2=2\alpha$, $N_1N_2=k\alpha^3$

Method of analysis

Let N_1 and N_2 be any two non-zero distinct integers such that

$$N_1-N_2=2\alpha \tag{1}$$

$$N_1N_2=k\alpha^3 \tag{2}$$

Eliminating N_2 between (1) and (2) , we have

$$N_1^2-2\alpha N_1-k\alpha^3=0. \tag{3}$$

Treating (3) as a quadratic in N_1 and solving for N_1 , we obtain

$$N_1=\alpha\left[1\pm\sqrt{1+k\alpha}\right] \tag{4}$$

The square-root part on the R.H.S of (4) is eliminated when $\alpha=k\beta^2\pm2\beta$.

Therefore, from (1) and (4) , we get four choices of N_1 , N_2 satisfying (1) and (2) and they are as illustrated below.

$$(N_1,N_2)=\begin{cases}\beta(k\beta+2)^2,k\beta^2(k\beta+2) & (5)\\ -k\beta^2(k\beta+2),-\beta(k\beta+2) & (6)\\ k\beta^2(k\beta-2),\beta(k\beta-2)^2 & (7)\\ -\beta(k\beta-2)^2,-k\beta^2(k\beta-2) & (8)\end{cases}$$

It is worth to note that, the choices (7) and (8) are nothing that the replacements of N_1 by $-N_2$ and N_2 by $-N_1$ in (5) and (6) respectively.

For simplicity and clear understanding, a few interesting relations among N_1 and N_2 given by (5) and (7) are exhibited below:

Properties for set (5):

$$k(N_1+N_2)=12P_{k\beta}^3$$

$$\frac{(N_1-N_2)^2}{N_1}\equiv 0(\bmod\ 4)$$

$\frac{kN_2^2}{N_1}$ is a cubical integer.

$[kN_1-6P_{k\beta}^3+1]$ is a perfect square.

Each of the following expressions represents a nasty number.

$$6[2k(N_1-N_2)+4]$$

$$2[2P_{k\beta+2}^5-kN_1]$$

$$6[kN_2-2P_{k\beta}^5]$$

Properties for set (7):

Each of the following expressions represents a perfect square.

$$8[k(N_1-N_2)+2]$$

$$2[12P_{k\beta}^3-8k\beta-k(N_1+N_2)]$$

$$\frac{k(N_1-N_2)^2}{N_2}+4$$

Each of the following expressions represents a nasty number.

$$6[kN_2-2P_{k\beta-2}^5]$$

$$2[2P_{k\beta}^5-kN_1]$$

Conclusion

In this paper we have presented two non-zero distinct integers N_1 and N_2 such that $N_1-N_2=2\alpha$, $N_1N_2=k\alpha^3$

As diophantine problems are rich in variety , one may attempt to find other choices of diophantine problems.

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